Measuring and Interpreting core inflation: evidence from Italy
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Provisional version

1. Introduction

Despite the important role that the concept and measurement of core inflation plays in the monetary policy and in the analysis of inflation process, the term appears to have no clear definition and a wide range of different techniques have been used in trying to measure it. Since 30 years (Eckstein, 1981, Blinder 1982) many theoretical and empirical works has been published and various Central Banks and National Statistical Institutes usually have been computing the core or underlying inflation but without any final agreement on the methods used to determine it.

Actually, as it has been already stated (Silver, 2006), the methods to be applied to measure the core inflation depend on the inflationary process, on the characteristic of the markets and on the economic situation of the country involved, which affect the characteristics of the distribution of the price changes and the measurement of the core inflation. For this reason, it is obvious that in order to propose measures of the core inflation for one specific country it is important to start from the analysis of the specific economic situation and the distribution of the price changes of that country by carrying out adequate experiments.

The aim of this paper is to present a framework for measuring and interpreting core inflation, for the purposes of monetary policy formulation, referred to consumer price in Italy and to assess the validity of the different measures of core inflation proposed. The final goal is to suggest to the Italian Institutions, namely Bank of Italy and the Italian national statistical institute (Istat), appropriate measures to estimate and analyse core inflation currently. Because the results of the measure of core inflation depend also on the number of (price) items considered, (as stated by Roger, 1997, Laflèche, 1997 and others) we have decided to carry out the empirical analysis using a very detailed data set of more than 500 monthly price indices for representative products from year 1996 to 2008.)

The paper is structured as follows. In the second section a short critical review and classification of the definitions of the core inflation and of the methods proposed to measure it together with criteria for their evaluation are presented. The characteristics of the consumer price indices (CPIs) computed in Italy, the data set used and the organisation of the analyses has been presented in the third section. The fourth section will be devoted to present the measure of core inflation or underlying inflation currently carried out by Bank of Italy and the Italian national statistical institute (Istat). While in the section fifth, sixth, seventh and eighth, respectively, the experiments implemented to measure the core and underlying inflation through the estimation of trend using time series, the use of exclusion-based methods and those carried out by using the stochastic approach and trimmed mean are presented. The empirical assessment of the different indicators of core inflation for a selected time period is presented. Finally, some concluding remarks are devoted to suggest the implementation and use of the more adequate measures of core inflation considering the Italian situation.
2. A brief review of the definitions and current methods of measuring core inflation

2.1 Definitions and methods
As already mentioned, there is no agreement on the definition of core inflation so that Roger (1998) stated that core inflation tends to be defined in terms of the particular method used to construct a practical measure rather than in terms of what the measure is trying to capture, even though, in general terms, the core inflation or underlying inflation can be defined as the kernel component of the inflation that manifests itself in medium-long run change in prices caused by monetary reasons only (Biggeri, 1998). A more precise definition of core inflation necessarily requires an economic model of how the prices and money are determined in the economy.

However, to have a guidance in order to understand the different alternatives, it is possible to refer to various critical reviews which have been recently published concerning the issues of defining core inflation, measuring it using alternative methods and the criteria for choosing among them (Roger, 1998; Fenwick, 2003; Rich and Steindel, 2005; Roberts, 2005; Silver, 2006; Wynne, 2008).

Starting from these works it is possible to refer, in general and except some shades, at two different broad concepts of core inflation:

(i) one concept considers core inflation as the \textit{persistent component} of measured inflation, and it is based on the distinction between the steady or persistent component of measured inflation, and intermittent or transient inflation; this definition, that is coherent with the definition of inflation given by Friedman (1963) and Laidler and Parkin (1975) consider the core inflation as the persistent element reflected in a common tendency (core inflation and \textit{trend} inflation are essentially synonymous), or to draw a distinction between \textit{price level} shocks (having only a temporary impact on measured inflation) and more persistent \textit{inflation} shocks. The distinction between what is a persistent component and a transient influence on inflation rate is however different by different authors (see, for example,: Eckstein, 1981 and Quah and Vahey, 1995).

(ii) an alternative concept of core inflation is defined as the \textit{generalised component} of measured inflation, focusing on the \textit{generality} of movements in prices, and is reflected in Arthur Okun’s definition of inflation as ‘‘...a condition of generally rising prices’’ (Okun, 1970) and in John Fleming’s definition of inflation as ‘‘...the rate at which the \textit{general level of prices} in [the] economy is changing’’ (Fleming, 1976). In this conception, measured inflation is viewed as comprising a generalised or core inflation component, associated with expected inflation and monetary expansion, plus a relative price change component, mainly reflecting supply disturbances. Relative price disturbances are regarded as ‘noise’ blurring the more general or ‘underlying’ evolution of prices.

The differences in the definition of core inflation obviously affect the methods for their measure. Many classifications of the methods have been proposed taking into account different purposes and criteria. According to the above definitions and the purposes of the estimation of core inflation, that could be limited to the knowledge and analysis of the trend and to predict inflation, the classification of the methods can be done referring to the characteristics of the data necessary to carry out the estimations:

1) methods that use \textit{times-series} of the computed aggregated inflation and/or of its components, in order to distinguish trend from temporary shocks;

2) methods that use of \textit{cross-section data on the distribution of price changes} for each month, in order to obtain adequate and robust estimation of core inflation for separately each month,
(or quarter an so on according to the available data), also for the analysis of the inflationary process\textsuperscript{1}.

In the group 1, it possible to include the methods that are trying to estimate the persistent element of inflation using \textit{time series statistical methods} to distinguish trend inflation from shocks. This approach rely on \textit{smoothing techniques}, including seasonal adjustment, to identify some kind of trend, as simple as averaging over time or more sophisticate smoothing techniques (up to Exponential Smoothing and the ARIMA models) involving the use of fixed or moving seasonal adjustment factors. The computation can be done using only the aggregate index of inflation to extract the unobserved signal of trend (or trend-cycle) or using the different components of the general index (sub-indices), in this case the trend for each disaggregate index is identified and then by aggregating the individual estimates the trend for the general index is obtained. Moreover, \textit{multivariate methods} can be applied, for example multivariate ARIMA models. Using these methods it is also possible to condition the estimate of core inflation on information contained in other variables, in line with the economic theory (Quah and Vahey used to this aim a structural bivariate VAR model including a measure of aggregate output)

The group 2 can be split into two sub-groups according to the underlying logic and the methods used for the estimation of the central measure of the distribution of price changes, in fact:

2.1. the estimation can be done considering that the changes of the price of \textit{some products are volatile or fleeting} (depending on the permanence or not of the supply and demand shock) and have no impact in the central measure of the core inflation.

2.2. the estimation of the central measure of the distribution of price changes has been carried out in the framework of the \textit{stochastic approach} to the theory of index numbers and the method used depends on the characteristic of that distribution.

The methods of the sub-group 2.1 are usually called \textit{Exclusion-Based Methods (EBM)}, because the estimation are carried out excluding specific products with \textit{prices that are believed particularly volatile} as energy products and unprocessed food or price shocks considered to be of a one-off nature and, therefore having only a temporary impact on the measured aggregate inflation rate (for example the impact of changes in indirect taxes, administrative prices, etc.). The basic idea is that this high volatility, mainly due to supply shocks, could hide core inflation movements meant as general tendency of inflation or “generalised inflation”. In this way, substantially, a computation of one or more sub-indices are carried out.

Moreover, the exclusion of products can also be decided by considering a \textit{measure of volatility} of prices changes, for example \textit{excluding part of the tails of the distribution} of price changes that are very far from the average. Variants of this approach involve \textit{re-weighting} or modifying the constituent prices in the relevant price domain in order to highlight the more generalised component of price changes and diminish the influence of more transient relative price shocks.

One general criticism to the application of the previous mentioned methods for the estimation of the core inflation is that the components that are found to be volatile may become relatively stable over time, and, the components established as not being volatile may become volatile. On the other end the prices are not fully flexible in the short run. Therefore in the application it is necessary to consider that the period over which the calculation is made will affect the results and may be useful to consider the full length of the series as well as a rolling (one or more) year period.

\textsuperscript{1} A Dynamic Factor Index which combines the information coming from both the time series of the computed rate of inflation and the distribution of price changes along the time period has been proposed by Bryan and Cecchetti (1993).
The methods included in the sub-group 2.2, starts from the analysis of the price changes distribution to define the best and robust estimation of the core inflation (average) in each month. Therefore, the underlying or core (true) rate of inflation is treated as an unknown parameter to be estimated from the individual prices (Clements et al. 2006). The objective is to substitute the movements of the individual prices with the “unique” real cause of inflation, the monetary one, practically corresponding to the central tendency if there are no other permanent causes of change in the prices (Biggeri, 1998). On the other hand with this reasoning it will be possible to consider the measure of inflation as an expected value and to find it we have to impose the approximation conditions stated in a model of the following general form:

\[ g(\pi_t) = \pi_t + \epsilon_t \]

where \( g(.) \) is a function of the change in the price of commodity \( i \) \((i=1,\ldots,n)\) which are composed by a systematic part and a temporary disturbance \( \epsilon_t \). Therefore, the term on the left side of the model is the estimation of measured rate of inflation \( \Pi_t \), while the first term on the right side is the estimation of core inflation \( \Pi_t^* \), the systematic part common to all prices.

By specifying the functional form of \( g(.) \) and making suitable hypotheses on the variance of the random variable \( \epsilon_t \), in addition to the usual \( E(\epsilon_t) = 0 \) it is possible to obtain any synthetic price index formula considering the characteristic of the distribution of price changes.

If, for example, we assume \( g(\pi_t) = \pi_t \) and \( Var(\epsilon_t) = \sigma^2 \), the estimation of \( \Pi_t^* \) is formally obtained as an arithmetic mean. If, with the same hypothesis regarding \( g(.) \), we assume \( Var(\epsilon_t) = \sigma^2 / w_{100} \) (where \( w_{rs} = p_r q_s / \sum_i p_i q_i \) with \( r \) and \( s \) referring to time) we have a Laspeyres index.

If we assume \( g(\pi_t) = \log(\pi_t) \) and \( E(\epsilon_t) = 0 \) \( Var(\epsilon_t) = \sigma^2 \) we get an unweighted geometric mean, and so on. In any case, to get most of the well known synthetic formulas, we have to assume that \( \text{Cov}(\epsilon_i, \epsilon_j) = 0 \) , \((i, j = 1,\ldots,n)\), a hypothesis seldom acceptable from an economic point of view.

As Diewert (1995) and other authors observed, the justifications presented for the variance assumptions in the stochastic approaches are sometimes rather weak and are not consistent with the observed behaviour of prices changes.

Because, very frequently, the price change distributions are in practice asymmetric (usually on the right) and leptokurtic, the sample mean is not an efficient estimator. Therefore a lot of proposals advanced by many authors refers to the methods to obtain efficient and robust estimation of the population or underlying mean for a variety of distributions. A variety of robust or “Limited Influence” estimators such as median-based or trimmed-mean as been suggested. As Roger (1997) stated, given the potentially infinite number of L-statistics to choose from, it is useful to distinguish between the most efficient estimator for a particular distribution and the most reliable estimator for a variety of distributions.

Also for the application of this approach, problems arisen when the degree of asymmetry and/or kurtosis in the distribution of price changes may change through time, in which case an estimator assuming an unchanging distribution will be biased. Therefore may be useful in practice consider two variants of trimmed means: one with a constant and one with a time-varying optimal trimming percentage, according to the characteristics of the actual distributions.

At the end of this short review of concepts of core inflation and of the methods to measure it, we can say that, as clearly appear from the many works on the field, the difference in conceptual approaches to core inflation, although introduced in literature long time ago, may not be very
different in terms of substance. Therefore the differences in definition should not be exasperate because there is a kind of continuum of shades between them and also between the measure proposed.

2.2. Desirable properties of a measure of core inflation and evaluation of its validity

All the above mentioned authors explain strength and weakness of the different approach proposed and alternative methods to estimate the underlying or core inflation, that is not useful to recall here. In any case, from the practical point of view, it is important that the proposed and used methods of estimation of core inflation satisfy some desirable properties.

Roger (1997) propose three criteria to be satisfied by any candidate measure of core inflation. The measure should be:

i. timely (if the measure is only available with a long lag, that will reduce its value to policy makers);

ii. efficient, robust and unbiased (ideally, the difference between the average rate of inflation of the core measure and the headline measure should be zero over a long time period since any systematic differences will impair the credibility of the measure); and

iii. verifiable (the measure should be easy to reproduce and track, to ensure its high credibility with the users).

Other authors have suggested more criteria, for example the measure should be: forward looking in some sense; have some theoretical basis (ideally in monetary theory); have information content in terms of forecasting the headline inflation rate and so on.

Most of the proposed methods satisfy some of the desirable properties, however a measure that fully meets the various desirable characteristics of a core inflation is probably impossible, also because depend on the purposes for which the measure is used. As for the construction of an index number, before choosing a measure of core inflation it is important to specify what we want to capture by using that measure; for example, if the interest is the cost of living, then it is not clear why we would ever want to exclude the effects of oil price increases or indirect taxes; and so on.

As stated by Silver (2006), given the lack of consensus, it is proposed that the choice of methods should depend on the purpose of the measure and, at least in part, should be data-driven for each country so that the methods adopted are tailored to the features of the evolution of that country’s economy. In this way the choice of measures can be justified on an objective, transparent basis.

In any case, at the end, the criteria to evaluate a proposed measure or to choose among more measures, usually, consist in computing some tests against benchmark values.

For example, according to the type of measure the benchmark value could be:

(i) the long term trend or trend-cycle of the measured inflation, computed through smoothing or other methods;

(ii) the hypothetic measure of the inflation for each month according the distribution of price changes.

Considering the above general framework, the unbiasedness can be assessed by using a synthesis of the distances of the estimation of core inflation from the benchmark measure can be obtained by using Root Mean Square Error or Mean Absolute Deviation or other similar measures.

Besides the evaluation of performance in comparison with a benchmark proxy, also standard deviations can be calculated to measure the reduction of volatility produced by the measure of core inflation with the respect to the measured inflation rate.

Finally, specific indicators can be used to assess the efficiency and robustness of the proposed methods to measure core inflation, especially in the field of the stochastic approach and trimmed-mean methods, as Rogers (1997), Bryan and Cecchetti (1997), Aucremanne (2000) and other authors pointed out.
3. The data set of CPIs available and the organisation of the experiments

3.1. The procedures for the construction of CPI in Italy

Istat, as many other NSIs, computes as measured inflation for the consumption of all population, the CPI\textsuperscript{2} using, at a higher-level, an Annual Chain Index of the Laspeyres type, that is based on the following general formulas:

\[
I^{12,y-1,m,y} = \sum_i w^{12,y-1}_i \frac{P^{m,y}_i}{P^{12,y-1}_i} = \sum_i w^{12,y-1}_i P^{12,y-1,m,y}_i, \quad \sum_i w^{12,y-1}_i = 1
\]

where: \( I^{12,y-1,m,y} \) denotes the overall CPI, or any higher-level index, from period \( 12,y-1 \) to period \( m,y \); \( m \) represents the generic month (\( m = 1, \ldots, 12 \)) and \( y \) the year; \( i \) represents a given good or service, denoted as elementary item, purchased for consumption by the households in a given outlet for which the prices are collected; \( P^{m,y}_i \) and \( P^{12,y-1}_i \) are the price of item \( i \) in time \( m,y \) and \( 12,y-1 \); \( P^{12,y-1,m,y}_i \) is the elementary price index for the \( i \) for the elementary expenditure aggregate; and \( w^{12,y-1}_i \) the weight attached to each of the elementary price indices, that should be based on the consumer expenditure share for the item \( i \) (defined as the relative importance of the item in terms of expenditure for consumption made by households). Therefore, the elementary items chosen and the weights are revised at the beginning of each year with reference to December of previous year, in order to approximate as closely as possible the consumption patterns, and they remain fixed for a sequence of 12 months.

In order to obtain the indexes referred to the base period 0, a calculus is performed for each level of aggregation of the indexes; specifically, the chained general price index referred to the base period 0 and the current period \( m,y \) is obtained as:

\[
f^{0,m,y} = f^{0,12,y-1} \cdot f^{12,y-1,m,y}
\]

Taking into account the objectives of the CPI, the characteristics of the variability of the prices’ movements and the operational and administrative reasons, the framework for the construction of the CPI is referred to a kind of multistage purposive stratified sampling design.

At the beginning of each year, Istat defines, relying on the purposive selection, representative elementary items (products or services; 533 for the year 2008) that consist of groups of products that are as similar as possible and relatively homogeneous also in terms of price movements, chosen according a lot of specific information, to be included in the fixed basket. The items selected should be ones for which price movements are believed to be representative of all the products within the elementary aggregate, that is considered a product stratum, and for which a system of weight should be estimated. Moreover, in same cases, the representative products are “composite products” so more items or products can be selected for price collection (1099 in year 2008).

Concerning the products and the computation of sub-indices at different level of aggregation of products, Istat uses the COICOP hierarchical classification (Classification of Individual Consumption by Purpose), as underlined by the International Labour Office (ILO et A., 2004) The

\textsuperscript{2} Currently Istat computes three different CPIs: (i) a National CPI for the all the households, (ii) a National CPI for the workers and employees households, (iii) the European Harmonised Indices of Consumer Prices (HICP). However, for the purpose of this paper the three indices can be considered equal because they have common territorial base, price collection and procedures of computation. Anyway, experiments have been carried out on indices calculated for national CPI.
above mentioned elementary aggregates, have been aggregated in sub-classes (equal to 205, in the year 2008), in classes (109), in groups (38), up to 12 divisions (chapters of expenditure)\(^3\).

Besides, it is important to observe that the products whose prices are typically seasonal (in particular fresh fruits and fresh vegetables) are submitted to a treatment in order to eliminate seasonal component before entering in the calculation of all items index in Italian CPI\(^4\).

**3.2. Data set and organisation of the experiments**

In order to compute different measures of core inflation, we decide to use the monthly consumer price indices at level of elementary aggregates for the period 1996-2008\(^5\). Because the indices are, as already stated above, Annual Chain Index of the Laspeyres type, the number of the indices for the elementary aggregates are the same during the months of each year, but a little bit different in different years (in any case more then 530), depending on the revision of the basket made every year. We decide to use the maximum level of disaggregation of the indices, because the findings are, usually, sensitive to the levels of aggregation of prices. Indeed, the level of aggregation of the components can influence the results: the aggregation might conceal extreme movements in certain subcomponent. In the experiments we used all the elementary indices computed each year because they refer to the real estimation made of the inflation rate and at the same time the comparison with the previous years are substantially correct because the substitution and/or the inclusion of new products have been made in order to maintain the representativeness of items inside the more aggregate level of the computed price indices (the 205 sub-classes).

In order to study and evaluate the behaviour of both the month-on-previous-month and year-on-year changes, two series of price changes (rate of inflation) have been calculated considering different time horizon \(k\) for the time series of each elementary indices and for the general CPI, starting from the computed \(P_{0,t,y}^{0,m,y}\) and \(I_{0,t,y}^{0,m,y}\).

Therefore, two time horizons have been used, where \(k\) was 12 and 1 respectively, and substituting \(t\) in the previous formula as for a generic month \(m\) of the \(y\) year, the computations as been made considering both the elementary price index by applying:

\[
\pi^k_{it} = \frac{P_{0,t}^{0,y}}{P_{0,t-k}^{0,y}} - 1 \quad \text{which becomes} \quad \pi^{12}_{it} = \frac{P_{0,t}^{0,y}}{P_{0,t-12}^{0,y}} - 1 \quad \text{when} \quad k=12 \quad \text{and} \quad \pi^1_{it} = \frac{P_{0,t}^{0,y}}{P_{0,t-1}^{0,y}} - 1 \quad \text{when} \quad k=1
\]

and the overall CPI by computing:

\[
\Pi^k_{it} = \frac{I_{0,t}^{0,y}}{I_{0,t-k}^{0,y}} - 1 \quad \text{which becomes} \quad \Pi^{12}_{it} = \frac{I_{0,t}^{0,y}}{I_{0,t-12}^{0,y}} - 1 \quad \text{when} \quad k=12 \quad \text{and} \quad \Pi^1_{it} = \frac{I_{0,t}^{0,y}}{I_{0,t-1}^{0,y}} - 1 \quad \text{when} \quad k=1
\]

Obviously, we can equivalently express: \(\Pi^k_{it} = \sum w^k_{i,t} \pi^k_{it}\) which becomes \(\Pi^{12}_{it} = \sum w^{12}_{i,t} \pi^{12}_{it}\) when \(k=12\) and \(\Pi^1_{it} = \sum w^1_{i,t} \pi^1_{it}\) when \(k=1\).

\(^3\) The following example shows the possible coverage of the different aggregates. The entire set of consumption goods and services covered by the NIC is divided into divisions, such as “food and non-alcoholic beverages”. Each division is then divided into groups such as “food”; then each group is further divided into classes, such as “fish”. Moreover, each class is divided into more homogeneous sub-classes, such as “fresh fish”. Finally, a sub-class may be further subdivided to obtain the elementary aggregates and inside of these to select the representative product such as “freshwater fish”.

\(^4\) The recently approved European Regulation for seasonal items is expected to produce changes in CPI seasonal profile.

\(^5\) For the years 1997-1999 the CPIs were computed using weights referred at year 1996 (as a fixed base index).
Therefore, we have computed relative price changes for the general index and for each elementary indices, considering the $\Pi_{t}^{12}$ comparison, comparing price indices in one month with the corresponding month in the previous year (thus removing, at least partially seasonal variations and the $\Pi_{t}^{i}$ comparison, which compares price indices in one month with the previous month

Using this data base a lot of experiments have been organised applying some of the methods usually used for the estimation of the core inflation, chosen considering the theoretical statements and the empirical results illustrated by many authors. In particular, we decide to compute and analyse the results of the following measures of underlying or core inflation:

i. Time series approach, using ARIMA model;
ii. Exclusion Based Methods, excluding products on the basis of some measure of volatility of their prices;
iii. Stochastic approach, using Median and Weighted median, Mean Percentile and, above all, Asymmetric Trimmed means

In order to assess the performance of the estimators, referring to the above mentions proprieties of a measure of core inflation, we will refer to specific methods.

In particular, concerning first the property of tracking trend inflation, a proxy of the trend-cycle obtained by the 12 months centred moving average of measured inflation, (with the 1st and 13th element weighted 0.5 as in Wagemann – Macaulay formula) have been considered as the benchmark, denoted $\Pi_{t}^{Bk}$.

Regarding the metric to gauge the deviation between the series, the evaluations have been carried out using the two following indicators:

- the Root Mean Square Error (RMSE)
  \[
  \sqrt{\frac{\sum_{t=1}^{T}(\Pi_{t}^{*k} - \Pi_{t}^{Bk})^2}{T}}
  \]
- the Mean Absolute Deviation (MAD)
  \[
  \frac{\sum_{t=1}^{T}|\Pi_{t}^{*k} - \Pi_{t}^{Bk}|}{T}
  \]

where $\Pi_{t}^{*k}$ is the estimated measure of core inflation.

The next criterion we considered, examines the reduction in volatility that is obtained by the different core measures, in order to quantify the efficiency gain. To this aim we applied the usual standard deviation and a short term volatility measure. In particular, we used the standard deviation of the first difference of the different estimators (and of the observed inflation) expressed by (Aucremanne, 2000):

\[
\nu(\Pi_{t}^{*k}) = \sqrt{\frac{1}{m-1} \sum_{i=2}^{m} \left( \Delta \Pi_{t}^{*k} - \frac{1}{m} \sum_{i=2}^{m} \Delta \Pi_{t}^{*k} \right)^2}
\]
Since it is also important that the measure is not significantly biased relative to the \textit{computed inflation} (because it is desirable that the core inflation measure not systematically understate or overstate observed inflation), we will use statistical tests to verify the hypothesis of unbiasedness.

Now, before considering the explanation of the measures implemented and the analyses of the results obtained, a brief review of the methods of estimation of the underlying or core inflation usually carried out in Italy is presented.

4. Overview of the current measures of core inflation in Italy

A lot of scientific works have been carried out by researchers in Italy to propose different underlying and core inflation measures. Among others, it appears to be worth to quote: the application of common trends approach (Bagliano, Morana, 1999), the computation of a measure using the seasonally adjusting all items excluding energy and unprocessed food (Cristadoro and Sabbatini, 2000); the application of a dynamic factor model (Cristadoro et al. 2001; Forni et al., 2000, 2001), estimation of a multi-sector model, which describes separately price dynamics in four sectors: industrial goods, services, energy and food. (Siviero and Veronese, 2007) and a review of the methods applied (Gallo et Al., 2002)

Moreover two public Institutions are computing and disseminating underlying or core inflation indices: Bank of Italy and Istat\footnote{Another public institution, Isae (Italian Institute for economic analysis), produced and disseminated a core inflation index, called “all items index excluding energy and unprocessed food”, until the end of 2004 together with a seasonal adjusted index that is still regularly published. .}

Bank of Italy regularly produces and comments either different sub indices that exclude different groups of products or a seasonally adjusted CPI. Apart from seasonally adjusted index, aimed to eliminate short term changes due to seasonal price movements, concerning core inflation, Bank of Italy adopts Exclusion Based Methods. Taking into account the two main indices disseminated by Bank of Italy, one index excludes all food and energy products and the other one also products whose prices are regulated. The starting point of the calculation are the 205 \textit{sub-classes} of COICOP classification mentioned in par. 3.

Focusing its attention on the inflation process Istat makes a lot of analyses every month computing \textit{sub-component} indices, such as for processed and unprocessed foods, for energy products (split between regulated and non regulated ones), for tobaccos, for goods (split among durables, non durables and semi durables), for services (split among regulated and non regulated ones) and for all items index excluding energy products, that give an indication of how widespread price increases and decreases of sub-component are, and their relative importance within the overall price change CPI.

Istat calculates, also, its own “all items excluding energy and unprocessed food products CPI” (EBM1) coherently with the Eurostat approach\footnote{Eurostat publishes monthly a big amount of indices referred to so called “special aggregates” (Eurostat Compendium of HICP, 2001) that are aimed to provide better tools to policy makers to understand the evolution of inflationary process in EU and in each country. Among these special aggregates indices Eurostat also publishes an “all items excluding energy and unprocessed food index”. Moreover other indices are calculated and disseminated excluding products that are expected to introduce volatility obscuring long term inflation. In particular it is worth to be cited “all-items excluding energy”, “all-items index excluding energy, food, alcohol and tobacco”, “all-items index excluding energy and seasonal food”. It has to be stressed that Eurostat does not call any of these indices as “core inflation” for it does not deem anyone of them as a core inflation measure.}, calling it “underlying CPI component”. Therefore EBM1 is based on a priori exclusion of products and it is not based on a specific analysis of products volatility. With reference to 2008 CPI basket, the products that are excluded by Istat are 42 (in terms of weight almost 15 per cent of the entire basket); and they belong to the following 10 of the 109
classes by which indices are classified: vegetables, fruit, fish and seafood, meat, electricity, milk, cheese and eggs, gas, liquid fuels, solid fuels, fuels and lubricants for personal transport.

All the products belonging to the COICOP classes listed above, are excluded from the calculation of CPI and their weights are redistributed among the remaining ones.

In order to show the results obtained, in Figure 1 it is reported the comparison among 12 months percentage rates of change ($\Pi_t^{12}$) of all items CPI, EBM1, energy products and unprocessed food indices. First graph compares the whole time series whereas second graph is focused on the last two years.

**Figure 1.** Italian all items CPI, EBM1, Energy products and unprocessed food prices indices. 
*Year 1997 – 2009. 12 months percentage rates of change, differences*
Concerning the whole period analysed (1997 – 2008), as it is expected, first of all it emerges that really unprocessed food and energy products prices show clearly a strong volatility that seems to provide a justification to their exclusion in order to calculate a core inflation index. Moreover, the time series of EBM1 $\Pi^{12}_t$ goes up and down with respect to $\Pi^{12}_t$ of all items CPI. Obviously these movements are partially depending on the evolution of both the components excluded: energy products on one hand and unprocessed food products on the other hand. EBM1 $\Pi^{12}_t$ is negatively correlated with energy products $\Pi^{12}_t$ (-0.23 it is the value of $\rho$ in the period 1997 – 2008), whereas it is positively correlated with unprocessed food $\Pi^{12}_t$ (+0.51 it is the value of $\rho$ in the period 1997 – 2008). But if correlation between EBM1 and energy products and EBM1 and unprocessed food is analysed by rolling intervals of 12 months, correlation coefficients evolve from negative to positive values and when EBM1 is positively correlated with energy products, the opposite is the sign of correlation with unprocessed food (figure 2), a part 2006 and 2008 when pattern of these 12 months $\rho$ is very similar.

Focusing our attention on the last period observed (2007 – 2009. March), starting from the end of 2008 when prices of energy products have sharply decreased, EBM1 has registered $\Pi^{12}_t$ higher than those ones registered by CPI. The differences between CPI and EBM1 have become negative, following the evolution of price index of energy products, with which the differences CPI-EBM1 are strongly correlated (0.98 it is the value of $\rho$ in the period 2007 – 2009, March, 0.87 over the whole period 1997 - 2008.

Generally speaking, on the basis of a preliminary analysis, EBM1 has reduced volatility of the all items CPI (standard deviation of $\Pi^{12}_t$ decreases from 0.54 to 0.40) and provides an useful tool to understand underlying movements.

The main question that clearly emerges, in order to better reduce volatility, concerns the exclusion or not of all the products belonging to COICOP classes listed above. The possibility of excluding some of them together with other products belonging to other classes that really bring a strong contribution to CPI volatility will be discussed in section. 6 by applying to Italian data different exclusion based methods.

**Figure 2. EBM1, Energy products and unprocessed food prices indices. Year 1997 – 2008 months rates of change, correlation coefficient**

5. Time series approach to core inflation measure
In the field of time series approach to estimate the trend of prices many methods are suggested. Univariate (moving average and ARIMA model) and multivariate approach (Dynamic Factor Index, Structural VAR; Bihan, Sedillot, 1999) have been applied. In this section attention is focused on the use of ARIMA models for Italian all items CPI.

The hypothesis that time series of consumer price indices include unobservable components as trend – cycle, seasonality and the irregular one provides a theoretical justification for procedures aimed to eliminate those components obscuring the very persistent inflation and in particular seasonality and irregular component.

Therefore all items CPI has been seasonally adjusted by TRAMO SEATS and then trend cycle component has been extracted.

The SARIMA model identified for the series is a $(2,1,0)(0,1,1)$:

$$(1-0.21326B-0.28563B^2)(1-B)(1-B^{12})Y_t = (1+0.70919B^{12})\alpha_t$$

whereas trend cycle component has been extracted adopting an IMA(3,3).

CPI raw data and trend cycle indices are showed in figure 3, where it is immediately evident how much it is strong the trend component in Italian CPI. This preliminary evidence is confirmed by both graphs in figures 4 and 5 that show immediately that trend cycle component removes short term excessive variability without modifying significantly 12 months rates of change.

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Figure 3. Italian raw and trend cycle data of all items CPI. Year 1996 – 2008. Indices, base 1995=100

Figure 4. Italian raw and trend cycle data of all items CPI. Year 2007 – 2008. 1 and 12 months percentage rates of change

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8 TRAMO (Time series Regression with ARIMA noise, Missing values and Outliers) SEATS (Signal Extraction in ARIMA Time Series Signal Extraction in ARIMA Time Series) is a statistical software aimed to implement time series analysis, estimating Seasonal ARIMA model and decomposing time series into the unobserved components (trend cycle, seasonality, irregular component) adopting ARIMA model also for the decomposition.

9 SARIMA: acronym for Seasonal Autoregressive Integrated Moving Average model. Seasonal ARIMA are the models that allow to model both regular and seasonal component of a time series.
Figure 5. Italian raw and trend cycle data of all items CPI. Year 2007 – 2008. 1 month percentage rates of change

Standard deviation calculated on time series respectively of all items raw and trend cycle indices confirm what emerges from graphical analysis. If we calculate the standard deviation on $\Pi_t^{12}$ measured on raw data and on trend cycle, the value obtained is the same whereas it decreases from 0.14 to 0.08 if it is calculated on of $\Pi_t^1$. 
This result is mainly due to the weakness of seasonality\textsuperscript{10} in CPI whose trend component is strongly prevailing. Therefore TRAMO SEATS mainly removes irregular movements that affect short term evolution of CPI but it does not change substantially its own seasonal evolution.

This is the reason why ARIMA approach applied to Italian CPI appears not to be consistent with the aim of searching persistent inflation but only with the aim of removing short term excessive variability.

Moreover as far as trend cycle component shows an evolution not so different with respect to raw CPI. Figure 6 shows a comparison among 12 months rates of change of raw indices, of trend cycle component and centred moving average that clearly better smoothes the original series.

**Figure 6. Italian raw, trend cycle and centred moving average data of all items CPI. Year 1997 – 2009. 12 months percentage rates of change**

6. Exclusion based methods to estimate core inflation

6.1 Some remarks concerning Exclusion Based Methods

The so called “Exclusion Based Methods” (EBM) are widely adopted and they focus on the exclusion of products or groups of products whose prices show high volatility with respect to other products: the basic idea is that this high volatility could hide core inflation movements meant as general tendency of inflation or “generalised inflation”.

The EBM can be grouped as follows:

- methods that exclude products for their own characteristics;
- data driven methods, that exclude products from inflation calculation on the basis of some measures of the volatility of their prices.

\textsuperscript{10} Weakness of seasonal component in Italian CPI is mainly due to the treatment to the products which prices of typically seasonal.
In the first group it is possible to include the techniques that exclude indirect taxes, externally generated inflation, no processed food and energy products, other ad hoc products exclusion. The aims and the grounds of these exclusions are very different:

- exclusion of fresh food and energy products;
- eliminating indirect taxes is aiming to purify inflation rate from the influence of changes in indirect taxes regime that in general affect inflation rate over 12 months periods;
- eliminating traded goods whose prices are strongly influenced by erratic movements in exchange rates, could allow to measure domestically generated inflation and providing useful information for policy makers;
- other ad hoc exclusions could be justified because they concern products for which price movements are strongly influenced by one off shocks due to administrative decisions (as those ones that determine tariffs for regulated services).

The exclusion of fresh food and energy products is worth of some specific remarks. This approach to core inflation is largely adopted by Central Banks and National Statistical Institute. The reasons for the exclusions of fresh food and energy products are rooted on their undue volatility or on the fact that this kind of products are beyond the control of monetary authorities (Blinder, 1997). Concerning the issue of volatility, in general the volatility of fresh food and energy products is expected to be evident above all in one month rates of change, $\pi_i^1$, whereas with reference to $\pi_i^{12}$ their volatility is less evident. Anyway the higher short term volatility is the reason why, in general, unprocessed food and energy products are excluded to calculate core inflation indicators.

With reference to the second group, the selection of the products to be excluded is based on the past behaviour of their own price changes on a month by month evaluation of their position in the distribution of the price changes of the entire basket of products. Both methods to select products to be excluded suffer of different inconvenient (Silver, 2006). Using the first way, a product that has been strongly volatile in the past, may become stable and vice versa. The second way, that is a completely data driven approach, suffers of another kind of drawback: it does not ensure that the volatility of the products excluded has some longevity. Moreover excluding a product from the calculation of the aggregate indices on the basis of the evaluation of its position in the distribution of price changes, it is only a way to cut outliers; if the product dropped out from the calculation is permanently out of the same tail of the distribution (for example it has been showing strong negative price changes permanently over the years), it seems not correct to exclude it from the calculation of a core inflation indicator because may be that it capture a clear different signal than volatility of their prices.

In order to deal with these questions, volatility analysis of the price indices of the products in the basket of Italian CPI has been carried out so that the current list of products excluded from the actual calculation of EBM1 made by Istat has been investigated and alternative measures of core inflation have been calculated and compared among them and with EBM1.

6.2 Volatility of products in 2008 Italian CPI basket

The Italian data set, that has been described in section 3, from 1996 to 2008, has been analysed. Volatility analysis has been carried out taking into account four main statistics:

1) how many times $\pi_i^{12}$ of a specific product fell outside a pre determined interval $(\mu \pm \sigma, \mu \pm 1.5\sigma, \mu \pm 2\sigma, \mu \pm 2.5\sigma)$;
2) how many times $\pi_i^1$ of a specific product fell outside a pre determined interval $(\mu \pm \sigma, \mu \pm 1.5\sigma, \mu \pm 2\sigma, \mu \pm 2.5\sigma)$;
3) standard deviation of $\pi_{it}^{12}$, registered by a specific product and calculated over the period of its permanence in Italian CPI basket;

4) standard deviation of $\pi_{it}^{1}$, registered by a specific product and calculated over the period of its permanence in Italian CPI basket.

Statistics 3) and 4) have been calculated to analyse the volatility over the time evolution aiming not to exclude products that show a stable signal to the general inflation.

The computation and analysis carried out, have two different objectives: (i) to check the volatility of the products that are still in Italian CPI basket and that, for the time being, are eliminated from the calculation of EBM1 for their belonging to the category of unprocessed food and energy products; (ii) to compute different indicators of core inflation excluding different groups of products. These last results will be resumed in paragraphs 6.3 and 6.4.

The main results obtained are reported in Appendix 1. As it is clear, the 42 products excluded to compute EBM1 have been ordered on the basis of the percentages of exclusions taking into account how many months each products has shown a $\pi_{it}^{1}$ out of the interval $\mu \pm 1.5\sigma$ of the whole distribution of the 1 month rates of change of all the products.

It emerges that among the 42 products that are currently excluded from EBM1 only 14 have a $\pi_{it}^{1}$ that at least 25% of times (months) have fallen outside the considered interval. The products become 15 if we refer to $\pi_{it}^{12}$ and only 9 of the latter ones should be excluded also on the basis of 1 month rates of change. Moreover if we have a glance to the standard deviations and the position of each products in the ranking (products have been ordered starting from the product with the highest standard deviation), a lot of products have a ranking beyond the position number 100.

In a few words volatility analysis does not allow to support the current choice carried out by Istat to exclude all the unprocessed food and energy products form the calculation of EBM1 (defined as indicator of underlying inflation). Actually this choice is rooted in the Eurostat approach briefly described in a footnote before. But the question is: is it possible to calculate together with the current indicator, another indicator that could better approximate core inflation as generalised inflation? This is the main issue with which we will try to deal with in the next parts of the paper.

### 6.3 Alternative indicators of core inflation calculated excluding products

Taking into account volatility analysis with reference to $\pi_{it}^{12}$ and described in the previous section, five different core inflation indices have been calculated: four of them excluding month by month products whose $\pi_{it}^{12}$ fell outside the interval defined respectively by $\mu \pm \sigma$, $\mu \pm 1.5\sigma$, $\mu \pm 2\sigma$, $\mu \pm 2.5\sigma$ of the distribution of the $\pi_{it}^{12}$ of all the products. The fifth index has been calculated excluding a fixed list of products (100 with reference to all the baskets from 1996 to 2008; 40 with reference to 2008 basket) for their relevant volatility along the time (products have been excluded for the whole period 1996-2008 if $\pi_{it}^{12}$ have fallen at least 25% times out of interval defined by $\mu \pm 1.5\sigma$ of the distribution of monthly rates of change).

Five experimental core inflation indicators have been named as it follows:

1) EBM2 (exclusion of products whose $\pi_{it}^{12}$ fell outside interval $\mu \pm \sigma$);
2) EBM3 (exclusion of products whose $\pi_{it}^{12}$ fell outside interval $\mu \pm 1.5\sigma$);
3) EBM4 (exclusion of products whose $\pi_{it}^{12}$ fell outside interval $\mu \pm 2\sigma$);
4) EBM5 (exclusion of products whose $\pi_{it}^{12}$ fell outside interval $\mu \pm 2.5\sigma$);
5) EBM6 (exclusion of products whose $\pi_{t}^{12}$ have fallen at least 25% times out of interval defined by $\mu \pm 1.5\sigma$).

Performances of the previous indicators together with EBM1, have been evaluated, comparing them with a long term trend measure of inflation. The comparison have been carried out as already illustrated in section 3 considering the proxy of the trend-cycle obtained by the 12 months centred moving average of measured inflation taking into account the whole period analysed, or rolling temporal windows respectively of 12, 24 and 36 months. The deviations from this trend has been measured by the Root Mean Square Error (RMSE) and the Mean Absolute Deviation (MAD).

Besides the evaluation of performance in comparison with a trend proxy, also standard deviations have been calculated to measure the reduction of volatility produced by the exclusion of products with respect to the volatility of CPI.

Finally an analysis has been sketched about the performances of different indicators during the sharp increase and the decrease of inflation in 2008.

6.4 Exclusion based method: the results of the experiment on Italian data

Table 1 resumes the main results obtained in terms of indicators, concerning different core inflation measures, including EBM1. RMSE and MAD.

All the RMSE and MAD elaborated confirm the best performance of the index of core inflation estimated excluding a standard group of products (EBM6)\textsuperscript{11}.

Also looking to standard deviation measures, calculated on the whole time series of 12 months rates of change, EBM6 standard deviation is clearly lower than that one of EBM1, produced by Istat and equal to the standard deviation of index of core inflation excluding products out of $\mu \pm 2\sigma$.

Figure 7 shows a comparison among rates of change of different measures of core inflation (EBM1 and EBM6) and of all items CPI. Rates of change calculated on EBM6, excepted the first part of time series, in general lie above rates of change registered by EBM1. Second graph focuses the attention on the last period of $\pi_{t}^{12}$ time series (2007-2008) where it is confirmed that EBM6 $\pi_{t}^{12}$ are generally higher than those ones of EBM1 and frequently also than all items $\pi_{t}^{12}$, in particular during 2007 and the end of 2008. It emerges that the more volatile products that have been excluded from EBM6 contribute either to push up or to keep low the inflation rate allowing to avoid a persistent under or over estimate of underlying prices movement that should be measured by core inflation indicator:

Focusing on the differences measured between $\Pi_{t}^{12}$ of raw data and $\Pi_{t}^{12}$ of EBM1 and the differences between $\Pi_{t}^{12}$ of raw data and $\Pi_{t}^{12}$ of EBM6, we can state that the latter one are more stable than the previous one (standard deviation equal 0.34 versus 0.38), providing another remark in favour of the choice, for what concerns the exclusion based methods, of indices calculated excluding products identified as the more volatile ones for significant time intervals.

\textsuperscript{11} 100 products have been excluded from the calculation of EBM and their choice have been carried out on the basis of their volatility either over the time or month by month. Combining these two points of view to the issue of volatility, some products that have been evaluated significantly volatile from the first point of view, have shown a stable belonging to one of the tail of the distribution of the 12 months rates of change, bringing a signal that should not be removed. Really the amount of products (6 on 100) and their weight (2.9% on the total weight of products excluded) have pushed to prefer an understandable and transparent choice (exclusion of products whose $\pi_{t}^{12}$ have fallen at least 25% times out of interval defined by $\mu \pm 1.5\sigma$) going on in excluding them even though their insertion in the calculation is expected to slightly improve core inflation measures.
Table 1. Core inflation indices performances in terms of Standard deviations, RMSE and MAD

<table>
<thead>
<tr>
<th></th>
<th>EBM1 index without energy and no processed food products (disseminated by Istat)</th>
<th>EBM2 Index core inflation excluding products out of μ±1dev.stand yearly rates of change</th>
<th>EBM3 Index core inflation excluding products out of μ±1.5 dev.stand yearly rates of change</th>
<th>EBM4 Index core inflation excluding products out of μ±2 dev.stand yearly rates of change</th>
<th>EBM5 Index core inflation excluding products out of μ±2.5 dev.stand yearly rates of change</th>
<th>EBM6 Index core inflation excluding a list of 100 high volatile products</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviations</td>
<td>0.16</td>
<td>0.11</td>
<td>0.14</td>
<td>0.11</td>
<td>0.13</td>
<td>0.11</td>
</tr>
<tr>
<td>Total RMSE</td>
<td>0.3481</td>
<td>0.3164</td>
<td>0.3752</td>
<td>0.3063</td>
<td>0.3138</td>
<td>0.2962</td>
</tr>
<tr>
<td>Total MAD</td>
<td>0.2839</td>
<td>0.2611</td>
<td>0.3177</td>
<td>0.2593</td>
<td>0.2581</td>
<td>0.2433</td>
</tr>
<tr>
<td>% min RMSE 12 months m.a.</td>
<td>17.42</td>
<td>12.88</td>
<td>6.82</td>
<td>12.12</td>
<td>21.97</td>
<td>28.79</td>
</tr>
<tr>
<td>% min MAD 12 months m.a.</td>
<td>19.70</td>
<td>12.12</td>
<td>8.33</td>
<td>7.58</td>
<td>20.45</td>
<td>31.82</td>
</tr>
<tr>
<td>% min RMSE 24 months m.a.</td>
<td>15.00</td>
<td>17.50</td>
<td>4.17</td>
<td>20.00</td>
<td>18.33</td>
<td>25.00</td>
</tr>
<tr>
<td>% min MAD 24 months m.a.</td>
<td>9.17</td>
<td>16.67</td>
<td>6.67</td>
<td>24.17</td>
<td>15.83</td>
<td>27.50</td>
</tr>
<tr>
<td>% min RMSE 36 months m.a.</td>
<td>17.59</td>
<td>22.22</td>
<td>0.00</td>
<td>23.15</td>
<td>7.41</td>
<td>29.63</td>
</tr>
<tr>
<td>% min MAD 36 months m.a.</td>
<td>16.29</td>
<td>20.58</td>
<td>0.00</td>
<td>21.43</td>
<td>6.86</td>
<td>27.43</td>
</tr>
</tbody>
</table>

Source: elaboration on Istat data

Some final remarks concerning the list of products of 2008 basket that have been excluded to calculate EBM6. First of all they are 42, but matching this list with the list of products currently excluded by Istat to calculate EBM1, only 15 are present in both of them. The 18 COICOP classes to which the 42 products excluded from EBM6 are recreational and sporting services, meat, oils and fats, vegetables, fruit, fuels and lubricants for personal transport, gas, electricity, equipment for the reception, recording and reproduction of sound, photographic and cinematographic equipment and optical instruments and pictures, information processing equipment, games, toys and hobbies, insurance connected with transport, passenger transport by railway, passenger transport by air, telephone and telefax equipment, telephone and telefax services. As it is expected the classes involved by products excluded from EBM6 are more than those ones involved by products excluded from EBM1, for the criterion of selection that does not exclude all the products belonging to a specific class but the more volatile ones that may belong to a wider spectrum of classes.

The results obtained could push to investigate more deeply properties of EBM6 that seems to remove some risks coming from completely data driven approach or from a priori selection of products to be excluded as it is now carried out by Istat.
7. Stochastic approach and the analysis of price change distributions

7.1 Obtaining efficient and robust estimation of the underlying mean

The stochastic approach to index numbers has, implicitly or explicitly, formed the basis of many recent attempts to improve upon existing core inflation measures (Wynne, 2008).

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12 In the academic literature, this approach is exemplified by the papers by Clements and Izan (1981, 1987) as well as a book by Selvanathan and Prasada Rao (1994). The research of Bryan and Pike (1991), Bryan and Cecchetti (1993,
In this approach the underlying or core (true) rate of inflation is treated as an unknown parameter to be estimated from the individual price changes (Clements et al. 2006).

In this context we can expressed CPI inflation $\Pi_t$, as a core inflation $\Pi_t^*$, plus a temporary disturbance $\epsilon_t$, as follows:

$$\Pi_t = \Pi_t^* + \epsilon_t$$

where $\epsilon_t$ is a zero mean random component.

On the other hand, each price change $\pi_{it}$ is made up of a systematic part that is common to all prices, $\Pi_t^*$, and a zero-mean random component $\epsilon_{it}$. Since the term $\Pi_t^*$ equals $E[\Pi_t]$ it is interpreted as the systematic part that is common to all price changes or the underlying rate of inflation.\textsuperscript{13}

Therefore, the observed distribution of price changes in any given period is thought of as a sample drawn from an unknown or underlying population distribution which may vary over time.

In other words, the individual prices are observed with error and the problem is a signal-extraction one of how to combine noisy prices so as to minimise the effects of measurement errors and to obtain a good estimate of the measure of central tendency in the population distribution from the observed sample distribution.

As Roger (1997) underlined, if we cannot observe the true or population distribution we are limited to an estimate of the underlying or population mean based on the sample price changes. In choosing an estimator of the population mean, three properties are highly desirable: unbiasedness, efficiency, and robustness.

If the underlying population distribution of price changes can be assumed to be symmetric, then the choice of estimator mainly involves considering the robustness and efficiency of alternative estimator. If the distribution is also asymmetric, then attention must also be paid to bias in the estimator.\textsuperscript{14}

The sample mean is the best estimator (lowest variance) linear unbiased estimator of the population mean if the distribution is Normal. Departures from normality can arise from either kurtosis, skewness, or a combination of both.\textsuperscript{15}

The most appropriate estimator of the mean of a symmetric population distribution depends mainly on the kurtosis of the distribution.

Indeed, if the population distribution of price changes is symmetric but is characterised by high kurtosis, then the sample median will be a much more efficient estimator of the population mean than the sample mean. Essentially this is due to the fact that the median is much less affected by extreme price movements than the mean and such outliers are far more common when the population distribution shows high kurtosis.

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1994), and Cecchetti (1997) has brought this approach to the attention of monetary policymakers in the United States, while the work of Quah and Vahey (1995), Blix (1995), and Fase and Folkertsma (1996) indicates that this alternative way of thinking about inflation measurement is also relevant among the national central banks in the EU.

\textsuperscript{13} With this interpretation, the change in the relative price of good $i$ is then $\pi_{it} - \epsilon_{it}$. Since $\Pi_t = \Pi_t^* + \epsilon_t$ implies that $\pi_{it} - \Pi_t^* = \epsilon_{it}$ and as $E(\epsilon_{it}) = 0$, it follows that the expected value of the change in the $i$-th relative price is zero, which means that, on average, all relative prices are constant. While this is obviously restrictive, the approach can be extended by adding a commodity specific parameter, which are not considered here.

\textsuperscript{14} It is worth noting that ever since the first systematic efforts to construct aggregate or composite price indices in the 1860s it has been observed that the cross-sectional distribution of price changes has not conformed to the Normal distribution. Instead the distribution has typically been found to be markedly leptokurtic and more often positively than negatively skewed (Jevons, 1863, Edgeworth, 1887)

\textsuperscript{15} The mixed Normal distribution is often used in literature to approximate leptokurtic distributions in order to analytically illustrate the impact of excess kurtosis on the efficiency of the various population mean estimators (Bryan et al. 1997, Roger, 2000)
In general, as the kurtosis of the distribution increases, the efficiency of estimators, as the sample mean, that place a high weight on observations in the tails of the distribution falls relative to estimators that place a low weight on observations in the tails.

For platykurtic distributions, the most efficient estimators place relatively high weight on observations in the tails, while for leptokurtic distributions, the most efficient estimators place relatively low weight on observations in the tails.

Such estimators are known as order statistics, because the weight attached to observations depends on their order or ranking in the distribution.

A common and particularly simple class of estimators that places a relatively low weight on observations in the tails of the distribution is the **trimmed-mean (or limited influence estimator)**. These measures involve zero-weighting of some proportion of the observations at each end of the distribution of observations. It follows that trimmed means are also more efficient estimators than means for leptokurtic distributions.

The **sample median**, which is an extreme form of a trimmed mean (being the 50 percent trimmed mean), is not necessarily the most efficient estimator of the population mean for every symmetric distribution showing excess kurtosis. To this respect, Bryan, Cecchetti and Wiggins (1997) found that the optimal trim may be as little as 5 to 10 percent. Basically this result arises from the fact that for the particular class of distribution considered, after trimming around 10 percent from each tail of the distribution, the residual distribution is approximately Normal, so that the optimal estimator is the mean of the remainder.

When the exact shape of the distribution is not known more emphasis should be placed on the **robustness** of the estimator for a range of distribution. If the population distribution of price changes is **asymmetric**, then measures of central tendency of price changes will no longer coincide. As a consequence, robust estimators based on the implicit assumption of symmetry will be chronically biased.

In a symmetric distribution, the population mean will coincide with the median or 50th percentile price change. In an asymmetric distribution the mean, whether of the Laspeyres or superlative kind, will correspond to a different percentile. If the distribution is right skewed, the mean will correspond to a percentile of the distribution somewhere between the 50th and 100th percentiles. If this percentile can be determined, then the sample value of that percentile can be considered as and estimator for the population mean, just as the sample median is used as an estimator of the population mean in a symmetric distribution.

Therefore, the **mean percentile** is the value of the price change of the percentile class in which the mean falls. It ensures the average of price changes in the underlying variable lines up with that corresponding to the target variable.

For the trimmed mean class of estimators, including the median, the bias problem can be minimized by **trimming the distribution asymmetrically**. However in this case, theory neither prescribes the optimal size of the trim, nor the extent to which is should be asymmetric.

Therefore, the question is how much of the distribution should be trimmed and in which way this should be done in order to find an asymmetric trimmed mean, which is not systematically biased relative to inflation.

Regarding the economic arguments which motivate the use of trimmed mean Mankikar and Paisley (2004) noted that knowing the source of the shock is crucial in determining whether it is wise to trim. So there is no a priori case for trimming on theoretical grounds. On the one hand, the economic motivation for using trimmed mean (Bryan and Cecchetti, 1994) are based on theoretical
models of price setting, namely on the Ball and Mankiw (1995) model\textsuperscript{16} where relative price shocks temporarily affect the aggregate price level, even though the long-run effect, when all prices have been adjusted, is zero. As trimmed means disregard the CPI items with a behaviour strongly different from the mean, they just exclude the effects of those shocks, and so they potentially record only trend inflation. This view justifies trimming a larger proportion on the right tail if the distribution is positively skewed.

On the other hand, Bakhshi and Yates (1999) argue that other economic models suggest that trimming following the way suggested by Bryan and Cecchetti (1994), may make things worse, not better. They develop two versions of a model in which the best indicator of core inflation should not put less but more weight on observations at the tail of the distribution, since these contain more, not less, information about the underlying path of the aggregate price level. For example, if there is staggered, time-dependent price setting, and there are demand shocks but not supply shocks, we would adjust the distribution of price changes in the opposite way to that suggested in Bryan and Cecchetti (1994) thus trimming a larger proportion on the left tail if the distribution is positively skewed.

It is clear at this point that the most robust and efficient estimator of the population or underlying mean of the distribution cannot be specified \textit{a priori}. It is essential to look at the empirical distribution first.

In other words, the evaluation should be data driven, that is the most robust and efficient estimator should be evaluated using data and acceptable criteria (Silver, 2006). Therefore the distribution of price changes must be examined in order to determine the best estimator of the underlying mean of the distribution.

Obviously, there are some difficulties in moving from the statistical theory that motivated the trimmed mean, to a practical measure (Bakhshi and Yates, 1999). As we have discussed above, the amount of the tails we should chop off depends on the amount of kurtosis in the population distribution, which is unknown and possibly changing over time.

In addition, whether we should trim the same amount from the top as from the bottom of the distribution depends on the population skewness, which is not known and possibly varying over time as well.

Therefore, we follow a stochastic approach to obtain the best estimator of core inflation for each specific distribution, concerning price changes in a specific period of time, which is based on a detailed analysis of the characteristics of each price change distributions in order to get specific information on the extent and on the way of trimming.

\subsection*{7.2 Analysis of price change distributions}

In this section we analyse the main characteristic of the price change distributions. For the purposes of this kind of analysis, the observed distribution of consumer price changes in a particular period is still thought of as being a sample, even if measurement was comprehensive and error free (Roger, 1997).

As Silver (2006) stated there is an extensive literature on findings of, and theoretical reasons to expect, non normal distribution of price changes. The consistent body of empirical evidence (Roger, 2000) finds the distribution of price changes to be skewed to the right and leptokurtic (fat-tailed).

\footnotesize{\textsuperscript{16} A particularly important implication of the menu cost model is that the chronic tendency towards right-skewness in the distribution of relative price changes is a consequence of positive trend or generalized inflation. The issue of whether right (left) skewness in the distribution of relative prices is truly chronic or simply a product of positive trend inflation is important or the measurement of core inflation. (Roger, 2000).}
In fact, over the past twenty years analyses of the distribution of consumer price changes have proliferated due to the increasing attention paid by central banks on the measurement and control of inflation\textsuperscript{17}.

Not surprisingly, we found that the price change distributions for Italy are characterized by skewness and kurtosis. However, the extent to which skewness and kurtosis depend on the horizon used to calculate inflation. As we have already specified, we analysed relative price changes $\Pi_t^k$, considering when $k=1$ the $\Pi_t^1$ comparison, which compares prices in one month with the previous month and when $k=12$ the $\Pi_t^{12}$ comparison, comparing prices in one month with the corresponding month in the previous year, thus removing, at least partially, seasonal (monthly) variations.

Skewness and kurtosis are measured using the following indicators:

$$S_t^k = \frac{m_{3t}^k}{\left[m_{2t}^k\right]^{3/2}}$$

and

$$K_t^k = \frac{m_{4t}^k}{\left[m_{2t}^k\right]^2}$$

where the central moments of order $r$ (with $r=2,3,4$) are expressed by:

$$m_{rt}^k = \sum_i w_{it}^k \left(\pi_{it}^k - \Pi_t^k\right)^r$$

Table 2 shows numerous descriptive statistics for cross-sectional distribution of monthly price changes at overlapping horizons of one and twelve months\textsuperscript{18}.

As can be easily seen, the statistical proprieties of the distribution of 1-month price changes are different to those of the 12-month distribution of price changes.

To this respect it is worth noting that the use of a core measure as an early-warning indicator has led some researchers to focus on short-term fluctuations in inflation in defining a core measure that takes full advantage of the timeliness of the data. For example, various U.S. researchers derive core measures based on monthly fluctuations (Bryan and Pike 1991; Bryan and Cecchetti 1993), while Roger (1997) emphasizes measures based on quarterly changes in inflation for New Zealand. However, the volatility in monthly data suggests that sole reliance on higher frequency data could lead to policy errors or unnecessary volatility in the instruments of monetary policy. Cecchetti (1997) reports that changing the growth calculation from a month-to-month to a quarter-over-quarter growth rate halves the noise in inflation. Obviously the frequency chosen depends on the methodology used to construct CPI in each country.

The distributions of price change in Italy, analysed during the period 1996-2998 considering both month-on-previous-month and year-on-year changes, are often skewed and leptokurtic. The large standard deviation of $\Pi_t^1$ and $\Pi_t^{12}$ (Table 2 and Figure 9) demonstrates the significant dispersion of price changes as well.


\textsuperscript{18} Table 2 reports the weighted measures calculated on each monthly cross-sectional distribution from 1996m1 to 2008m12. Although the discussion in this paragraph focuses exclusively on the weighted measures, we have also calculated equally weighted measures. We found that, as underlined by Roger (1997), the resulting equally weighted measures differ substantially from the unequally weighted indicators of skewness and kurtosis.
Table 2. Summary statistics for Price Change Distributions

<table>
<thead>
<tr>
<th></th>
<th>One month ahead</th>
<th>12 months ahead</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( k=1 )</td>
<td>( k=12 )</td>
</tr>
<tr>
<td><strong>Mean Inflation rate</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.19</td>
<td>2.32</td>
</tr>
<tr>
<td>Std.dev</td>
<td>0.14</td>
<td>0.54</td>
</tr>
<tr>
<td>Min</td>
<td>-0.35</td>
<td>1.29</td>
</tr>
<tr>
<td>Max</td>
<td>0.53</td>
<td>4.16</td>
</tr>
<tr>
<td><strong>Std.dev of Inflation rate</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.15</td>
<td>3.78</td>
</tr>
<tr>
<td>Std.dev</td>
<td>0.46</td>
<td>0.72</td>
</tr>
<tr>
<td>Min</td>
<td>0.42</td>
<td>2.63</td>
</tr>
<tr>
<td>Max</td>
<td>2.93</td>
<td>6.33</td>
</tr>
<tr>
<td><strong>Skewness of Inflation rate</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>2.00</td>
<td>1.03</td>
</tr>
<tr>
<td>Std.dev</td>
<td>6.21</td>
<td>1.38</td>
</tr>
<tr>
<td>Min</td>
<td>-12.56</td>
<td>-2.23</td>
</tr>
<tr>
<td>Max</td>
<td>16.99</td>
<td>6.17</td>
</tr>
<tr>
<td><strong>Kurtosis Inflation rate</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>96.14</td>
<td>18.40</td>
</tr>
<tr>
<td>Std.dev</td>
<td>77.25</td>
<td>10.52</td>
</tr>
<tr>
<td>Min</td>
<td>5.75</td>
<td>7.38</td>
</tr>
<tr>
<td>Max</td>
<td>386.55</td>
<td>87.33</td>
</tr>
</tbody>
</table>

It is worth noting that as the horizon becomes longer, from one-month to twelve month basis, skewness falls. This is evident from Figure 8, which compare 1-month and 12-month price changes (expressed as percentage). On average, skewness seems to be a major problem for distribution of one-month changes, where the mean value is 2.0 while the mean of \( S^1_{12} \) is equal to 1.03. However, this suggests that both the distributions of relative price changes at horizons \( k=1 \) and \( k=12 \) are positively skewed on average, indicating that, typically, the right-hand tail of the distribution is longer than the left-hand tail.

This figure is higher than the one found for the USA (0.21) by Bryan et al. (1997) but similar to the ones found for Portugal (0.83) by Marques and Mota (2000), for Australia (0.7) by Kearns (1998) and for Ireland (0.8) by Meyler (1999).

Moreover, the standard deviation of \( S^1_{12} \) is 6.21 while that of \( S^1_{12} \) is 1.38. In particular, the plot of the skewness coefficients of \( S^1 \) and \( S^1_{12} \) (Figure 10) illustrates that in nearly all months the distribution of price changes is skewed. The distribution is more often positively skewed than negatively skewed. The skewness switches from the right to the left during the year 2004 and 2007.

As already mentioned, among the important characteristics it can be underlined that both the price change distributions considered are always leptokurtic.

It is evident from Figure 11 which depicts the kurtosis coefficients \( K^1 \) and \( K^{12} \). The kurtosis ranges from an average value of 18.40 for 12-month price changes to 96.14 for 1-month price changes. Therefore the kurtosis tends to decrease for higher horizons \( k \) as well. Aucremanne (2000) found similar result for Belgian data and also stressed that the level of kurtosis tends to decrease for higher aggregation levels.

Nonetheless, the kurtosis coefficient is often very large. Indeed, it is greater than 5.75 in every month, demonstrating that the distribution of 12-month price changes is always leptokurtotic (that is, more fat-tailed than a normal distribution). This indicates that in a typical month, a large proportion of the CPI basket may experience price changes significantly different from the mean
inflation rate. This fact suggests the use of trimmed mean as an estimator of the population mean, as it is more efficient than the sample mean. However, it is important to consider that these figures may be sensitive to the desaggregation level of the CPI. Therefore, we conducted a similar analysis considering two more aggregated levels of CPI, namely the classes and groups using the COICOP hierarchical classification (Classification of Individual Consumption by Purpose) concerning products, as underlined by the International Labour Office (ILO et al., 2004). The results obtained shown that skewness and kurtosis tend to be less pronounced when the distribution is analysed at a higher level of aggregation.

Finally, it is worth noting that the conclusions drawn on asymmetry and kurtosis of the price changes distribution were obtained with non-robust estimators, namely $S_k^t$ and $K_k^t$, as they tend to underestimate the true importance of the tails of the distributions. This is the so-called masking phenomenon which is basically due to the fact that both indicators are computed using the sample mean and standard deviation, which are themselves influenced by the occurrence of outliers. However, as underlined by Marques and Mota (2000) one should expect that the true skewness and kurtosis coefficients to be higher than those measured by using central moments.

Figure 8. Mean Inflation rates

Figure 9. Standard deviation of Inflation rates

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19 Aucremanne (2000) and Silver (2006) suggested the use of some alternative measures for skewness and tail weight.
Regarding correlation analyses of the moments of the price change distributions, Tables 3 and 4 demonstrate that the mean inflation rate is positively correlated with the skew of the distribution of price changes. The relationship is higher concerning the 1-month price change distribution. This result is consistent with a wide body of literature (Ball and Mankiw, 1995; Balke and Wynne, 1996, 2000).

Table 3 Correlation of moments-1month price changes

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>-0.072</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>0.437</td>
<td>-0.001</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-0.076</td>
<td>0.114</td>
<td>0.204</td>
<td>1.000</td>
</tr>
</tbody>
</table>
The dispersion of 12 month price change distribution is positively related to the mean rate of inflation (0.39), while the standard deviation concerning the 1-month price changes shows a negative, although very low, correlation coefficient with the inflation rate (-0.072).

The skewness and kurtosis are positively related, in particular concerning the 12-month price changes. This implies that the fat-tails of the distribution are often not symmetric. In other words, periods characterised by strong asymmetry are also periods in which the kurtosis is higher (and vice versa). This figure is similar to the one found for Australian price changes by Roger (1998) where the correlation between skewness and kurtosis was 0.41, for the Portuguese case by Marques and Mota (2000), where the correlation was found on graphical examination and for Ireland by Meyler (1999), where the correlation coefficient was 0.24.

These Authors underlined that there could be two main explanations for this correlation. On one hand, if the distribution is positively (negatively) skewed the samples tend to be skewed to the right (left) indicating that the right (left) hand tail is the longest. As the kurtosis evaluates the relative importance of the tails it will tend to be higher the stronger is the asymmetry.

On the other hand, sample skewness may be generated by the kurtosis of the distribution. In fact, according to Bryan et al. (1997) when the distribution is leptokurtic one is more likely to obtain a draw from one of the tails that is not balanced by an equally extreme observation in the opposite tail. In this case the higher the kurtosis the higher the probability of getting a skewed sample, even if the underlying distribution is symmetric.

This finding has important practical consequences as it implies that it is not possible to isolate or separately correct the effect of skewness and kurtosis for a given sequence of sample.

Since the one-month price changes contain substantial high frequency noise we will focus on 12-month price changes to obtain measures of core inflation. Therefore, in order to minimise the effect of seasonality on the cross-sectional distribution, we use year-on-year rates.

By using skewness and kurtosis coefficients, the normality of the distribution has been tested statistically. Not surprisingly, the null hypothesis, that the 12-month CPI price changes have a normal distribution, has been comprehensively rejected in all months. Although the results with respect to the mean and standard deviation, as well as the skewness and the kurtosis indicators, provide ample evidence for the presence of differences in the monthly cross-section of Italian CPI changes over the period 1997-2008, it is interesting to examine if the twelve distributions over a calendar year show similar characteristics.

By pooling the normalised monthly distribution of 12-month price changes over a calendar year we can obtain the approximated population distribution for that year. The normalisation necessarily eliminates information about changes in the means and variances, but, by pooling monthly data, more precision is gained in estimating skewness and kurtosis of the distribution.

The results shown in Appendix 2 are interesting. It is apparent that the shape of the distributions is different from the Normal distribution in each year. Moreover the various distributions of price changes in the period 1997-2008 are quite different.

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20 The normality of a distribution has been tested by using the Jarque-Bera statistic, defined as: $JB = N \left[ \frac{S^2}{6} + \frac{1}{4} (K - 3)^2 \right]$, which has a chi-squared distribution with two degrees of freedom under the null hypothesis, where $N$ is the sample size. In all other quarters, the null hypothesis can be rejected at a significance level of 0.001.
Figure 12 shows cumulative the twelve normalised frequency monthly distributions pooled over each calendar year considered in the period analysed. From this figure the differences among price change distributions appear more clearly. In particular, in the period 1997-2003 the distributions are rather different from each other. With the exception of the distribution in the year 1998 they are also fairly dissimilar from the Normal distribution. On the other hand, during the period 2004-2008 the basic shape of the distribution of 12-month price changes seem to became similar despite comparatively different average inflation rates and different evolution of the economic situation.

In conclusion, we can reasonably state that these findings strongly support our methodological approach to obtain an optimal trimming percentage for each month separately.

Figure 12. Cumulative frequency distribution of 12-month price changes of CPI (pooled, in standard deviation from mean)
8 Stochastic approach: constructing a measure of core inflation using asymmetric trimmed means

8.1 Some remarks on finding the optimal trimmed mean

As we have ascertained in the previous section, the distribution of consumer price changes in Italy is non normal, always leptokurtic and skewed, more often to the right. Therefore these distinctive features of the distribution of price changes need to be considered when constructing measure of core inflation, based on the stochastic approach. This section considers the construction of a statistically-efficient measure of core inflation given the high degree of kurtosis in the price change distribution whilst correcting for skewness in the distribution as well.

By considering the issue of the estimation of the central tendency of inflation from a stochastic perspective we will search the best estimator among different trimmed means\(^{21}\).

When the distribution of price changes is asymmetric the optimal trim need to be asymmetrical, because, symmetric trimmed means, including the median, may result in a series that consistently understates (overstates) inflation. However, finding an optimal asymmetric trimmed mean is a controversial methodological issue in the literature. Difficulties concern both the method used to select the optimal trim and the way of the trimming, i.e. which side of the distribution should be characterized by a higher percentage of trimming.

On the first problem, Marques and Mota (2000) among several trimmed means select the less volatile estimator, represented by the 10 per cent asymmetric trimmed mean centred on the 51.5\(^{th}\) percentile, that is the percentile that allow to trim 11.5 per cent from the left-hand tail and 8.5 per cent from the right-hand tail of the ordered price change distribution. Kearns (1998) and Meyler (1999) compute an almost infinity of trimmed means changing both the trimming percentage from each tail (between zero and 50 per cent) as well as the percentile in which the trimmed mean is centred on. Both authors select the asymmetric trimmed mean that minimises the average absolute deviation and/or the mean square error relative to some benchmark, usually a centred moving average.

Aucremanne (2000) after having computed the trimmed means for all the percentiles between 50\(^{th}\) and 60\(^{th}\) as a first step he selected as the optimal trimmed means the ones for which the null of Normality was rejected according to the Jarque- Bera statistics. Among these, the optimum trimmed mean is chosen as the one that minimises the average absolute error relative to the inflation rate.

Regarding the second matter, when the distribution of price changes is positively skewed, some authors suggest that the trimming should be less severe on the right-hand side (therefore one should trim more on the opposite side of the distribution), due to the fact that price changes with large positive values are an important part of measured inflation. In this way the asymmetric trimmed mean will be not systematically biased relative to inflation (Silver, 2006, Marques and Mota, 2000). On the other hand, various authors (for example, Bakhshi and Yate, 1999, Meyler, 1999) justify trimming more from the right side of the distribution, when the distribution of price changes is positively skewed, in order to minimize the variance of the resulting estimator. Moreover, considering large price changes as noise a larger proportion of the top of the distribution should be

\[^{21}\] As it is known the trimmed mean can be computed from

\[ \Pi_{i} = \sum_{k} w_{i}^{k} \pi_{i}^{k} \]

by excluding a given percentage of the highest and lowest price changes. For example, taking into account the weight, the 20 per cent symmetric trimmed mean is obtained by excluding the 10 per cent lowest price changes and the 10 per cent highest price changes, thus considering only 80 per cent of the central distribution.
trimmed in order that over repeated draws, the expected impact of sampling errors on the mean is zero.

Our approach is to examine both methods with different objectives. First we try to find an asymmetric trimmed mean which is not systematically biased relative to inflation, then we look at the estimator with minimum variance.

Moreover, since the shape of the distribution change periodically, any one trimmed mean may not always be the best representation of core inflation. Therefore our suggestion is to relax the implicit assumption that the trimming percentage remain constant over time and allow it to vary in line with the characteristics of the cross-section distribution of price changes. In other words, we select an optimal trimmed mean for each month in the period 1997m1-2008m12 since the assumption that the kurtosis and skewness of price change distribution are time invariant seems to be not plausible in the Italian case.

Considering the first objective of finding an asymmetric trimmed mean which is not systematically biased relative to inflation, the optimal trimmed mean should satisfy the condition $\Pi_t - \Pi_t^* = \varepsilon_t$ set out in Marques and Mota. (2000) as a requirement for a core inflation measure.

In this context the main concern is to preserve the information derived by the shape of the distribution as it is part of measured inflation, and not have the trimming throw out such information with the noise (Silver, 2006). Therefore, it is required that the trimming be less harsh on the right-hand side, when the distribution is skewed to the right, and less hard on the left-hand side, when the distribution is skewed to the left.

One way to correct for this bias, following Roger (1997 and 2000), is not to center the trim on the 50th percentile, but to center it on the mean percentile.

The strategy adopted was to construct a range of trimmed mean estimators with trim varying from 0 to 100 per cent in steps of five per cent. As already mentioned, to allow for the skewness found in the distribution, we centred the trimmed mean on the mean percentile.

The bias can be eliminated by removing a larger proportion of the trim from the tail opposite to the direction of the skew. The larger is the average degree of skewness, expressed by the location of the mean percentile, the larger is the proportion of the trim that must be taken from the left-hand tail to avoid average rate bias.

Considering our second objective of finding an asymmetric trimmed mean which minimises the variance, a larger percentage should be trimmed from the right hand tail of the distribution when the distribution is positively skewed in order to remove noise or temporary disturbances.

The starting point to find an optimal trim, among a range of trimmed mean estimators with trim varying from 0 to 100 per cent in steps of five per cent, is represented by the mean percentile, but in this case, following Meyler (1999) the percentage to be removed from the lower half of the distribution and from the upper half of the distribution is determined by using the following rule:

- Lower half of the distribution = total percent of trim*[1-mean percentile]
- Upper half of the distribution = total percent of trim*[mean percentile]

The several trimmed means calculated in both methods are then compared with the established benchmark represented, as said in par.3, by the 13 month moving average of the mean inflation rate. The aim is to find the trimming percentage that minimises the value of the Absolute Deviation from the benchmark.

Based on the previous discussion, the next section will illustrate the results found carrying out a study on the Italian 12-month price changes in the period 1997-2008 by using the stochastic approach.

Heath et al. (2004) extended Aucremanne’s (2000) use of the Jarque-Bera test statistic as a basis for selecting the level of trim that jointly removes skewness and excess kurtosis. The level of trim required to do this would vary each period and the Jarque-Bera statistic is used to decide between measures with varying percentages of trim and with varying central percentiles (between 40 and 60 percent).
8.2 Stochastic approach: Robust estimators

8.2.1 Median and Weighted median

Bryan and Pike (1981) were the first to suggest the use of the median. In their view, the median of changes in the components of the U.S. CPI is a better representation of the trend rate of inflation than the weighted average. Since only the order, not the values, of the various price changes is used in its calculation, the median is a central tendency statistic that is greatly independent of the data’s distribution. The median also has the appealing property of lying close to the majority of price changes than does any alternative measure as it minimizes the mean absolute deviation of the data. This approach was then picked up by Bryan and Cecchetti (1993), who examined the weighted median, which is the value that separates the ordered sample into two parts, with the sum of the weights of each part being equal to 50 per cent. As already mention although the weighted median is theoretically the measure least affected by outlying relative price movements, it is unlikely to be the most efficient as it gives potentially informative observations a zero weights. However, as an order statistic, the weighted median will be a more robust measure of the tendency of the individual price changes that make up the distribution than the weighted mean if the distribution of price changes is non-normal.

Figure 13 plots the Median (50\textsuperscript{th} percentile) and the weighted median against the actual 12-month inflation rate over the period examined.

**Figure 13 . Median and Weighted Median (12-month CPI changes)**

The weighted median and particularly the median appear to be systematically lower than 12-month CPI changes, except in 2007, which suggests that they are doing more than just excluding relative price disturbances.
8.1.2 Sample mean percentile

To begin with, Figure 14 shows the evolution of the mean percentile of the 12-month price change distribution. The main feature to note is that the mean percentile is almost always above the 40th percentile, which provides an indication of the good degree of representativeness of the mean rate of inflation. For the sample period the average mean percentile is 54th (depicted in Figure with the red line).

In particular, Figure 14 illustrates the extent to which the mean can be pulled away from the central mass of price changes by price changes in the tails (right or left) of the distribution.

We found a positive sample correlation, equal to 0.59, between the mean percentile and the mean inflation rate, which is consistent with the proposition that higher average or trend inflation will produce greater skewness.

An implication of the positive correlation is that the use of a time-invariant percentile price change, as an estimator of the population mean price change, will tend to understate the trend rate of inflation if the trend is rising, and overstate it when the trend rate is decreasing, at least over the short term.

Figure 14. Sample Mean Percentile (12 month CPI changes)

This series is the 100% trimmed mean centred on the average mean percentile (54th) and it appears as a natural candidate for a core inflation measure (Marques and Mota, 2000).

Figure 15 plots the 54th against the actual 12-month inflation rate over the period examined. However the results should be interpreted under the assumption that the asymmetry of the price changes distribution can be assumed constant across the sample period. Although, in the Italian case, the empirical evidence tend to not support this assumption, the results can be helpful in

---

23 This figure is similar to the one computed by Kearns (1998) concerning Australian data (52th) and by Marques and Mota (2000) concerning Portuguese data (56th)
understanding the need for searching an asymmetric mean satisfying not only the unbiased property but other important properties that a core inflation measure should satisfy.

Figure 15. 54th Percentile (12-month CPI changes)

8.2.3 Asymmetric trimmed mean

Figure 16 plots the 12-month movements in CPI inflation and the optimal trimmed mean measure of inflation, (TRIM1) obtained applying the first procedure we suggested. It is apparent the over the period 1997-2008 the series have vary similar means and tend to move together. As the optimal trimming vary with the characteristics of the price change distribution it is worth noting that the average percentage of trimming over the period 1997-2008 is equal to 77.1%. Therefore, it seems that with the aim of minimising the bias relative to the recorded inflation we increase the amount of trimming and in so doing we do not change the volatility of the corresponding trimmed mean. This outcome could be an indicator that we are trimming too much, thus excluding to much information, in particular regarding the left hand side of the distribution if the distribution is skewed to the right, which is fundamental for the definition of a trend measure of inflation.

The asymmetric means TRIM2 obtained following the second procedure described in the previous section, are plotted in Figure 17. It seems that this measure of core inflation meet both the condition of unbiased and reduction in volatility. Comparing to the weighted median it shows a lower systematic downward bias relative to the CPI inflation recorded. The average percentage of trimming over the period 1997-2008 is equal to 25.8% and concerns particularly the right hand tail of the distribution in case of negative asymmetry. We will test the properties of unbiasedness and reduction in volatility in the next section.
Figure 16. Asymmetric Trimmed Mean -TRIM1 and CPI inflation rates (12 month CPI changes)

Figure 17. Asymmetric Trimmed Mean- TRIM2, Weighted Median and CPI Inflation
8.3 Assessing the performance of the estimators considered

An assessment of the estimators constructed in the previous section is made on the basis of three evaluation criteria, described in section 3, which try to clarify the statistical conditions that a suitable underlying inflation indicator should satisfy. Bias is initially assessed informally by comparing the average of underlying inflation with that of CPI inflation over a given period. Of course, the existence or extent of bias observed may depend on the specific period over which the calculation is performed. But it is nonetheless a useful indicator of which measures have bias properties that merit further investigation.

Table 5 reports the historical average of difference between the estimator and observed inflation together with information concerning the variability of the different indicators.

Table 5 Unbiasedness and volatility of estimators

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Average Difference</th>
<th>Standard deviation</th>
<th>Standard deviation of the first difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer CPI Inflation</td>
<td>-0.536</td>
<td>0.536</td>
<td>0.164</td>
</tr>
<tr>
<td>54th Percentile</td>
<td>-0.013</td>
<td>0.400</td>
<td>0.164</td>
</tr>
<tr>
<td>Median</td>
<td>-0.467</td>
<td>0.462</td>
<td>0.134</td>
</tr>
<tr>
<td>Weighted Median</td>
<td>-0.172</td>
<td>0.382</td>
<td>0.081</td>
</tr>
<tr>
<td>TRIM1</td>
<td>-0.002</td>
<td>0.531</td>
<td>0.121</td>
</tr>
<tr>
<td>TRIM2</td>
<td>-0.096</td>
<td>0.420</td>
<td>0.154</td>
</tr>
</tbody>
</table>

After that we test whether the bias is statistically significant by estimating the equation:

\[ \Pi_t = \beta_0 + \beta_1 \Pi_t^* + \epsilon_t \]

and testing the joint null hypothesis that \( \beta_0 = 0 \) and \( \beta_1 = 1 \) for each of the estimators considered. The results are shown in Table 6.

Table 6 Test for Unbiasedness

<table>
<thead>
<tr>
<th>Indicator</th>
<th>F statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>54th Percentile</td>
<td>1.25</td>
<td>0.2910</td>
</tr>
<tr>
<td>Median</td>
<td>99.88</td>
<td>0.0000</td>
</tr>
<tr>
<td>Weighted Median</td>
<td>22.86</td>
<td>0.0000</td>
</tr>
<tr>
<td>TRIM1</td>
<td>0.26</td>
<td>0.7727</td>
</tr>
<tr>
<td>TRIM2</td>
<td>30.40</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

We found that over the period 1997-2008 only two measures assessed pass the test for unbiasedness with respect to CPI inflation, namely the 54th percentile and the TRIM1. This is not unexpected since as we have underlined above this two measures should satisfies \( \Pi_t - \Pi_t^* = \epsilon_t \) by construction. The Median and the Weighted Median being symmetric estimators are biased thus confirming the chronic skewness of the price change distributions.

The next question we address is whether the various measures we have considered are less volatile than CPI. This criterion examines the reduction in volatility that is obtained by the different core
measures, in order to quantify the efficiency gain. We consider the usual standard deviation and a short term volatility measure, reported in section 3.

As Table 5 shows, considering the standard deviation, the volatility of each of the five indicators is below that of the observed inflation. We can draw similar conclusion considering the high frequency volatility measure. With the exception of 54th Percentile, all of the measures assessed reduce the variability.

This confirms the idea that a substantial efficiency gain can be obtained by making use of robust estimators.

The TRIM2 does not produce a high reduction in the variability as we expected.

The third criterion we discuss is which measure of core inflation that best tracks trend inflation, expressed by the 13 month (with the 1st and 13th element weighted 0.5 as in Wagemann – Macaulay formula) moving average of the mean inflation. The various trimmed means can be compared using the MAD and the RMSE between the trimmed mean 12-month rate of inflation and the proxied trend series. The MAD penalises all deviation from the trend series equally, while the RMSE places a higher penalty on those deviations further from the trend.

We found that found that the mean absolute deviation (MAD) provide different results from the RMSE, as noted by Bakhshi and Yates (1999).

Indeed, the results reported in table 7 indicate that, considering the MAD, among the different indicators assessed the weighted median does by far the best job in tracking trend inflation.

On the other hand, the 54th Percentile seems to be the most efficient estimator considering the RMSE which penalises large errors more heavily.

<table>
<thead>
<tr>
<th></th>
<th>MAD</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>54th Percentile</td>
<td>0.1831</td>
<td>0.2293</td>
</tr>
<tr>
<td>Median</td>
<td>0.1080</td>
<td>0.3286</td>
</tr>
<tr>
<td>Weighted Median</td>
<td>0.0899</td>
<td>0.2999</td>
</tr>
<tr>
<td>TRIM1</td>
<td>0.2140</td>
<td>0.4626</td>
</tr>
<tr>
<td>TRIM2</td>
<td>0.4397</td>
<td>0.6631</td>
</tr>
</tbody>
</table>

9. Concluding remarks

Many theoretical and empirical works has been published and various Central Banks and National Statistical Institutes usually are computing the core or underlying inflation but without any final agreement on the methods to measure it.

Actually, the methods to be used to measure the core inflation depend on the inflationary process, on the characteristic of the markets and on the economic situation of the country involved, which affect the characteristics of the distribution of the price changes and the measure of the core inflation.

After a brief review of the definitions and a presentation of a classification of the different approaches, a lot of empirical analyses has been carried out in order to assess different possible measures of core in inflation in Italy, using a very detailed data set of more than 500 monthly price indices for representative products from year 1996 to 2008.

The results obtained are very interesting and the most important of them can be resumed as follows for the different methods used.
In the time series approach the computation of a SARIMA model appears not to be consistent with the aims of searching persistent inflation but only with the aim of removing short term excessive variability.

In the field of EBM, the check of the real volatility of the price of the products excluded by Istat in computing EBM1 measure of the underlying inflation shows that among the 42 products that are currently excluded only 14 have a $\pi_i$ that at least 25% of times (months) have fallen outside the considered interval. In short volatility analysis does not allow to support the current choice carried out by Istat to exclude, as suggested by Eurostat, all the unprocessed food and energy products form the calculation of EBM1.

The five experimented measures of core inflation excluding products whose $\pi_i^{12}$ fell outside, the interval of the distribution of price changes, respectively, $\mu \pm \sigma$ (EBM2), $\mu \pm 1.5\sigma$ (EBM3), $\mu \pm 2\sigma$ (EBM4), $\mu \pm 2.5\sigma$ (EBM5) and whose $\pi_i^{12}$ have fallen at least 25% times out of interval defined by $\mu \pm 1.5\sigma$ (EBM6), give interesting results about the preference of measure EBM6 in comparison with EBM1.

With reference to the stochastic approach, the analysis of price changes distribution $\pi_i^{12}$ and $\pi_i^1$, showed that both are often skewed and leptokurtic. In general, as the horizon becomes longer, from one-month to twelve month basis, skewness falls.

It is clear that the shape of the distributions is different from the Normal distribution in each year. The distribution is more often positively skewed than negatively skewed. The skewness switches from the right to the left during the year 2004 and 2007.

The skewness and kurtosis are positively related, in particular concerning the 12-month price changes. This findings are similar to the one found for Australian price changes by Roger (1998) where the correlation between skewness and kurtosis was 0.41, for the Portuguese case by Marques and Mota (2000), where the correlation was found on graphical examination and for Ireland by Meyler (1999), where the correlation coefficient was 0.24.

For the sample period the average mean percentile is 54th; this finding is similar to the one computed by other authors.

Concerning the measures of core inflation, the weighted median and particularly the median appear to be systematically lower than 12-month CPI changes, except in 2007, which suggests that they are doing more than just excluding relative price disturbances.

Regarding the issue of obtaining asymmetric trimmed mean in order to reduce biased and volatility, we suggested the use of two indicators TRIM1 and TRIM2, obtained following two different procedures. To this respect, we found that over the period 1997-2008 only two measures assessed pass the test for unbiasedness with respect to CPI inflation, namely the 54th percentile and the TRIM1. The Median and the Weighted Median being symmetric estimators are biased thus confirming the chronic skewness of the price change distributions.

Considering the standard deviation, the volatility of each of the five indicators is below that of the observed inflation.

The measure of core inflation that best tracks trend inflation, expressed by the 13 month moving average of the mean inflation, the weighted median. On the other hand, the 54th Percentile seems to be the most efficient estimator considering the RMSE which penalises large errors more heavily.

Concluding, we can state that in Italy, as in many other countries, the results show that no measure of core inflation performs well in all empirical tests, although some measure such as trimmed mean or weighted median, perform fairly satisfactorily against the different evaluation criteria.

The results obtained could suggest to investigate more deeply properties of EBM6 that seems to remove some risks coming from completely data driven approach or from a priori selection of products to be excluded as it is now carried out by Istat and the use of the stochastic measure in interpreting correctly the process of inflation. In particular it is important to investigate the use of...
\( \pi_{it}^1 \) and \( \pi_{it}^{1,2} \), taking into account the possible different objective of the analysis. Computation of core inflation measure choosing weights for individual prices that are inversely proportional to the volatility of those prices, could be experimented, although theies methods are not easy to explain to the users.

Anyway, it is obvious possible to compute other core inflation measures, but the most important thing is to stimulate a discussion among researchers and Italian Institution, Bank of Italy, Istat and Isae, in order to decide which of the measures proposed in this paper could computed and disseminated currently.

References


Flemming, J (1976), Inflation, Oxford University Press (London)


### Appendix 1

**Analysis of volatility of products eliminated from Italian 2009 CPI to calculate EBM1. Years 1996 – 2008**

<table>
<thead>
<tr>
<th>n.</th>
<th>product group of products</th>
<th>% of exclusion on the basis of 12 month rates of change volatility out of $\mu \pm 1.5 \text{dev. stand}$</th>
<th>% of exclusion on the basis of 1 month rates of change volatility out of $\mu \pm 1.5 \text{dev. stand}$</th>
<th>Standard deviation ranking (descending order) 1 month rates of change</th>
<th>Standard deviation ranking (descending order) 12 month rates of change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Tomatoes for sauces vegetables</td>
<td>66.0</td>
<td>86.81</td>
<td>11.27</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Ananas fruit</td>
<td>38.9</td>
<td>69.44</td>
<td>6.13</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>Potatoes vegetables</td>
<td>68.8</td>
<td>57.84</td>
<td>4.32</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>Lemons fruit</td>
<td>22.9</td>
<td>56.25</td>
<td>2.92</td>
<td>34</td>
</tr>
<tr>
<td>5</td>
<td>Onions vegetables</td>
<td>44.4</td>
<td>47.22</td>
<td>2.70</td>
<td>44</td>
</tr>
<tr>
<td>6</td>
<td>Celereacs vegetables</td>
<td>27.8</td>
<td>37.50</td>
<td>2.33</td>
<td>52</td>
</tr>
<tr>
<td>7</td>
<td>Diesel Fuels and lubricants for personal transport</td>
<td>57.6</td>
<td>35.42</td>
<td>2.26</td>
<td>54</td>
</tr>
<tr>
<td>8</td>
<td>Unleaded petrol Fuels and lubricants for personal transport</td>
<td>49.3</td>
<td>34.72</td>
<td>2.26</td>
<td>55</td>
</tr>
<tr>
<td>9</td>
<td>Diesel for heating Liquid fuels</td>
<td>47.9</td>
<td>27.78</td>
<td>2.18</td>
<td>59</td>
</tr>
<tr>
<td>10</td>
<td>Banana fruit</td>
<td>23.6</td>
<td>27.78</td>
<td>1.60</td>
<td>103</td>
</tr>
<tr>
<td>11</td>
<td>Automotive gas Fuels and lubricants for personal transport</td>
<td>55.6</td>
<td>25.69</td>
<td>1.66</td>
<td>96</td>
</tr>
<tr>
<td>12</td>
<td>Grapefruits fruit</td>
<td>28.5</td>
<td>25.69</td>
<td>1.64</td>
<td>100</td>
</tr>
<tr>
<td>13</td>
<td>Dry garlic vegetables</td>
<td>21.5</td>
<td>25.69</td>
<td>1.66</td>
<td>93</td>
</tr>
<tr>
<td>14</td>
<td>Sea fish (fished) fish and seafood</td>
<td>0.0</td>
<td>25.00</td>
<td>1.70</td>
<td>89</td>
</tr>
<tr>
<td>15</td>
<td>Gas for heating gas</td>
<td>48.6</td>
<td>19.44</td>
<td>1.24</td>
<td>142</td>
</tr>
<tr>
<td>16</td>
<td>Rabbit (fresh) meat</td>
<td>20.1</td>
<td>19.44</td>
<td>1.65</td>
<td>97</td>
</tr>
<tr>
<td>17</td>
<td>Gas for cooking gas</td>
<td>38.9</td>
<td>16.67</td>
<td>1.71</td>
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</tr>
<tr>
<td>18</td>
<td>Sheep and goat's meat (fresh) meat</td>
<td>7.6</td>
<td>15.97</td>
<td>1.08</td>
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</tr>
<tr>
<td>19</td>
<td>Electricity electricity</td>
<td>38.3</td>
<td>13.33</td>
<td>1.33</td>
<td>120</td>
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<tr>
<td>20</td>
<td>Fresh shelfishes fish and seafood</td>
<td>0.0</td>
<td>12.50</td>
<td>1.46</td>
<td>113</td>
</tr>
<tr>
<td>21</td>
<td>Turkey breast meat</td>
<td>36.1</td>
<td>10.42</td>
<td>1.14</td>
<td>167</td>
</tr>
<tr>
<td>22</td>
<td>Chicken (fresh) meat</td>
<td>31.9</td>
<td>9.72</td>
<td>1.48</td>
<td>112</td>
</tr>
<tr>
<td>23</td>
<td>Chicken breast meat</td>
<td>22.2</td>
<td>9.72</td>
<td>0.92</td>
<td>106</td>
</tr>
<tr>
<td>24</td>
<td>Fresh vegetables vegetables</td>
<td>20.8</td>
<td>7.64</td>
<td>0.88</td>
<td>205</td>
</tr>
<tr>
<td>25</td>
<td>Fresh fruits fruit</td>
<td>34.0</td>
<td>6.94</td>
<td>0.81</td>
<td>222</td>
</tr>
<tr>
<td>26</td>
<td>Gas cylinders gas</td>
<td>26.4</td>
<td>6.25</td>
<td>0.71</td>
<td>246</td>
</tr>
<tr>
<td>27</td>
<td>Pork meat (with bones) meat</td>
<td>23.6</td>
<td>4.86</td>
<td>0.77</td>
<td>233</td>
</tr>
<tr>
<td>28</td>
<td>Fresh fish (from fish breeding) fish and seafood</td>
<td>0.0</td>
<td>4.17</td>
<td>0.96</td>
<td>186</td>
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<tr>
<td>29</td>
<td>Fresh molluscs fish and seafood</td>
<td>0.0</td>
<td>4.17</td>
<td>1.19</td>
<td>149</td>
</tr>
<tr>
<td>30</td>
<td>Pork meat (without bones) meat</td>
<td>16.7</td>
<td>2.78</td>
<td>0.61</td>
<td>258</td>
</tr>
<tr>
<td>31</td>
<td>Lubricating Oil Fuels and lubricants for personal transport</td>
<td>11.8</td>
<td>2.78</td>
<td>1.15</td>
<td>162</td>
</tr>
<tr>
<td>32</td>
<td>Horse meat meat</td>
<td>8.3</td>
<td>2.08</td>
<td>0.37</td>
<td>357</td>
</tr>
<tr>
<td>33</td>
<td>Fresh milk milk, cheese and eggs</td>
<td>2.1</td>
<td>0.69</td>
<td>0.35</td>
<td>376</td>
</tr>
<tr>
<td>34</td>
<td>Milk (long life) milk, cheese and eggs</td>
<td>0.7</td>
<td>0.00</td>
<td>0.31</td>
<td>414</td>
</tr>
<tr>
<td>35</td>
<td>Veal fresh meat meat</td>
<td>0.0</td>
<td>0.00</td>
<td>0.30</td>
<td>427</td>
</tr>
<tr>
<td>36</td>
<td>Beef, first quality (without bones) meat</td>
<td>0.0</td>
<td>0.00</td>
<td>0.22</td>
<td>583</td>
</tr>
<tr>
<td>37</td>
<td>Beef, second quality (without bones) meat</td>
<td>0.0</td>
<td>0.00</td>
<td>0.29</td>
<td>451</td>
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<tr>
<td>38</td>
<td>Minced beef meat</td>
<td>0.0</td>
<td>0.00</td>
<td>0.23</td>
<td>553</td>
</tr>
<tr>
<td>39</td>
<td>Sweet water fish fish and seafood</td>
<td>0.0</td>
<td>0.00</td>
<td>0.54</td>
<td>275</td>
</tr>
<tr>
<td>40</td>
<td>Chicken eggs milk, cheese and eggs</td>
<td>0.0</td>
<td>0.00</td>
<td>0.28</td>
<td>463</td>
</tr>
<tr>
<td>41</td>
<td>Salad packages vegetables</td>
<td>0.0</td>
<td>0.0</td>
<td>0.7</td>
<td>247.0</td>
</tr>
<tr>
<td>42</td>
<td>Solid fuel solid fuels</td>
<td>0.0</td>
<td>0.0</td>
<td>0.5</td>
<td>305.0</td>
</tr>
</tbody>
</table>

*Source: elaboration on Istat data*
Appendix 2

Frequency distribution of 12-month changes of CPI-1997

Price changes in standard deviation from mean

% of distribution

Frequency distribution of 12-month changes of CPI-1998

Price changes in standard deviation from mean

% of distribution

Frequency distribution of 12-month changes of CPI-1999

Price changes in standard deviation from mean

% of distribution

Frequency distribution of 12-month changes of CPI-2000

Price changes in standard deviation from mean

% of distribution
Frequency distribution of 12-month changes of CPI-2005

Frequency distribution of 12-month changes of CPI-2006

Frequency distribution of 12-month changes of CPI-2007

Frequency distribution of 12-month changes of CPI-2008