RECENT DEVELOPMENTS IN THE RETAIL PRICE INDEX

by

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Introduction

The paper considers two aspects of the retail prices index (RPI) of recent interest.

The first concerns the representativeness of the RPI, that is the extent to which the RPI is representative of the experience of a wide range of household types, and in particular low income households, bearing in mind the rapid inflation in recent years. The paper discusses the main technical principles involved, which of course are much concerned with the weighting pattern of the index. It also gives the results of some practical studies which shed light on the issue.

The second aspect concerns a technical feature about the way the RPI is constructed. It also involves weighting but at a much more detailed level, that is the method of combination of price quotations at the item level of the index, for example, the 500–800 quotations collected each month for sugar, or a brand of petrol or a loaf of bread. This involves first the method of stratifying the quotations and then secondly the choice of formula for combining the quotations within each stratum. Some recent technical developments are described which break new ground in an area that hitherto has received surprisingly little attention.

Representativeness of the RPI

There is considerable interest in the degree to which the RPI is representative of a wide range of households. It is often asked whether rising prices, especially in times of acute inflation, have affected some groups of households more than others, for example those with low incomes or with large families. Some recent work, together with existing data, shed light on this. The broad conclusion is that the impact of rising prices on different households is remarkably similar and that the RPI is representative over a wide range. First are examined some general technical considerations as to what might be expected. Then results of some practical studies are discussed.

Table 1

Patterns of expenditure by income of household

Commodity or service (%)	*Weekly income of household*							
	Under £25	*£26 to £40*	*£40 to £60*	*£60 to £70*	*£70 to £80*	*£80 to £90*	*£90 to £100*	*Over £100*
Food	30	29	27	25	24	23	22	19
Alcoholic drink	2	4	4	5	5	5	6	6
Tobacco	3	4	4	5	4	4	4	3
Housing	19	16	12	13	12	13	11	12
Fuel and light	11	9	7	7	6	6	6	5
Durable household goods	4	4	6	6	7	7	7	8
Clothing and footwear	5	7	7	8	8	7	9	9
Transport and vehicles	6	7	11	12	14	14	14	15
Miscellaneous goods	8	8	8	8	8	7	8	8
Services	9	9	10	9	8	9	10	11
Meals out	2	2	3	3	3	3	4	4

Note: The individual percentages do not sum to 100 in all cases due to rounding. No adjustments have been made for under-recording of alcoholic drink and tobacco.

Source: Family Expenditure Survey, 1976.

The relative experience of rising prices of different household groups compared with the movement shown by the RPI depends on: (a) the variation in their spending patterns compared with the pattern of the general RPI "basket" of goods and services and (b) the variation in price increases for different groups of goods and services.

The variation in spending patterns is shown in Tables 1–3. The variation with income is shown in Table 1 where the gradual shift as income rises clearly emerges. Over a broad area in the middle ranges of income there are only small and gradual differences, with the main divergence occurring at the top and bottom of the distribution. Because of the difference in pattern at the upper end of the range, the 3–4 per cent of households whose heads have the highest incomes are omitted from the households covered by the RPI. At the same time, at the bottom end of the scale, where the spending pattern begins to differ materially from the general run, low income pensioner households are also excluded from the households covered by the RPI (low income pensioner households are those with at least three-quarters of their income coming from National Insurance retirement or similar pensions). The expenditure patterns for the low income pensioner households compared with households covered by the general RPI are shown in Table 2. These indicate clearly the greater proportion spent on food and fuel by the pensioner households. This also applies to low income households in general as can be seen in Table 1.

Table 2

RPI weights for General Index and pensioner households (1979) (excluding housing)

	General Index households	*Two-person persioner households*	*One-person pensioner households*
Food	263	439	436
Alcoholic drink	88	42	20
Tobacco	50	54	31
Fuel and light	67	135	173
Household durables	73	40	42
Clothing and footwear	93	69	67
Transport and vehicles	163	58	33
Miscellaneous goods	78	85	88
Services	67	67	90
Meals out	58	11	20
Total	1,000	1,000	1,000

Source: Department of Employment.

Table 3

Patterns of expenditure by type of household

Household groups	*Housing*	*Fuel, light and power*	*Food and meals out*	*Alcoholic drink*	*Tobacco*	*Clothing and footwear*	*Durable household goods*	*Other goods*	*Transport and vehicles*	*Services*	*Miscellaneous*	*Total of all expenditure*
All households	14	6	25	5	4	8	7	7	13	10	1	100
Retired households (1 or 2 adults)	22	10	28	3	3	6	5	7	7	10	0	100
Non-retired households:												
2 adults:												
All	15	6	22	5	4	7	8	7	15	10	1	100
Quarter with lowest incomes	15	7	28	5	5	6	7	7	13	7	0	100
Quarter with highest incomes	15	4	19	5	2	8	8	8	16	14	1	100
2 adults, 1 or 2 children:												
All	15	6	25	4	4	8	7	8	13	9	1	100
Quarter with lowest incomes	13	7	29	4	6	8	7	7	11	7	1	100
Quarter with highest incomes	15	5	22	4	2	10	7	8	15	11	1	100
2 adults, 3 or 4 children:												
All	15	6	28	4	4	8	6	8	11	9	1	100
Quarter with lowest incomes	11	9	32	5	6	6	7	8	9	6	1	100
Quarter with highest incomes	14	5	26	4	2	9	7	8	11	12	2	100
Other compositions	12	5	25	6	4	9	7	7	14	10	1	100
Households whose head was:												
Professional, etc., employee	15	5	21	4	2	8	7	8	16	13	1	100
Clerical employee	16	5	23	4	3	9	8	8	13	10	1	100
Manual employee	12	6	26	6	5	8	7	7	14	8	1	100
Self-employed	13	6	23	4	3	9	8	8	13	12	1	100

Note: Adjustments have been made for under-recording of alcoholic drink and tobacco.
Source: Family Expenditure Survey.

Since it is well known that moderate differences in weights often have little effect on index numbers, these data on spending patterns suggest that, apart from the households with low incomes and those at the top end of the income range, the experience of the middle ranges of household would be very similar and vary little from that of the general RPI. Table 3 shows the pattern of expenditure for different household types and a similar picture emerges. Because of the relatively moderate differences in the pattern of expenditure weights for different groups of households, it will be necessary for there to be marked variations in the price increases for different groups of goods and services if the price experience of the different household groups is to vary very significantly. The differential movement of the prices of broad groups of commodities and services over the period 1962–78 is shown in Table 4. There is a tendency for

Table 4

Increases in retail prices 1962–78 and 1970–77

	Average percentage increase per year	
	Jan. 1962–Jan. 1979	*Jan. 1970–Jan. 1978*
Food	9·5	15·4
Alcoholic drink	7·3	10·3
Tobacco	7·3	11·2
Housing	8·9	11·9
Fuel and light	9·1	14·0
Durable household goods	6·6	10·8
Clothing and footwear	6·5	10·7
Transport and vehicles	8·2	13·6
Miscellaneous goods	8·4	13·0
Services	9·0	13·2
Meals out	(Not available)	16·0
All items	8·5	13·1

Source: Retail Prices Index.
Note: The periods have been chosen to correspond as closely as possible with those in Tables 5 and 6.

prices to move broadly together, though over this period some differences are apparent. In particular prices of food, fuel, light and services have risen somewhat faster than the average, while those of clothing and footwear, durable household goods, alcoholic drink and tobacco have risen less fast. In broad terms therefore it might be reasonable to suppose that some households could be somewhat more affected by rising prices than others. In particular, the greater proportion of expenditure by low income households on food and fuel suggests that prices for these households could have risen a little faster than the general experience over the period. In individual years the variation in prices can be much greater, with the consequent possibility of greater divergence, in the short term, of the price experience of different households.

Results of some practical studies are now discussed. In passing, it might be noted that in index number work, theory and actual data march hand in hand in shedding light on issues and problems that arise.

Pensioner Indices

The official retail prices indices for one- and two-person pensioner households for the period 1962–79 are shown in Table 5 together with the

Table 5

Retail prices indices, excluding housing

		General index households		*One-person pensioner households*		*Two-person pensioner households*	
		Index	*% increase over previous year*	*Index*	*% increase over previous year*	*Index*	*% increase over previous year*
January 1962=100							
1962	QI	100·2		100·2		100·2	
1963	QI	103·1	2·9	104·4	4·2	104·0	3·8
1964	QI	104·1	1·0	105·4	1·0	105·3	1·3
1965	QI	108·9	4·6	110·4	4·7	110·5	4·9
1966	QI	113·3	4·0	114·3	3·5	114·6	3·7
1967	QI	117·1	3·4	118·8	3·9	118·9	3·8
1968	QI	120·2	2·6	122·9	3·5	122·7	3·2
1969	QI	128·1	6·6	129·4	5·3	129·6	5·6
1970	QI	134·5	5·0	136·9	5·8	137·0	5·7
1971	QI	146·0	8·6	148·5	8·5	148·4	8·3
1972	QI	157·4	7·8	162·5	9·4	161·8	9·0
1973	QI	168·7	7·2	175·3	7·9	175·2	8·3
1974	QI	190·7	13·0	199·4	13·7	199·5	13·9
January 1974=100							
1974	QI	101·5	13·0	101·1	13·7	101·1	13·9
1975	QI	123·5	21·7	121·3	20·0	121·0	19·7
1976	QI	151·4	22·6	152·3	25·6	151·5	25·2
1977	QI	176·8	16·8	179·0	17·5	178·9	18·1
1978	QI	194·6	10·1	197·5	10·3	195·8	9·4
1979	QI	211·3	8·6	214·9	8·8	213·4	9·0
Annual average			8·4		8·9		8·8

Source: Department of Employment.

corresponding index for general households. Details of their construction were given in the *DE Gazette* (1969) but the main difference from the General Index is that housing costs are excluded. The reason for this is that it is not possible to construct a satisfactory price indicator of these costs for a specific group of households (whereas it is possible for all households); a large proportion of the pensioner households receive rent or rate rebates. Another feature of the pensioner indices is that they reflect the different types and smaller pack sizes of some goods bought by pensioners. The problem of a separate housing indicator for pensioner and other household types is discussed in the next section.

The dominant feature of the table is the close similarity between the rates of increase of prices shown by all three indices. It is clear that the differences in spending patterns, and in price increases for different groups of goods and services, are insufficient to produce marked differences in price experience of the low-income pensioner households compared with the general experience. Over the 17 years the General Index (excluding housing) increased by an average of 8·4 per cent per annum, only 0·4–0·5 per cent less than the 8·9 and 8·8 per cent for the indices for one- and two-person pensioner households, respectively. Many of the figures in individual years are similarly close, though occasionally divergences can arise, for example the one-person pensioner index increased by 25·6 per cent in the year to the first quarter, 1976, compared with 22·6 per cent for the General Index households. In the previous year the relative movements went the other way, with the one-person pensioner index increasing by 20 per cent compared with 21·7 per cent for General Index households. Fluctuations in food prices were a factor in these movements.

If it had been possible to include housing in the pensioner indices, it is likely, from studies of the period since 1970, that the difference between the pensioner indices and the General Index would have narrowed. This is because of the slower rate of increase of housing costs for pensioner households, as a result of rate and rent rebates, compared with that for households in general. A more extensive examination of the effects of rising prices on low income households was published in the *DE Gazette* (1979).

Price Indicators for Different Household Types

Recent special studies by the Department of Employment also throw some light on the effect of rising prices on a wide range of household types, including those covered by the General Index, for the period 1970–77. These were published in the *DE Gazette* (1978, 1979). Special price indicators were constructed, using as weights expenditure data for different household types from the Family Expenditure Survey (FES)

and price data from the RPI. The indicators do not have the same precision as the RPI and indeed there are substantial differences in their construction but they give a good indication of the relative movements in prices between the household groups. The household groups were analysed by whether retired or not, and by household composition and size, occupation of household head and income.

The indicators for each year to 1977 are calculated by first revaluing at 1970 prices the expenditure for the year and then expressing the year's expenditure as a percentage of the revalued expenditure. The revaluation was performed at the section level – about 90 categories of expenditure corresponding to those of the FES data used for the RPI weights, such as gas, beef and veal, cheese, beer and cider, footwear, furniture, meals out. The price indicator I_{it} for household group i in period t is defined as:

$$I_{it} = \frac{\sum_j E_{ijt}}{\sum_j (E_{ijt}/P_{jt})}$$

where (a) E_{ijt} is the expenditure within the section j by the household group i in period t and (*b*) P_{jt} is the RPI price index for expenditure section j for period t relative to 1970.

The price indicators are implicit Paasche-type index numbers, as the index for each year (with 1970=100) is based on the current year's expenditure pattern. The RPI, on the other hand, is a chained Laspeyres index, with the expenditure weights updated each year from the latest available FES data. The same RPI section price index was used for each household group in revaluing expenditure in that section; strictly it might be preferable to have had a separate price index appropriate to each household group but this was not possible and is unlikely to have had much effect on the overall price indicator for a household group.

For housing, however, special procedures were adopted because in two major respects the RPI section price indices were clearly inappropriate as applying uniformly to expenditures for household groups (though they are valid for General Index households as a whole). In the absence of information to enable separate housing price indices to be constructed an assumption had to be made. The procedure chosen was to calculate an expenditure total for housing for each household group and then to assume that price increases accounted for the whole of the increase in expenditure, apart from the increase over the period in the volume (or quality) of housing purchased. This was estimated to be about 1 per cent per annum. The procedure is not wholly satisfactory but is preferable to omitting housing altogether and should help to reduce distortions in the comparisons between different household type. Price indicators inclusive and exclusive of housing were both calculated.

The reasons why the RPI section indices could not be applied to individual household groups were: (a) for owner-occupiers, the RPI has since 1975 included the mortgage interest payment net of tax relief as the section price index. This information is not available with sufficient accuracy from the FES so, for the special price indicators, expenditure was measured by an imputed rent, based on rateable value of the dwelling; this in fact was the practice with the RPI prior to 1975, and (b) rent and rate rebates, received direct by recipients from the local authorities rather than as part of social security, are treated in the RPI as a reduction in the cost of rent and rates and the RPI section indices therefore reflect the average reduction from this source. When considering the housing costs of different household groups, the reduction in rent and rates due to rebates varies, with the rebates directed at lower income families, so that the RPI section indices would be inappropriate.

Over the period 1970–77 prices increased more rapidly than the longer period considered earlier. Table 4 shows average annual percentage increases in the RPI group price indices. Prices of food, fuel and light and meals out increased more rapidly than average while those of tobacco, alcoholic drink, durable household goods and clothing and footwear increased less rapidly than average.

Table 6

Price indicators for types of household, 1977 (1970 = 100)

	Including housing	*Excluding housing*
All households	248	244
Retired households:		
1 or 2 adults	252	252
Non-retired households:		
2 adults:		
All	250	243
Quarter with lowest incomes	251	247
Quarter with highest incomes	248	242
2 adults, 1 or 2 children:		
All	249	243
Quarter with lowest incomes	248	246
Quarter with highest incomes	250	242
2 adults, 3 or 4 children:		
All	253	246
Quarter with lowest incomes	249	248
Quarter with highest incomes	248	246
Other compositions	247	244
Households whose head of household was:		
Professional, etc., employee	249	243
Clerical employee	250	242
Manual employee	247	243
Self-employed	248	244

Source: Department of Employment.

A summary of the price indicators for household groups for 1970–77 is shown in Table 6. What emerges clearly is that the dispersion between the price indicators for the different types of household is very small. For all households the 1977 indicator (inclusive of housing) is 248 (1970 = 100) and the range of values for the groups shown is 247–253, −0·5 to +2 per cent, around this figure. Exclusive of housing the range is a little wider, 242–252, or −1 to +3 per cent around an overall value of 244. For retired households the indicator, inclusive of housing, is 252, or 2 per cent above the overall figure. Retired households in this exercise are defined as those in which over half the total income comes from retired people. This group, accounting for 21 per cent of households in 1977, has wider coverage than that of the retail prices indices for one- and two-person pensioner households, as described above.

An Index with "Democratic" Weights Compared with the General RPI

The pattern of expenditure used in the general RPI is obtained by aggregating the expenditure of all index households. As explained above, these exclude only the low income pensioner households and those with the top 3–4 per cent of incomes. Aggregation of the expenditure of households in this way is the normal method of constructing national retail prices index numbers, and it means that the aggregate corresponds to the overall basket of goods and services bought by index households. Within this aggregate, expenditure of households with higher incomes has greater weight than that of households with lower incomes so that the overall aggregate expenditure has a pattern which corresponds to that for households above the centre of the range. As seen before from Table 1, the pattern of expenditure changes only very gradually in the middle ranges of income and there are only small differences in the pattern of expenditure over a substantial span of incomes in the central areas of distribution. The pattern for the RPI aggregate expenditure differs only moderately from that of households with average incomes. It follows that index numbers based on these two patterns of expenditure will differ only slightly.

Calculations show that if the overall pattern for the RPI were recalculated with equal weights to each household's pattern (the so-called "democratic" index), the pattern so calculated would differ only a little from that used in the RPI, and accordingly an index based on it would likewise differ little from the RPI. Over a period from January 1975 to January 1979, calculations based on somewhat simplified procedures show the democratic index to have increased by around 0·1 per cent per annum faster than the RPI. It is sometimes argued that the RPI should be weighted by this democratic method. But although it might be correct in a narrow technical sense to say that the pattern of aggregate expenditure on which the general RPI is based corresponds most closely to that of households with incomes above the middle of the income distribution of households, in fact the difference in this pattern from that of households with average incomes is slight, with a negligible effect on the index.

General Representativeness

The conclusion is, from both the theoretical considerations and the results of practical studies, that the RPI is representative of price changes experienced by the vast majority of households.

Combination of Price Quotations for Individual Items

The remainder of this paper is concerned with a weighting problem at the most detailed level of the index calculations, that is the method by which the price quotations obtained for individual items are combined. For example, up to 800 quotations for granulated sugar are collected up and down the country, from different kinds of shop, and the problem is how these should be combined to give an index for this item to be combined with indices for other items in building up the RPI. The quotations are handled in two stages. First they are stratified (according to region and form of retail organization) and then, within each stratum, the individual quotations have to be combined by an appropriate formula. These two stages, of stratification and of the formula to be used in each stratum, are discussed in turn. First, however, the broad structure of the RPI is described as background to the problem.

General Structure of the RPI

The retail prices index since its inception in modern form in 1962 has been constructed as a Laspeyres index with annual links. The weights of the 95 *sections* into which the RPI is divided are up-dated each year from the most recently available information from the Family Expenditure Survey (FES). This is a continuous survey of households, with about 7 000 cooperating households in a year supplying detailed information on their expenditure over a two-week period. The 95 sections define the "basket of goods" whose changing cost is measured by the RPI from month to month. For most sections, the weights are derived from a single year's FES expenditure data for the year ending in the June of the

previous year. The RPI sections depend on the FES classifications of expenditure and many are quite narrowly defined; for example, among foods, bread, beef, fish, fruit. Among many non-food sections, there is often a broader coverage, for example, furniture, radio/television/gramophones, men's outer clothing, men's footwear, medicines/surgical goods. Nevertheless, even for foods, for pricing purposes the sections cover a great many distinguishable articles purchased across the country. It is necessary to select a sample of representative items within each section for pricing each month. Relatively few items are selected for pricing within each of the sections, usually of the order of 5–10. They are selected as representatives of the range of articles in the section, on the basis that it covers relatively similar goods with relatively similar rates of price change. For example, the beef section has about seven items for pricing, from sirloin steak down to fore ribs, and men's outer clothing has about nine items from suits to jeans and overalls. The item indices are weighted together to form the section price index, using a variety of trade and other sources for the weights as the FES information is not generally sufficiently detailed. The full structure of the RPI is shown in Figure 1.

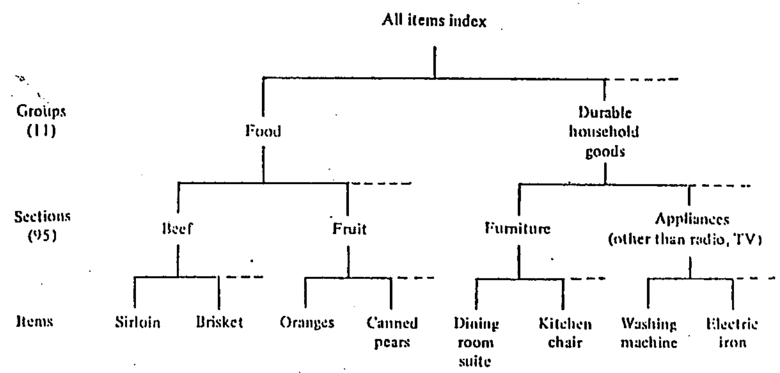

Figure 1

Price changes are measured in general by recording in 200 towns or local areas across the UK the price of the same article in the same retail outlet at each successive date of collection (normally the Tuesday nearest the middle of each month). It is the process of careful matching of articles and retail outlets for pricing that is the best guarantee of an accurate estimate of the "pure price changes", so as to exclude any element due to a change in the shop sample or in the quality of the article priced. The price quotations collected number about 150 000 each month.

Maintaining and improving the quality of the RPI involves keeping the selection of shops and of items up to date. As the market develops, some goods evolve with the appearance of more modern designs or, again, some goods disappear altogether while other completely new goods appear. Such developments as the growing importance of panti-hose, table wine or do-it-yourself home repairs are introduced in the RPI while items of reduced importance are dropped. There is a balance to be struck between the desirability of increasing the number of indicators and the additional cost. Shopping patterns also change over time, and the increasing importance of multiple retail stores, supermarkets and, more recently, hypermarkets has been reflected in the RPI shop sample, and in its method of compilation, as is discussed later.

The RPI is essentially a comparison between the current cost and the base cost of a fixed basket of goods and services reflecting consumption in the base period. In order to calculate the index the components of household expenditure are valued at current and January prices. The current value is obtained by multiplying the January expenditure by a price index measuring the change in price between January and the current month. This procedure is followed at successive levels of aggregation in the RPI. The Laspeyres index for all items is thus computed as a weighted average of price changes rather than as a ratio of two expenditures. That is, the Laspeyres formula for the price change between periods 0 and t:

$$L_{0t}=\frac{\sum_i P_{it}Q_{0t}}{\sum_i P_{0t}Q_{0t}}$$

is used in the form

$$L_{0t}=\sum_i W_{0t}\frac{P_{it}}{P_{0t}}$$

where

$$W_{0t}=\frac{P_{0t}Q_{0t}}{\sum_i P_{0t}Q_{0t}}$$

and P_{0t} Q_{0t} are the price and quantity respectively of item i in period 0.

Stratification of Price Quotations

In the scheme for collecting price quotations, there are three main types of sample: (a) large samples, often of 500–800 quotations, from 200 local offices; (b) medium samples, of 75–200 quotations, some from local offices, and some by postal enquiry, and (c) small samples of quotations from a few firms or even a single firm or body. In a number of other cases the price information is virtually complete, for example, postage, licence fees, gas and electricity charges.

In the case of the large samples, 500–800 quotations are obtained by about 200 local unemployment benefit offices of the Department of Employment, from up to five retailers in each town or local area. Where possible, in each area, one of the retailers is a local cooperative society, and another a branch of a chain store; self-service shops are also represented.

The combination of individual quotations in the past used to be a mixture of both ratios of averages and averages of relatives, usually with the ratios of averages at lower levels of aggregation, for example, within towns. Since 1966, when the processing was computerized, price relatives were combined at all the stages of aggregation. The price relatives for towns were divided into five groups:

(a) Greater London (25 local areas)
(b) Towns with a population of 200 000 or over (26 areas)
(c) Towns with a population of 50 000–200 000 (56 areas)
(d) Towns with a population of 10 000–50 000 (46 areas)
(e) Towns with a population of under 10 000 (47 areas).

The grouping by population and the selection of the localities within each population group were made in such a way as to give adequate representation to different types of localities throughout the country with some allowance for the rather lower purchasing power in the smaller towns. The price relatives for the towns within each group were averaged together and the price relatives for the five groups were averaged together, with equal weights.

In February 1978, a new stratification scheme for the price quotations was introduced, together with a change in the formula for combining the quotations. Making use of data from the FES and the 1971 Census of Distribution, the following possible stratification factors were considered:

(a) regions, either: (i) the 12 regions of the UK; or (ii) five cells, Scotland, Wales, Northern Ireland, Greater London and the rest of England;
(b) form of retail organization, multiples, cooperatives and independents (and possibly departmental stores for some goods);
(c) type of shop, supermarket, non-supermarket, self-service and counter service (and possibly departmental stores);
(d) town's size, the four groups noted above, excluding Greater London.

An analysis was undertaken to assess the variability of the prices and of the price changes with respect to these stratification factors, with a view to selecting those factors with the greater proportion of variation between, rather than within, the strata. The data on price levels suggested that form of retail organization was the most important factor, and

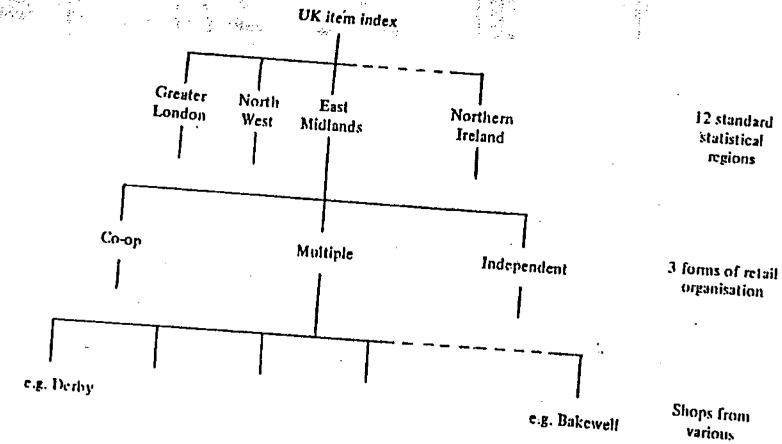

Figure 2

for some items regions were also significant, but that after taking these into account, there was no significant variation between the town size groups. The data on variation of price changes yielded less clear indications on the appropriate choice of strata. It was decided to adopt a stratification scheme using the 12 regions and three forms of retail organization. This stratification scheme is shown in Figure 2.

The Problem of Combining Price Quotations

Given the stratification of the samples of quotations for individual items in the RPI, it is necessary to consider the formula to be used in combining the quotations within each stratum. A stratum, in the case of large samples, might typically consist of some 20 quotations, representing a form of organization (for example, multiples, cooperatives or independents) within a region. Such a stratum might be the quotations for sugar in the North-West region sold by independent retailers. No information is available to weight these prices, that is the quantities associated with them. These will vary, although the variation will have been reduced to some extent by the stratification by form of organization.

The items in the index are of varying degrees of homogeneity. Some, such as sugar, butter and petrol, are relatively uniform whereas others, such as men's suits, can vary considerably in quality. Most of the argument which follows is concerned with the former kind but some reference is made subsequently to the problem of less homogeneous items.

Though the problem might appear very simple, this is far from the case. Surprisingly little work appears to have been done on it. The paper explores a number of considerations, both theoretical and practical, and

both simple and more complicated. It might perhaps be noted at the outset that the choice of formula does not make a large difference to the index but nevertheless in view of the importance of the RPI the problem needs to be examined thoroughly and the index compiled in the best possible way.

Two Simple Formulae

A simple practical first step, given that no information on weights is available, is to try using simple formulae with equal weights. It is in fact common practice in the absence of weighting information to use one of two simple formulae. These are: (a) the ratio of averages, A, =

$$\frac{(1/n)\sum P_t}{(1/n)\sum P_0}$$

and (b) the average of relatives, R, =

$$\frac{1}{n}\sum\left(\frac{P_t}{P_0}\right)$$

There are other, more complex, formulae but these have the practical merits of convenience and simplicity of concept.

It is found that the two formulae give different answers, and R is nearly always higher, though not necessarily by very much. An illustration of the differences can be seen in the top two lines of Table 7 (though these differences are bigger than what would occur in practice since they are not based on data which has been stratified).

The different answers require a choise to be made. It is necessary to ask why they differ, which is preferable and whether indeed some different formula should be used.

Starting from Laspeyres

If one starts with the Laspeyres formula as the basis for which an estimate is being sought, one has

$$L_{0t}=\frac{\sum_i P_{ti}Q_{0i}}{\sum_i P_{0i}Q_{0i}}=\sum_i W_{0i}\left(\frac{P_{ti}}{P_{0i}}\right)$$

where

$$W_{0i}=\frac{P_{0i}Q_{0i}}{\sum_i P_{0i}Q_{0i}}.$$

In the absence of information on the weights a simple and common assumption is to take them as equal. Equating the quantity weights

$Q_{0t} = Q$ for all i (and dropping the subscript i)

$$L_{0t} = A_{0t},$$

the ratio of averages,

$$= \frac{\sum p_t}{\sum p_0}$$

and equating the expenditure weight $W_{0t} = W$ for all i

$$L_{0t} = R_{0t},$$

the average of relatives,

$$= \frac{1}{n} \sum \left(\frac{p_t}{p_0}\right)$$

If it so happened that the balance or structure of the sample of quotations corresponded precisely to the structure of the universe with regard to either quantities or expenditures in the shops then equal weights would be entirely appropriate, using the ratio of averages in the former case and the average of relatives in the latter.

In practice the quantities or turnover appropriate to the shops are unknown. They are selected on more general criteria, so as to be as representative as possible of the required coverage, but there are practical constraints as the voluntary cooperation of retailers to the regular collection of prices is involved. For food items in the RPI, the shops are stratified by the form of retail organization to which they belong and this is related to turnover, and within a stratum, as food prices have a relatively small dispersion, the alternative weighting assumptions are equally well or poorly met; if quantities are roughly equal, so will expenditures be roughly equal.

One can also consider the implicit weighting assumption in the two formulae in the light of possible consumer behaviour. It was noted above that the ideal basis for selecting the shop sample is for it to be representative of the base period quantities or expenditure, but this is of necessity a matter of judgement as detailed sales data are not available. The RPI is a chained index and having taken a sample of shops the formula is applied to the data within each annual link of the Laspeyres index and chained to previous index values. In this situation equal quantity weights in the formula A would be repeated in each link, as would the equal expenditure weights in the formula R. These amount to somewhat different assumptions about the relationship over time between price and quantities.

Equal quantity weights imply that despite possibly different rates of price increase, the relative quantities sold remain the same. This is, in fact, equivalent to an unchained index. Equal expenditure weights

imply that for any increase in a relative price within a link, there will be a compensating decrease in the relative quantity sold. Neither assumption appears particularly satisfactory in a situation of substantial variations in price changes to which consumers may to some extent be reacting. If the sources of variation are temporary, this probably limits the scope for the consumer to react. However, given the uncertainty it cannot be said with confidence which of the two assumptions, of equal quantity weights or of equal expenditure weights, is to be preferred.

The relationship between the formulae and the Laspeyres index may be shown by deriving expressions involving the unknown quantity and expenditure weights as follows (see Appendix 1)

$$L_{0t} = A_{0t}\left[\frac{1 + C(P_t)C(Q_0)\rho(P_t, Q_0)}{1 + C(P_0)C(Q_0)\rho(P_0, Q_0)}\right] \quad (1)$$

$$L_{0t} = R_{0t}\left[1 + C\left(\frac{P_t}{P_0}\right)C(W_0)\rho\left(\frac{P_t}{P_0}, W_0\right)\right] \quad (2)$$

where C and ρ are respectively the coefficient of variation and the correlation coefficient. They show that the extent to which either formula can give a biased estimate of the Laspeyres index will depend on the relationships between the actual prices and the quantities sold at those prices and that the stratification and selection of the shops are particularly important. They do not, however, provide any clear-cut evidence to support the choice of one formula over the other.

This approach highlights the importance of weighting information and the benefits to be obtained from structuring the shops and their price quotations into strata with similar turnovers. If this is the case the coefficients of variation of Q and W, in equations (1) and (2) tend to be smaller and the potential for bias is reduced whichever formula is used.

Dispersion and Negative Correlation

Before developing further the analysis of the problem an important feature of the price data – their dispersion – needs to be discussed. One factor giving rise to dispersion is the variation from one shop to another in timing of the implementation of price increases. This produces temporary dispersion in the array of quotations. In addition, temporary price cutting occurs, causing the quotations for individual shops to ebb and flow about some longer term mean or trend level. Or again, there may be variation in the goods for which prices are collected, or errors of observation or reporting, notwithstanding the precautions taken to avoid these things happening. This short-term fluctuation about the longer-term position for each shop we describe as "noise", by analogy with the fuzz on a radar screen.

In addition to these zones of variation about individual shop means, there will be dispersion of the shop means about the overall mean. For reasonably homogeneous items such as food, and especially in cells involving cooperatives and multiples where regional pricing policies may be pursued, the shop means will be close to the overall mean but for less homogeneous items – men's suits say – there will be a wider spread. So the dispersion can be thought of as having two components, that reflecting the longer term tendency for individual shop means to differ from the general mean and a shorter term fluctuation about the longer term position for each shop.

This ebb and flow, or "hunting", of prices about a trend position, which we have called "noise", clearly involves increases in price being more likely in prices currently relatively low and reductions (or relatively smaller increases) more likely in prices currently relatively high. This *negative correlation* between price changes and price levels is an observed feature of the data which applies to many items. The correlation is hardly ever positive, at least for long. This is perhaps not very surprising, because a persistently positive correlation would mean that the dispersion of price quotations would increase indefinitely.

For food items, the price dispersion within an RPI item in a particular month, as measured by the coefficient of variation, is typically 5–15 per cent and for some foods such as fresh vegetables and some meat may be 25 per cent. For non-food items with wider quality differences the dispersion may be much greater.

In practice the two formulae A and R, when applied to the same price data, usually give different results. The difference is frequently significant and usually R exceeds A. The formulae are related as follows:

$$A=R\left[1+C\left(\frac{P_t}{P_0}\right)C(P_0)\rho\left(\frac{P_t}{P_0},P_0\right)\right].$$

They are equal only if one of the coefficients of variation is zero or if the correlation term is zero. R exceeds A when the correlation between the base period prices P_0 and the price relative (P_t/P_0) is negative and this indeed can be expected to be the case when there are random fluctuations in the prices.

Effects of Price Dispersion

The formulae are now considered in relation to price dispersion. The prices P_{0t} and P_{tt} may be written as follows:

$$P_{0t}=\bar{P}_0(1+x_{0t})$$
$$P_{tt}=\bar{P}_t(1+x_{tt})$$

The x_{0t} reflect the dispersion of the prices relative to the mean price $\bar{P}_0$. The standard deviation of x_{0t} is the coefficient of variation of P_{0t}. The x_{tt} are the corresponding values at time t. If the price dispersion is due solely to quality variation and the prices are all increasing at the same rate, $x_{0t}=x_{tt}$. If the price dispersion is due solely to random variation then x_{0t} and x_{tt} are independent random variables.

The two formulae are related in the following approximate expression. This is derived (see Appendix 1) from the Maclaurin expansion by neglecting terms in x^3 or higher powers, a reasonable assumption for many items in practice.

$$\frac{R_{0t}}{A_{0t}}=1-\frac{1}{n}\sum\frac{x_0(1+x_t)}{1+x_0}$$

(dropping the subscripts i)

$$\simeq 1+\frac{1}{n}\sum(x_0^2-x_0x_t)$$

$$=1+\Delta \qquad (3)$$

where R_{0t} is the average of relatives and A_{0t} is the ratio of averages.

If a significant part of the price dispersion arises from noise, $\sum x_0^2 \gg \sum x_0x_t$ and $R_{0t}/A_{0t}\simeq 1+(1/n)\sum x_0^2$. R_{0t} would exceed A_{0t} by a term $(1/n)\sum x_0^2$, which depends only on the price dispersion in the base period. A typical price dispersion of 10 per cent (as measured by the coefficient of variation) leads to R_{0t} exceeding A_{0t} by up to 1 per cent.

The effects of price dispersion when using the formulae A_{0t} and R_{0t} in both chained and unchained indices can now be compared. The chain index between time 0 and time t using A is:

$$CA_{0t}=\frac{\sum P_1}{\sum P_0}\cdot\frac{\sum P_2}{\sum P_1}\cdots\frac{\sum P_t}{\sum P_{t-1}}=\frac{\sum P_t}{\sum P_0}$$

and using R is:

$$CR_{0t}=\frac{1}{n}\sum\left(\frac{P_1}{P_0}\right)\frac{1}{n}\sum\left(\frac{P_2}{P_1}\right)\cdots\frac{1}{n}\sum\left(\frac{P_t}{P_{t-1}}\right)$$

If the dispersion of prices is similar from one period to the next, so that the noise term for each link of the chain is Δ, then

$$\frac{CR_{0t}}{CA_{0t}}\simeq(1+\Delta)^t$$

This indicates that the chained index based on the average of relatives would diverge indefinitely from that based on the ratio of averages.

The chained index using the average of relatives formula would also diverge from the unchained index using the same formula:

$$\frac{CR_{0t}}{R_{0t}} \simeq \frac{(1+\Delta)^t}{1+\Delta}$$

This property of the chained average of relatives formula can be shown to be undesirable most easily if it is assumed that there is no trend in prices. Suppose all the prices in all the periods are above a lower level PL and below a higher level PH. The maximum price increase possible by any standpoint is PH÷PL. The chained index using the ratio of averages formula would be bounded, but the chained index using the average of relatives formula would exceed the bound after sufficient time periods. A similar argument would apply if there is also a price trend. It is interesting to note in passing that the unchained index using the formula R is less affected by noise than the chained index. The index (R_{0t}) of price change involving the base period prices is affected, as seen from equation (3), but the effect tends to disappear from the index (R_{st}) of the price change between two subsequent periods:

$$R_{st} = \frac{R_{0t}}{R_{0s}} \simeq \frac{A_{0t}(1+\Delta)}{A_{0s}(1+\Delta)} \simeq A_{st}$$

The two formulae may be compared in relation to Irving Fisher's time reversal test (1922). This test requires that when an index is calculated forwards and backwards in time, the result should be unity:

$$I_{0t}I_{t0} = 1$$

It can be seen that the ratio of averages passes the test:

$$A_{0t}A_{t0} = \frac{\sum P_t}{\sum P_0} \cdot \frac{\sum P_0}{\sum P_t} = 1$$

whereas the average of relatives does not:

$$R_{0t}R_{t0} = \frac{1}{n}\sum\left(\frac{P_t}{P_0}\right)\frac{1}{n}\sum\left(\frac{P_0}{P_t}\right) \neq 1$$

Moreover the product $R_{0t}R_{t0}$ is always greater than unity except when there is zero dispersion among the price relatives. Using the approximate formulae derived above

$$R_{0t}R_{t0} \simeq (1+\Delta)^2$$

It may be noted at this point that many types of index numbers used in practice do not pass this test, particularly, the Laspeyres and Paasche indices, i.e.

$$L_{0t}L_{t0} = \frac{\sum P_tQ_0}{\sum P_0Q_0} \cdot \frac{\sum P_0Q_t}{\sum P_tQ_t} \neq 1$$

and

$$P_{0t}P_{t0} = \frac{\sum P_tQ_t}{\sum P_0Q_t} \cdot \frac{\sum P_0Q_0}{\sum P_tQ_0} \neq 1$$

It is not the purpose of the paper to debate the relative merits of these forms of index numbers where the quantity weights are available and where other considerations apply. Nevertheless on common-sense grounds, and other things being equal, a measure of price change at the stratum level that does not satisfy the test would seem less attractive.

Difference Between R and A in Terms of Implied Weights

It is also possible to look at the difference between R and A by reference to their implied weights.

The two formulae can be written:

$$R = \frac{1}{n}\sum\left(\frac{P_t}{P_0}\right) = \frac{1}{\sum P_0}\sum\left(\frac{\sum P_0}{nP_0}\right)P_t$$

and

$$A = \frac{(1/n)\sum P_t}{(1/n)\sum P_0} = \frac{1}{\sum P_0}\sum P_t$$

In this form, the difference lies in the coefficients given to P_t in the numerator; whereas in A they are equal, in R they are not quite equal but represent a systematic set of considerable interest, in which quotations with a low P_0 are given somewhat enhanced weight and those with a high P_0 are given somewhat reduced weight.

An initial point to note about the weights is that whether or not the weights of individual quotations should be varied depends, as pointed out earlier in the paper, on the balance of the sample in relation to the universe, e.g. whether bigger outlets (which often have lower prices) are under- or over-represented. This is a matter which it is hoped to examine in due course (by the Department, not in this paper). In the meantime, however, the systematic arrangement of weights so that they are inversely proportional to prices as they happened to be in January, and then rearranged in successive Januaries, appears highly arbitrary in relation to the balance of the sample.

A major feature of the average of relatives formula is that the implied coefficients of P_t as derived above, have a systematic interaction with the observed negative correlation of price changes and price levels. The weights are systematically constrained so that, for example, greater weight is given to quotations for which the January price is relatively low while at the same time it is known that such quotations are likely to increase relatively more (and vice versa for relatively high January prices). In circumstances where "noise" is present, when prices temporarily diverge from their trend position and then return to it, the end position on prices is, by definition, the same as at the beginning. This would not show, however, in the case of the average of relatives formula, because, as one passed January, the weights would be changed in the systematic way already described and this would lead, in respect to those price fluctuations in the noise cycle which take place after January, to a higher measure of change than if the weights had been left unchanged. This problem is avoided if the weights are kept constant over the noise cycle as would be the case if the ratio of averages were used.

It would of course be possible to reduce the negative correlation, and hence the amount by which the average of relative exceed the ratio of averages, if, instead of calculating the relatives in relation to the latest January each year, they were taken back over a period of years. However it would not be entirely eliminated and at the same time other problems would be introduced, for example, how to justify assessing the change in prices since the latest January as the ratio of two movements from a more distant year. Another way to reduce the noise in the base period would be to take the average prices over a year instead of at the single point in January. This would still mean however that the weights would have the systematic feature already described. The question of weighting for individual shops needs investigation.

Formulae Other than A and R

This paper has focused attention on the average of relatives and ratio of averages formulae because they arise naturally out of the Laspeyres form of index number, they are commonly used, they are relatively simple to understand and are easy to compute. There are however other possible formulae. In particular the geometric mean (G) of price relatives (or the ratio of the geometric means of the prices) is sometimes used by index compilers.

Consideration of the Paasche and Fisher forms of index number yields two further possibilities, the harmonic mean (H) of the price relatives and the geometric mean ($\sqrt{HR}$) of the mean and harmonic mean of the price relatives. The derivation of these formulae and their relation to the Paasche and Fisher indices is given in Appendix 1.

The behaviour of these formulae can be considered in the situation of price dispersion in a similar manner to that developed above for A and R. Approximate relationships between each and the ratio of averages (A) can be derived using MacLaurin's expansion and ignoring terms in x (the relative price dispersion) in powers of 3 or greater.

These are:

(a) The geometric mean (G)

$$\frac{G}{A} = \left(\Pi \frac{1+x_t}{1+x_0}\right)^{1/n}$$

$$\simeq 1 + \frac{1}{2n} \sum (x_0^2 - x_t^2)$$

(b) The harmonic mean of the price relatives (H)

$$\frac{H}{A} = 1 \bigg/ \left[1 - \frac{1}{n} \sum \frac{x_t(1+x_0)}{1+x_t}\right]$$

$$\simeq 1 - \frac{1}{n} \sum (x_t^2 - x_0 x_t)$$

(c) The geometric mean of H and R

$$\frac{\sqrt{HR}}{A} \simeq 1 + \frac{1}{2n} \sum (x_0^2 - x_t^2)$$

It may be concluded from these approximations that: (i) G and $\sqrt{HR}$ are approximately equal and, provided the dispersions at the two times are roughly equal, differ by little from A. It is unlikely that there would be a sustained difference in the dispersions, though short-term variations would occur, and (ii) if the cross-product term $\sum x_0 x_t$ is less than the variance term $\sum x_0^2$, which is likely to be the case, especially where the dispersions x_0 and x_t are comparable, (though on occasion A may be larger), H will usually be less than A.

Table 7 gives the index numbers calculated by the five different formulae for a selection of RPI items typical of those with large samples of price quotations. The table confirms the expected results and for these items the average of relatives (R) is always the largest and the harmonic mean of relatives (H) always the smallest. The geometric mean and $\sqrt{HR}$ are almost identical and neither differ much from A. Since G and $\sqrt{HR}$ are more cumbersome to compute, are not commonly used, and are less comprehensible to the general public this paper has concentrated on the choice between R and A. It should be noted that the prices used in compiling Table 7 were drawn from the national sample of prices and were not stratified. The effect of this will have been to exaggerate the dispersions and therefore the differences between the formulae.

Another Consideration

An important consideration in the construction of the retail prices index or any other consumer price index is that the resultant index should be

Table 7

Comparison of item indices based on alternative formulae January 1974 (January 1973 = 100)

Formula	*Flour*	*Instant coffee*	*Pork sausages*	*Large eggs*	*Standard eggs*	*Bread*	*NZ butter*	*Beef sirloin*
1. Average of relatives (R)	133·02	110·78	118·07	218·07	235·25	127·00	95·98	106·74
2. Ratio of averages (A)	131·78	110·57	117·62	214·96	233·34	126·76	95·62	105·88
3. Geometric mean (G)	131·56	110·36	117·59	216·43	234·14	126·79	95·64	105·76
4. Geometric mean of R and H ($\sqrt{HR}$)	131·58	110·37	117·59	216·43	234·13	126·79	95·64	105·78
5. Harmonic mean of relatives (H)	130·16	109·96	117·07	214·69	233·01	126·57	95·30	104·82

Source: Sample of price data collected for the RPI.

acceptable to the general public. This acceptability applies not only to the "all items" index but also to the individual item indices. Whilst for many items such as men's suits, where prices reflect substantial variations in quality, the concept of an average price has little meaning, for others such as sugar there is a general interest in the average price (per pound). It is not unreasonable to expect the change in prices as measured by the item indices to be consistent with the changes in these average prices.

The Department publishes average prices for a selection of food items and it is desirable from a presentational point of view that the price indices for these items should be consistent with the average prices over time, especially in the longer term. From the foregoing analysis of the behaviour of the two formulae it is clear that if the average of relatives formula is used to construct stratum indices within a chain-linked item index then the resultant index is likely to move progressively above the average price over a number of links. The use of the ratio of averages formula for constructing stratum indices would produce an item index that would move closely in line with the average price in both the short and long term.

One reason why average prices are avoided in the compilation of price indices is the possibility that the sample of prices collected in the current period may not cover the same quantity/quality mix of goods as covered by the base period sample. This problem can be avoided in calculating stratum indices by ensuring that the ratio is formed from "matched" average prices. That is, the current prices are taken from the same shops and for the identical items as in the base period; if a particular price quotation is not obtained in one month the corresponding price is removed from the base period average. All the prices obtained can nevertheless be used to calculate the average price for any item in a particular month and accordingly slight differences might occasionally occur between movements in these averages and the item indices.

Choice Between the Formulae A and R

To summarize the arguments so far, which have related essentially to relatively homogeneous items, the dominant factor is the negative correlation, that is, for many items, relatively low prices in the base period are associated with above average price rises and relatively high prices are associated with below average price rises. The effect of this negative correlation between price levels and price changes on the two commonly used formulae, the average of relative prices (R) and the ratio of average prices (A), is that R exceeds A. Empirical evidence suggests that this is usually the case in practice. It has been shown that a chained index based on the average of relatives would diverge upwards indefinitely from an index based on the ratio of average prices. This clearly is an undesirable feature of R. Other aspects of the two formulae and their behaviour which have been examined are generally less conclusive but are not inconsistent with a preference for A rather than R, nor do other formulae offer advantages over A. There is the further advantage that the ratio of averages formula provides item indices consistent with the long run movements in average prices.

Consideration of Less Homogeneous Items

The case of less homogeneous items is now considered, that is where there are two sources of dispersion of prices. The first source is the "noise" discussed in the previous sections. The second source is a longer term tendency for a quotation from a particular outlet to be higher (or lower) than the stratum mean because for example, either the outlet sells the same article but consistently at relatively high or low prices or because the quality of the article covered by the quote is above or below average (the specification of some of the items for which quotations are obtained is decided locally). The price P_{0tj} is considered to have three components:

$$P_{0tj}=\bar{P}_{0\cdot\cdot}+(\bar{P}_{0t\cdot}-\bar{P}_{0\cdot\cdot})+(P_{0tj}-\bar{P}_{0t\cdot})$$

$$=\bar{P}_{0\cdot\cdot}(1+y_{0t}+e_{0tj})$$

The variation of P_{0tj} about the stratum mean is expressed as the random variation within a group of closely related quotations about a common subsidiary mean $\bar{P}_{0t\cdot}$ (resulting in the term e_{0tj}) and the variation of the means $\bar{P}_{0t\cdot}$ about the stratum mean (the terms y_{0t}). The previously derived relationship between the two formulae R and A becomes, in cases where the price dispersion is not too large and the approximation justified:

$$\frac{R}{A}\simeq 1+\frac{1}{n_y}\sum(y_{0t}^2-y_{0t}y_{tt})+\frac{1}{n_y}\sum\frac{1}{n_e}\sum(e_{0tj}^2-e_{0tj}e_{ttj})$$

With closely defined items such as sugar, there is no "quality" variation and the stratification removes much of the persistent variation of price level between outlets so that the term in the e's dominates the term in the y's. The cross product term ($e_{0tj}e_{ttj}$) tends to be small relative to the term e_{0tj}^2 and the expression is similar to that in x_{0t} discussed in the previous section. Where prices are dispersed because of quality variation, both the terms are likely to be significant. The cross product term $y_{0t}y_{tt}$, though substantial and positive, will not usually entirely offset the term y_{0t}^2, and the expression involving noise will also be present.

Thus in the case of these less homogeneous items in as far as noise is present the ratio of average formula A is still to be preferred. As regards other sources of price dispersion the average of relatives R has traditionally been used to prevent a few high prices from dominating the index; in a given sample or stratum A gives a higher weight than R to changes in the higher prices. However the disadvantages of R cannot be ignored in this situation.

This aspect of the problem requires further examination; it is however, a problem of stratification rather than a question of formula. If the major sources of price variation can be identified and a suitable stratification scheme drawn up then, providing approximate strata weights can be found, the strata indices can be computed using A. Alternatively it may be possible to ensure that the sample of price quotations is appropriately balanced and approximately self weighting. In this way the likelihood of A over weighting changes in the higher prices is reduced.

Application to the RPI

The methods by which the RPI item indices have been compiled in the past grew largely out of a series of decisions taken by the Technical Working Party of the RPI Advisory Committee. Over the period of the index both the ratio of averages and the average of relatives formulae have been used. The ratio of averages was used to compile town price indices until the work was computerized and separate price relatives were calculated for each shop. Until recent years the differences between the formulae were of little significance; prices were relatively stable with fewer and smaller price increases. However, with more rapid rates of inflation the difference between the two has become noticeable for some items.

Studies by the Department in early 1977 revealed opportunities for improving the method of construction of the individual item indices by taking advantage of the accumulated experience of the previous system and by making use of certain information that had become available from the Census of Distribution and from the Family Expenditure Survey. The development in grading and branding of products had also made it possible to obtain prices for closely defined items.

In the autumn of 1977 the RPI Advisory Committee agreed that the stratification of price quotations by form of retail organization and geographical region, for which weighting information could be obtained (from the above sources), would yield more reliable estimates of price changes than hitherto. They also agreed, on the basis of arguments presented in this paper, that the ratio of averages formula should be used in the case of homogeneous items to combine the price quotations within the new strata. The Committee recognized that the arguments in favour of the ratio of averages formulae also applied in the case of non-homogeneous items but felt that other considerations were important in these cases and further investigations were necessary.

The effect of the changes on the index has been calculated for the period January 1975 to January 1977. Over that period the "all items" index calculated on the old method increased by 43·7 per cent. If the proposed new stratification and formula had been used for the homogeneous items to which it is now applied the increase in the "all items" index would have been 43·3 per cent. Thus the effect on the "all items" index is small, though there are larger effects on individual item indices.

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Appendix 1 Derivation of Formulae

(a) The Laspeyres Index

$$L_{0t}=\frac{\sum_i P_{ti}Q_{0t}}{\sum_i P_{0t}Q_{0t}}=\sum_i W_{0t}\left(\frac{P_{tt}}{P_{0t}}\right)$$

where

$$W_{0t}=\frac{P_{0t}Q_{0t}}{\sum_i P_{0t}Q_{0t}}$$

Using the standard result that

$$\frac{1}{n}\sum xy=\bar{x}\bar{y}[1+C(x)C(y)\rho(x,y)]$$

where C and ρ are the coefficient of variation and correlation coefficient, respectively, L_{0t} may be expressed in terms of the ratio of averages formula A_{0t}:

$$L_{0t}=\frac{\bar{P}_t\bar{Q}_0}{\bar{P}_0\bar{Q}_0}\frac{[1+C(P_t)C(Q_0)\rho(P_t; Q_0)]}{[1+C(P_0)C(Q_0)\rho(P_0, Q_0)]}-\frac{A_{0t}[1+C(P_t)C(Q_0)\rho(P_t, Q_0)]}{1+C(P_0)C(Q_0)\rho(P_0, Q_0)}$$

L_{0t} may also be expressed in terms of the average of relatives formula R_{0t}:

$$L_{0t}=\sum W_0\left(\frac{P_t}{P_0}\right)=n\bar{W}_0\overline{\left(\frac{P_t}{P_0}\right)}\left[1+C(W_0)C\left(\frac{P_t}{P_0}\right)\rho\left(W_0,\frac{P_t}{P_0}\right)\right]$$

$$=R_{0t}\left[1+C(W_0)C\left(\frac{P_t}{P_0}\right)\rho\left(W_0,\frac{P_t}{P_0}\right)\right]$$

(b) The Paasche Index

$$S_{0t}=\frac{\sum_i P_{ti}Q_{ti}}{\sum_i P_{0t}Q_{ti}}=\frac{1}{\sum_i W_{ti}(P_{0i}/P_{ti})}$$

Similarly, the Paasche index is related to the ratio of averages formula:

$$S_{0t}=\frac{A_{0t}[1+C(P_t)C(Q_t)\rho(P_t, Q_t)]}{1+C(P_0)C(Q_t)\rho(P_0, Q_t)}$$

and to the harmonic mean of relatives formula, H_{0t} where

$$H_{0t}=\frac{n}{\sum(P_0/P_t)}$$

$$S_{0t}=\frac{H_{0t}}{1+C(W_t)C(P_0/P_t)\rho(W_t, P_0/P_t)}$$

(c) The Fisher Index

$$F_{0t}=\sqrt{L_{0t}S_{0t}}$$

may be related to the ratio of averages formula:

$$F_{0t}=A_{0t}\left[\frac{1+C(P_t)C(Q_0)\rho(P_t, Q_0)}{1+C(P_0)C(Q_0)\rho(P_0, Q_0)}\times\frac{1+C(P_t)C(Q_t)\rho(P_t, Q_t)}{1+C(P_0)C(Q_t)\rho(P_0, Q_t)}\right]^{1/2}$$

It may be related to the two formulae R and H:

$$F_{0t}=\sqrt{HR}\left[\frac{1+C(W_0)C(P_t/P_0)\rho(W_0, P_t/P_0)}{1+C(W_t)C(P_0/P_t)\rho(W_t, P_0/P_t)}\right]^{1/2}$$

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(d) Relationships Among the Formulae A, R, G, H

(i) A and R

$$A=\frac{\sum P_t}{\sum P_0}=\sum\left(\frac{P_0}{\sum P_0}\right)\left(\frac{P_t}{P_0}\right)=\sum\left(\frac{1+x_0}{n}\right)\left(\frac{1+x_t}{1+S_0}\right)\frac{\bar{P}_t}{\bar{P}_0}$$

where

$$P_t=\bar{P}_t(1+x_t)$$

$$P_0=\bar{P}_0(1+x_0)$$

Rewriting the standard result used above as:

$$\frac{1}{n}\sum zY=\bar{z}\bar{Y}\left[1+\frac{1}{n}\sum\left(\frac{z-\bar{z}}{\bar{z}}\right)\left(\frac{y-\bar{y}}{\bar{y}}\right)\right]$$

$$A=R\left[1+\frac{1}{n}\sum\left\{\frac{x_0}{R}\left(\frac{1+x_t}{1+x_0}A-R\right)\right\}\right]$$

$$=R\left(1+\frac{A}{nR}\sum\frac{x_0(1+x_t)}{1+x_0}\right)$$

therefore

$$\frac{R}{A}=1-\frac{1}{n}\sum\frac{x_0(1+x_t)}{1+x_0}$$

(ii) *A and H*

$$H=n\Big/\sum\left(\frac{P_0}{P_t}\right)$$

Similarly to (i) above:

$$A^{-1}=\frac{\sum P_0}{\sum P_t}=H^{-1}\left(1+\frac{H}{nA}\sum\frac{x_t(1+x_0)}{1+x_t}\right)$$

therefore

$$\frac{H}{A}=1\Big/\left(1-\frac{1}{n}\sum\frac{x_t(1+x_0)}{1+x_t}\right)$$

(iii) *A and G*

$$G=\left(\Pi\frac{P_t}{P_0}\right)^{1/n}=\frac{\bar{P}_t}{\bar{P}_0}\left(\Pi\frac{1+x_t}{1+x_0}\right)^{1/n}=A\left(\Pi\frac{1+x_t}{1+x_0}\right)^{1/n}$$