

**AXIOMATIC AND ECONOMIC APPROACHES TO
ELEMENTARY PRICE INDEXES**

by

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Abstract:

The paper explores various approaches to the problems involved in collecting and aggregating price and quantity information at the lowest level of aggregation. The axiomatic approaches of Eichhorn and Dalén are explored as are the economic approaches of Reinsdorf and Moulton. The unit value approach of Walsh and Davies is recommended if price and quantity information is available. Finally, the paper catalogues various sources of bias in consumer price indexes and concludes that U.S. consumer price inflation had an upward bias in the .75 to 1.7% per year range during the 1980s.

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1. INTRODUCTION

"Though the problem might appear very simple, this is far from the case. Surprisingly little work appears to have been done on it."

A.G. Carruthers, D.J. Sellwood and P.W. Ward [1980; 16]

"There is an abundant literature, both theoretical and descriptive, on the computation of consumer price indexes above the basic aggregation level, but little is written about their derivation below that level. In this respect, the index makers resemble those chefs who only allow their dishes to be presented to patrons at a certain stage of preparation, without sharing how they have been mixed and simmered in the kitchen."

Bohdan J. Szulc [1987; 11]

In an important paper, Marshall Reinsdorf [1993] used Bureau of Labor Statistics data to compare the growth of average prices in the U.S. with corresponding official Consumer Price Index growth rates. He found that the official index for food showed average annual increases during the 1980's of 4.2% per year while the weighted mean of average prices grew at only 2.1% per year. For gasoline, Reinsdorf found that average prices fell during the 1980's about 1% per year more than the official CPI components for gasoline. Thus it appeared that the CPI components for food and gas were biased upwards by about 2% and 1% per year respectively during the 1980's.

Reinsdorf [1993; 246] attributed the above results to outlet substitution bias; that is, consumers switched from traditional high cost retailers to new discount stores in the case of food and to self serve gas stations from full service stations in the case of gasoline. The existing methodology used by statistical agencies in compiling price indexes does not pick up this shift of purchasers from high to low cost suppliers.¹

The more recent paper by Reinsdorf and Moulton [1994]² presents an alternative explanation for Reinsdorf's earlier results: when the BLS moved to probability sampling of prices in 1978, the micro price quotations were aggregated together using an index number formula that generates an upward bias. In section 2 below, we discuss index number formulae that are used to aggregate prices at the finest level of disaggregation and we provide Irving Fisher's [1922; 383] intuitive

explanation for the Reinsdorf-Moulton empirical results.

In section 3 below, we draw on the work of Eichhorn [1978] and pursue Dalén's [1992] axiomatic approach for finding an appropriate functional form for an elementary level price index. These price indexes depend only on prices for the two periods under consideration, whereas traditional bilateral index number theory depends on prices and quantities pertaining to the two periods. We find that the ratio of average prices formula appears to be the elementary price index counterpart to the Fisher [1922] ideal index of bilateral index number theory.

In section 4, we add additional axioms to those which were introduced in section 3 to obtain new axiomatic characterizations for the Jevons (geometric) and Dutot (ratio of average prices) elementary price indexes.

In section 5, we use the economic exact index number approach to obtain (weak) justifications for the use of the Jevons and Dutot indexes.

In section 6, at the individual outlet level, we endorse the approach suggested by C.M. Walsh [1921a; 88] and George Davies [1924] [1932] who advocated the use of unit values to represent average prices during a single period. This approach to aggregation at the elementary level was also suggested by Szulc [1987; 13] and Dalén [1992; 135]. The computer revolution makes the Walsh-Davies approach more feasible to implement.

In section 7, we briefly discuss sampling problems.

In section 8, we review some of the empirical literature that calculated alternative elementary price indexes and we find that the theoretical inequalities between these indexes that were first derived by Fowler [1978; 356] and Carruthers, Sellwood and Ward [1980; 20-21] seem to hold in practice.

In section 9, we briefly review the recent literature on sources of bias in consumer price indexes.

Section 10 concludes with a number of recommendations to Statistical Agencies.

2. THE PROBLEM OF AGGREGATING PRICE QUOTES AT THE LOWEST LEVEL

"Who ever heard, for instance, of Carli and of Dutot as authorities on the subject?"
F.Y. Edgeworth [1901; 404] commenting on C.M. Walsh [1901]

In order to provide an intuitive explanation for the empirical results of Reinsdorf and Moulton [1994], it is necessary to introduce a bit of notation and define a few index number formulae. We assume that the Statistical Agency is collecting price quotations on a commodity at the lowest level of aggregation where information on quantities purchased is not available.³ Assume that the physical and economic characteristics of the good are homogeneous and that N price quotes on it are collected in periods 0 and 1 respectively. Denote the period t vector of price quotes as $p^t = [p_1^t, p_2^t, \dots, p_N^t]$ for $t = 0, 1$. Define an elementary price index as a function of the 2N prices $[p_1^0, \dots, p_N^0; p_1^1, \dots, p_N^1] = [p^0; p^1]$. Examples of specific functional forms for elementary price indexes are⁴:

- (1) $P_{CA}(p^0, p^1) = \sum_{n=1}^N (1/N)(p_n^1/p_n^0)$;
- (2) $P_{JE}(p^0, p^1) = \prod_{n=1}^N (p_n^1/p_n^0)^{1/N}$;
- (3) $P_{DU}(p^0, p^1) = \sum_{n=1}^N (1/N)p_n^1 / \sum_{n=1}^N (1/N)p_n^0$.

P_{CA} is the arithmetic mean of the price ratios p_n^1/p_n^0 (first suggested by Carli [1804] in 1764); P_{JE} is the geometric mean of the price ratios (first suggested by Jevons [1884] in 1863) and P_{DU} is the arithmetic mean of period 1 prices divided by the arithmetic mean of period 0 prices (first suggested by Dutot [1738]).

Reinsdorf and Moulton [1994] point out that the starting point for the BLS method of aggregating elementary price quotes resembles the Carli price index P_{CA} defined by (1).⁵ In actual BLS practice, a more complicated formula than (1) is used,⁶ but as a very rough approximation, we can say that the elementary components of the U.S. CPI are computed using (1).

Reinsdorf and Moulton [1994] used official U.S. BLS aggregation techniques to construct consumer price index components for June 1992 to June 1993 and they compared these simulated components to corresponding indexes that aggregated the elementary level price quotes using the geometric mean formula (2). Omitting housing, they found that their simulated "official" CPI

exceeded the corresponding geometric mean CPI by about .5% for the year.⁷

Of course, if precisely (1) and (2) were being compared, we would always have

$$(4) \quad P_{CA}(p^0, p^1) \geq P_{JE}(p^0, p^1)$$

since an arithmetic mean is always equal to or greater than the corresponding geometric mean.⁸

Moreover, the less proportional that prices are in the two periods (i.e., the more variable are prices), the greater the inequality in (4) will be.

It is likely that the inequality (4) explains a large portion of the empirical results in Reinsdorf and Moulton [1994]. However, at this stage, it is not clear why we should prefer the geometric average of the price relatives to the corresponding arithmetic average.

An explanation for our preference can be found in the work of Dalén [1992] who adapted the traditional bilateral test approach to index number theory⁹ to the present situation where information on quantities is missing. Dalén [1992; 138] suggested that a reasonable functional form P for an elementary price index should satisfy the following time reversal test:¹⁰

$$(5) \quad P(p^0, p^1)P(p^1, p^0) = 1;$$

i.e., if prices in period 2 are identical to prices in period 0, then the price change going from period 0 to 1 should be exactly offset by the price change going from period 1 to 2. It can be verified that the geometric mean price index P_{JE} defined by (2) satisfies (5) but the arithmetic mean price index P_{CA} defined by (1) will be biased upwards; i.e.,

$$(6) \quad P_{CA}(p^0, p^1) P_{CA}(p^1, p^0) \geq 1,$$

with a strict inequality if p^0 is not proportional to p^1 .¹¹ Irving Fisher [1922; 66 and 383] seems to have been the first to establish¹² the upward bias of the Carli price index P_{CA} and he made the following observations on its use:

"In fields other than index numbers it is often the best form of average to use. But we shall see that the simple arithmetic average produces one of the very worst of index numbers. And if this book has no other effect than to lead to the total abandonment of the simple arithmetic type of index number, it will have served a useful purpose."

Fisher [1922; 29-30]

Unfortunately, Irving Fisher's warning about the use of the arithmetic mean of price ratios as a functional form for an elementary price index was forgotten, not only by the compilers of the U.S. CPI as the work of Reinsdorf and Moulton [1994] shows, but it was also forgotten by the compilers of the Swedish CPI for a short period in 1990 as was noted by Dalén [1992; 139].¹³ Thus in view of its upward bias, the use of the Carli price index P_{CA} for aggregating elementary price quotes is definitely not recommended; the use of the geometric index P_{JE} defined by (2) or the average price index P_{DU} defined by (3) is definitely preferable since they both satisfy the time reversal test (5).

Dalén [1992] initiated an axiomatic approach to the determination of the functional form for aggregating price quotations at the lowest level where quantity information is not available. It turns out that the work of Eichhorn [1978; 152-160] is relevant in developing this approach. In the following sections, we attempt to integrate the work of Dalén and Eichhorn in order to obtain axiomatic justifications for the use of either the Jevons price index P_{JE} or the Dutot index P_{DU} .

3. AN AXIOMATIC APPROACH TO ELEMENTARY PRICE INDEXES

"So, also, while it seems theoretically impossible to devise an index number, P , which shall satisfy all of the tests we should like to impose, it is, nevertheless, possible to construct index numbers which satisfy these tests so well for practical purposes that we may profitably devote serious attention to the study and construction of index numbers."

Irving Fisher [1911; 200]

"But the problem of axiometry always involves at least two periods. There is a first period, and there is a second period which is compared with it. Price-variations have taken place between the two, and these are to be averaged to get the amount of their variation as a whole. But the weights of the commodities at the second period are apt to be different from their weights at the first period."

Correa Moylan Walsh [1921a; 90]

In the usual test approach to bilateral index number theory,¹⁴ the price index $P^*(p^0, p^1, q^0, q^1)$ is regarded as a function of the price vectors (p^0, p^1) and the quantity vectors (q^0, q^1) that pertain to the two periods under consideration. In the present section, we follow the example of Eichhorn [1978; 152-160] and Dalén [1992] and regard the price index $P(p^0, p^1)$ as a function of only the two price vectors.¹⁵

The test approach to index number theory is well developed in the $P^*(p^0, p^1, q^0, q^1)$ situation. In this context, the individual period t prices and quantities, p_t^t and q_t^t , can refer to different commodities. In our present elementary price index context, we assume that all of the price and quantity quotes pertain to the same commodity in the different time periods. We shall also interpret Dalén's [1992; 138] equal weights approach to mean that $q_n^0 = q_n^1 = k$ for $n = 1, 2, \dots, N$; i.e., that each price quote in each period refers to a constant amount k of the commodity that is being purchased. Our approach to deriving meaningful axioms for $P(p^0, p^1)$ in the present context can now be explained: we simply set $P(p^0, p^1) = P^*(p^0, p^1, k1_N, k1_N)$ ¹⁶ where P^* is a traditional bilateral price index that depends on prices and quantities. We now go through Diewert's [1992; 214-223] list of 20 tests that characterize the Fisher [1922] ideal bilateral price index and adapt them to the present situation, giving these tests a new interpretation in some cases. Since some of the 20 tests involve variations in quantities, they are of course not applicable in the present situation where we are holding all quantities constant. Following this procedure, we obtain the axioms T1 to T11 listed below. We shall also use the work of Eichhorn [1978; 152-160] to establish some relationships between the various tests or axioms.

In what follows, we assume that the elementary price index P is a well defined function of the $2N$ positive price quotes for periods 0 and 1, p^0 and p^1 . If prices are equal in the two periods, we denote the common vector of prices as $p = (p_1, \dots, p_N)$.

T1: Continuity: $P(p^0, p^1)$ is a continuous function defined for strictly positive price vectors $p^0 \gg 0_N$ and $p^1 \gg 0_N$.

T2: Identity: $P(p, p) = 1$;

i.e., if the price vectors are equal during the two periods, then the price index should equal 1.

T3: Monotonicity in Current Period Prices: $P(p^0, p^1) < P(p^0, p)$ if $p^1 < p$;

i.e., if any period 1 price increases, then the price index increases.

T4: Monotonicity in Base Period Prices: $P(p^0, p^1) > P(p, p^1)$ if $p^0 < p$;
i.e., if any period 0 price increases, then the price index decreases.

T5: Proportionality in Current Prices: $P(p^0, \lambda p^1) = \lambda P(p^0, p^1)$ if $\lambda > 0$;
i.e., if all period 1 prices are multiplied by the positive number λ , then the initial price index is also multiplied by λ .

T6: Inverse Proportionality in Base Prices: $P(\lambda p^0, p^1) = \lambda^{-1} P(p^0, p^1)$ if $\lambda > 0$;
i.e., if all base period prices are multiplied by the positive number λ , then the resulting price index is equal to the original price index divided by λ .

T7: Mean Value Test: $\alpha \leq P(p^0, p^1) \leq \beta$ where $\alpha = \min_i \{p_i^1/p_i^0: i = 1, \dots, N\}$ and $\beta = \max_i \{p_i^1/p_i^0: i = 1, 2, \dots, N\}$;
i.e., the price index lies between the smallest and largest outlet price ratios, $p_n^1/p_n^0, n = 1, \dots, N$.

Proposition 1 (Eichhorn [1978; 155])¹⁷: Tests T1, T2, T3 and T5 imply test T7.

Proof: Let $p^0 >> 0_N$ and $p^1 >> 0_N$. From the definitions of α and β , we have:

$$(7) \quad \alpha p^0 \leq p^1 \leq \beta p^0.$$

Using T2 and the definition of α , we have

$$\begin{aligned} \min_i \{p_i^1/p_i^0\} &= \alpha P(p^0, p^0) \\ &= P(p^0, \alpha p^0) \quad \text{using T5} \\ &\leq P(p^0, p^1) \quad \text{using (7), T1 and T3.} \end{aligned}$$

Similarly, using T2 and the definition of β , we have:

$$\begin{aligned} \max_i \{p_i^1/p_i^0\} &= \beta P(p^0, p^0) \\ &= P(p^0, \beta p^0) \quad \text{using T5} \\ &\geq P(p^0, p^1) \quad \text{using (7), T1 and T3.} \end{aligned}$$

Q.E.D.

T8: Positivity: $P(p^0, p^1) > 0$;

i.e., the price index is positive.

Proposition 2: Test T8 is implied by tests T1, T2, T3 and T5.

Proof: Tests T1, T2, T3 and T5 imply T7 and thus $P(p^0, p^1) \geq \alpha = \min_i \{p_i^1/p_i^0: i = 1, \dots, N\}$.

The desired result now follows from the positivity of the prices.

Q.E.D.

T9: Dimensionality: $P(\lambda p^0, \lambda p^1) = P(p^0, p^1)$ for $\lambda > 0$;

i.e., if we change the units of measurement for each commodity in all outlets by the same positive number λ , then the elementary price index remains unchanged.

The term dimensionality test is used by Eichhorn [1978; 153] and Dalén [1992; 137].

However in the present context where the N commodities are regarded as identical, this test could be regarded as a specialization of Fisher's [1911; 411] [1922; 420] commensurability test, due originally to Pierson [1896; 131].

Proposition 3 (Eichhorn [1978; 155]): If P satisfies T5 and T6, then it also satisfies T9.

The proof of the above Proposition is immediate. Eichhorn [1978; 153-155] also shows that the tests T2, T3, T4, T5 and T6 are independent. Propositions 1 - 3 above show that the tests T7, T8 and T9 are consequences of the first 6 tests.

Tests T3, T4, T5 and T9 were proposed by Dalén [1992] as was the following test:

T10: Time Reversal: $P(p^1, p^0) = 1/P(p^0, p^1)$;

i.e., if the data for periods 0 and 1 are interchanged, then the resulting price index should equal the reciprocal of the original index.

Proposition 4: If P satisfies T8 and T10, then it also satisfies T2.

Proof: Using T10, we have $P(p, p) = 1/P(p, p)$ or $[P(p, p)]^2 = 1$. Test T8 rules out the case $P(p, p) = -1$ so we must have $P(p, p) = 1$.

Q.E.D.

Proposition 5: If P satisfies T8, T10 and one of T3 or T4, then it satisfies both T3 and T4.

Proof: Suppose P satisfies T3, T8 and T10 and let $p^0 < p$. Then by T3, $P(p^1, p^0) < P(p^1, p)$.

Using T8, this inequality becomes:

$$(8) \quad 1/P(p^1, p^0) > 1/P(p^1, p).$$

Using T10, we have

$$\begin{aligned} P(p^0, p^1) &= 1/P(p^1, p^0) \\ &> 1/P(p^1, p) \quad \text{by (8)} \\ &= P(p, p^1) \quad \text{by T10} \end{aligned}$$

which establishes T4. The proof that T4 implies T3 is similar.

Q.E.D.

Proposition 6: If P satisfies T10 and one of T5 or T6, then P satisfies both T5 and T6.

The proof of the above Proposition is straightforward.

The next two tests are symmetry or invariance tests. Test T11 is Fisher's [1922; 63] commodity reversal test applied to our present situation.

T11: Symmetric Treatment of Outlets: $P(p^0, p^1) = P(\bar{p}^0, \bar{p}^1)$ where \bar{p}^0 and \bar{p}^1 denote the same permutation of the components of the price vectors p^0 and p^1 respectively;

i.e., if we change the ordering of the outlets from which we are obtaining the price quotes for the two periods, then the elementary price index remains unchanged.

T12: Permutation or Price Bouncing Test: $P(p^0, p^1) = P(\bar{p}^0, \bar{p}^1)$ where \bar{p}^0 and \bar{p}^1 denote (possibly different) permutations of the components of the price vectors p^0 and p^1 ;

i.e., if the ordering of the price quotes for both periods is changed (in possibly different ways), then the elementary price index remains unchanged.

Obviously, T11 is the special case of T12 where the permutations of the prices for the two periods are restricted to be the same. Thus T12 implies T11.

Test T12 is due to Dalén [1992; 138]. He justified this test by noting that the price index should show no change if prices "bounce" in such a manner that the outlets are just exchanging prices with each other over the two periods.¹⁸

The above axioms all seem quite reasonable in the present context. However, they do not suffice to pin down the functional form for the elementary price index; for example, both (2) and (3) satisfy all of the above axioms. In order to have the axiomatic approach lead to a specific index number formula, it is necessary to add more tests.

In the usual bilateral test approach to index number theory where the price index $P^*(p^0, p^1, q^0, q^1)$ is a function of both prices and quantities, we have the Paasche and Laspeyres bounding test¹⁹; i.e., $P^*(p^0, p^1, q^0, q^1)$ lies between the Laspeyres price index $P_L^*(p^0, p^1, q^0, q^1) = p^1 \cdot q^0 / p^0 \cdot q^0$ and the Paasche price index $P_P^*(p^0, p^1, q^0, q^1) = p^1 \cdot q^1 / p^0 \cdot q^1$. If we adapt this test to the current situation where we assume that quantities are all equal (i.e., $q^0 = q^1 = k1_N$), then we obtain the following inequalities:

$$(9) \quad p^1 \cdot 1_N / p^0 \cdot 1_N \leq P(p^0, p^1) \leq p^1 \cdot 1_N / p^0 \cdot 1_N.$$

Since the upper and lower bounds in (9) are identical, it can be seen that adding the (modified) Paasche and Laspeyres bounding test to our list of tests leads directly to $P(p^0, p^1) = p^1 \cdot 1_N / p^0 \cdot 1_N$; i.e., we have

$$(10) \quad P(p^0, p^1) = P_{DU}(p^0, p^1)$$

where P_{DU} is the Dutot price index defined earlier by (3).

The conclusion (10) can be reached by adapting another traditional bilateral test to the present situation. The constant quantities test²⁰ says that if the quantity vectors in the two periods are identical so that $q^0 = q^1 = q$, then $P^*(p^0, p^1, q, q) = p^1 \cdot q / p^0 \cdot q$. In our present circumstances, we have $q = k1_N$, so $P(p^0, p^1) = P^*(p^0, p^1, k1_N, k1_N) = p^1 \cdot 1_N / p^0 \cdot 1_N$. Thus the constant quantities test leads directly to (10) as the appropriate functional form for the elementary price index.

Thus the Dutot elementary price index, which satisfies all of the above axioms, has a very reasonable axiomatic justification. Moreover, P_{DU} appears to be the elementary price index counterpart to the bilateral Fisher ideal price index, $P_F^*(p^0, p^1, q^0, q^1) = [P_L^* P_P^*]^{1/2}$, since the axioms stated in this section (with the exception of T12) are all elementary counterparts to the 20 bilateral axioms used by Diewert [1992; 221] to characterize the Fisher ideal index.

In the following section, we provide an axiomatic justification for the geometric mean elementary price index, P_{JE} defined by (2).

4. ADDITIONAL AXIOMS FOR ELEMENTARY PRICE INDEXES

"In that work the best methods of averaging price-variations were put to various tests -- among them the circular test, never before used for measuring their comparative errors, and unfortunately never since made use of ..."

Correa Moylan Walsh [1921a; 108] commenting on Walsh [1901]

"The only formulae which conform perfectly to the circular test are index numbers which have constant weights, i.e., weights which are the same for all sides of the 'triangle' or segments of the 'circle,' i.e. for every pair of times or places compared."

Irving Fisher [1922; 274]

In order to further restrict the functional form for the elementary price index, we impose what Fisher [1922; 413] called the circularity test (due originally to Westergaard [1890; 218-219]):

T13: Circularity: $P(p^0, p^1)P(p^1, p^2) = P(p^0, p^2)$ for all p^0, p^1, p^2 ;

i.e., the price index going from period 0 to 1 times the index going from 1 to 2 is equal to the price index going directly from period 0 to period 2.

Proposition 7 (Eichhorn [1978; 156]): If P satisfies T13 and T8, then it also satisfies the identity test T2 and the time reversal test T10.

Proof: Using T13 with $p^0 = p^1 = p^2 = p$, we have $[P(p,p)]^2 = P(p,p)$. Using T8, we can divide by $P(p,p) > 0$ and conclude that T2 is satisfied. Using T13 with $p^2 = p^0$ implies

$$\begin{aligned} P(p^0, p^1)P(p^1, p^0) &= P(p^0, p^0) \\ &= 1 \quad \text{by T2.} \end{aligned}$$

Dividing by $P(p^0, p^1) > 0$, we conclude that $P(p^1, p^0) = 1/P(p^0, p^1)$.

Q.E.D.

Thus the circularity test is not independent of our previous tests. Eichhorn also showed that circularity significantly restricts the class of admissible elementary price indexes as the following Proposition shows.

Proposition 8 (Eichhorn [1978; 156-157]): If P satisfies T8 and T13, then there exists a positive function of N positive variables, $m(p)$, such that

$$(11) \quad P(p^0, p^1) = m(p^1)/m(p^0).$$

Proof: Using circularity, we have for every $p \gg 0_N$, $p^0 \gg 0_N$ and $p^1 \gg 0_N$:

$$P(p, p^0)P(p^0, p^1) = P(p, p^1) \text{ or}$$

$$P(p^0, p^1) = P(p, p^1)/P(p, p^0) \quad \text{using T8}$$

$$= P(1_N, p^1)/P(1_N, p^0) \quad \text{setting } p = 1_N$$

$$= m(p^1)/m(p^0)$$

where we define $m(p) = P(1_N, p)$.

Q.E.D.

Corollary: If in addition: (i) P satisfies T1, then m is continuous; (ii) P satisfies T3, then m is increasing; i.e., $m(p^0) < m(p^1)$ if $p^0 < p^1$; (iii) P satisfies T5, then m is (positively) linearly homogenous; i.e., for $p \gg 0_N$ and $\lambda > 0$, $m(\lambda p) = \lambda m(p)$; (iv) P satisfies T11, then m is a symmetric function of its N variables, and (v) $m(1_N) = 1$.

Proof: Parts (i) - (iii) are immediate if we use the representation $m(p) = P(1_N, p)$ for m . For part (iv), let \bar{p} be a permutation of p and suppose that P satisfies T11. We have

$$m(p) = P(1_N, p)$$

$$= P(\bar{1}_N, \bar{p}) \quad \text{by T11}$$

$$= P(1_N, \bar{p}) \quad \text{since } \bar{1}_N = 1_N$$

$$= m(\bar{p}).$$

To prove part (v), note that Proposition 7 implies that the identity test T2 holds. Thus $m(1_N) = P(1_N, 1_N) = 1$ using T2.

Q.E.D.

The proof of part (iv) of the Corollary shows that to obtain the symmetry of m , we do not require Dalén's permutation test T12; the weaker symmetric treatment of outlets test T11 will suffice.

Using the Propositions noted above, it can be seen that if P satisfies T1, T3, T5, T8, T11 and T13, then $m(p) = P(1_N, p)$ is a homogenous symmetric mean²¹; it is a continuous, increasing, linearly homogenous and symmetric function of N variables that has the following mean property:²²

$$m(\lambda 1_N) = \lambda.$$

Thus under these axioms, the elementary price index $P(p^0, p^1)$ must be the ratio of the homogenous symmetric means, $m(p^1)/m(p^0)$, where the same functional form is used in the numerator and denominator.

The class of symmetric means is a huge class of functions. Thus we consider additional axioms for P to satisfy in order to further restrict m . In the following axiom, recall that the vector of period 1 prices is $p^1 = [p_1^1, p_2^1, \dots, p_N^1]$.

T14: Consistency in Aggregation: $P(p^0, p^1) = P(p^0, M(p_1^1, p_2^1), M(p_1^1, p_2^1), p_3^1, \dots, p_N^1)$ for some M and all $p^0 \gg 0_N$ and $p^1 \gg 0_N$ where the function of two variables $M(p_1, p_2)$ is a symmetric mean; i.e., M is a continuous, increasing and symmetric function of two variables which has the property $M(\lambda, \lambda) = \lambda$ for all λ .

The meaning of T14 can be explained as follows. We compute $P(p^0, p^1)$ using a two stage aggregation procedure. In the first stage, the mean of the first two period 1 prices, p_1^1 and p_2^1 , is computed using the aggregator function M ; i.e., we compute the mean price $M(p_1^1, p_2^1)$. Then replace each of p_1^1 and p_2^1 by this mean price and compute the second stage price index as $P(p^0, M(p_1^1, p_2^1), M(p_1^1, p_2^1), p_3^1, \dots, p_N^1)$. The axiom T14 requires that this second stage price index be equal to the original price index, $P(p^0, p^1)$.²³

Proposition 9: Let $N \geq 3$ and suppose $P(p^0, p^1)$ satisfies T1, T3, T5, T8, T11, T13 and T14. Then there exists an r such that $P(p^0, p^1) = m(p^1)/m(p^0)$ where m is defined as

$$(12) \quad m(p_1, p_2, \dots, p_N) = \phi^{-1} [\sum_{i=1}^N (1/N)\phi(p_i)]$$

where the function of one variable ϕ is defined as

$$(13) \quad \phi(z) = \alpha + \beta f_r(z), \beta \neq 0$$

for some constants α and β and f_r is defined as

$$(14) \quad f_r(z) = \begin{cases} z^r & \text{if } r \neq 0; \\ \ln z & \text{if } r = 0. \end{cases}$$

Proof: The proof follows directly from the Corollary to Proposition 8 above and Proposition 10 in Diewert [1993b; 381].

Q.E.D.

The results in this section can be summarized as follows. Suppose that the elementary price index $P(p^0, p^1)$ satisfies the axioms T1, T3, T5, T8, T11, T13 and T14. Then the elementary price index can be written as follows:

$$(15) \quad P(p^0, p^1) = M_r(p^1)/M_r(p^0)$$

where the mean of order r function, M_r , is defined for each number n as follows:²⁴

$$(16) \quad M_r(p_1, \dots, p_N) = \begin{cases} [\sum_{i=1}^N (1/N)p_i^r]^{1/r} & \text{for } r \neq 0; \\ \prod_{i=1}^N p_i^{1/N} & \text{for } r = 0. \end{cases}$$

Moreover, the P defined by (15) and (16) will satisfy all of the tests T1 to T14 that have been listed thus far.

We still have to add at least one additional axiom to determine the parameter r which appears in (16). The following axiom is a natural one in the present context.

T15: Dependence on Relative Prices: $P(p^0, p^1) = F(p_1^1/p_1^0, p_2^1/p_2^0, \dots, p_N^1/p_N^0)$ for some F and all $p^0 \gg 0$ and $p^1 \gg 0_N$;

i.e., the elementary price index P depends only on the relative prices found in the N outlets for the two periods, p_n^1/p_n^0 for $n = 1, 2, \dots, N$.

Proposition 10: Let $N \geq 3$ and suppose that the elementary price index $P(p^0, p^1)$ satisfies T1, T3, T5, T8, T11, T13, T14 and T15. Then $P(p^0, p^1) = P_{JE}(p^0, p^1)$. Moreover, the Jevons geometric price index P_{JE} defined by (2) above satisfies all 15 tests, T1 - T15.

Proof: From Proposition 9 above, $P(p^0, p^1)$ is defined by (15) and (16). The only way the resulting P can be consistent with T15 is to have $r = 0$. Verifying that P_{JE} satisfies all 15 tests is straightforward.

Q.E.D.

It may not be clear why T15 is a desirable property for an elementary price index so we will replace it by the following axiom:

T16: **Commensurability:** $P(\lambda_1 p_1^0, \dots, \lambda_N p_N^0; \lambda_1 p_1^1, \dots, \lambda_N p_N^1) = P(p_1^0, \dots, p_N^0; p_1^1, \dots, p_N^1) = P(p^0, p^1)$
for all $\lambda_1 > 0, \dots, \lambda_N > 0$;

i.e., if we change the units of measurement for every commodity in each outlet, then the elementary price index remains unchanged.

The motivation for this test might be that we are no longer certain that the commodity for which we are collecting price quotes is really homogeneous across outlets and hence we would like our price index to be invariant to changes in the units of measurement of these possibly outlet specific commodities.²⁵

Proposition 11: Let $N \geq 3$ and suppose that the elementary price index $P(p^0, p^1)$ satisfies T1, T3, T5, T8, T11, T13, T14 and T16. Then $P(p^0, p^1) = P_{JE}(p^0, p^1)$. Moreover, the Jevons geometric price index P_{JE} defined by (2) above satisfies all 16 tests, T1 - T16.

Proof: Analogous to the proof of Proposition 10.

Q.E.D.

Proposition 10 or 11 provides a reasonable axiomatic foundation for the use of the geometric elementary price index $P_{JE}(p^0, p^1)$.²⁶

If we replace T15 or T16 by the following axiom, we obtain an axiomatic characterization for the Dutot price index.

T17: Weak Additivity: $P(p^0, p^1 + k1_N) = P(p^0, p^1) + P(p^0, k1_N)$ for all $p^0 \gg 0_N$, $p^1 \gg 0_N$ and $k > 0$;

i.e., if we add the same positive constant k to each of the period 1 prices p^1_n , then the resulting price index is equal to the sum of the original price index $P(p^0, p^1)$ plus the price index which results when we replace p^1 by the vector of constant prices $k1_N$, $P(p^0, k1_N)$.

Proposition 12: Let $N \geq 3$ and suppose that the elementary price index $P(p^0, p^1)$ satisfies T1, T3, T5, T8, T11, T13, T14 and T17. Then $P(p^0, p^1) = P_{DU}(p^0, p^1)$. Moreover, the Dutot price index P_{DU} satisfies all of the tests T1 - T17 except T15 and T16.

Proof: Applying Proposition 9 above, we have $P(p^0, p^1) = M_r(p^1)/M_r(p^0)$ where the mean of order r , M_r , is defined by (16). Thus we have for some r and for all $p \gg 0_N$:

$$(17) \quad P(1_N, p) = M_r(p)/M_r(1_N) = M_r(p).$$

Since P satisfies T17, letting $p^0 = 1_N$, we have for all $k > 0$:

$$P(1_N, p^1 + k1_N) = P(1_N, p^1) + P(1_N, k1_N) \text{ or}$$

$$(18) \quad M_r(p^1 + k1_N) = M_r(p^1) + M_r(k1_N) \quad \text{using (17)}$$

$$= M_r(p^1) + k \quad \text{using (16)}.$$

Equation (18) says that M_r is translatable.²⁷ Since $M_r(p)$ is a linearly homogenous separable mean that is also translatable, it follows from a result in Diewert [1993b; 384] that r must equal 1; i.e., we must have

$$(19) \quad M_r(p) = (1/N) 1_N \cdot p = \sum_{i=1}^N (1/N) p_i.$$

Thus $P(p^0, p^1) = M_1(p^1)/M_1(p^0) = 1_N \cdot p^1 / 1_N \cdot p^0 = P_{DU}(p^0, p^1)$. It can be verified that P_{DU} satisfies all of the tests except T15 and T16.

Q.E.D.

Propositions 10-12 in this section provide axiomatic justifications for the geometric and arithmetic average elementary price indexes P_{JE} and P_{DU} defined by (2) and (3) above.

In the following sections, we consider some alternative approaches to the determination of the functional form for an elementary price index.

5. ECONOMIC APPROACHES TO ELEMENTARY PRICE INDEXES

"Dr. Laspeyres urges, if I read him aright, that as the value of gold meant its purchasing power, we ought to take the simple arithmetic average of the quantities of gold necessary for purchasing uniform quantities of given commodities. There is certainly some ground for the argument. But it may be urged with equal reason that we should suppose a certain uniform quantity of gold to be expended in equal portions in the purchase of certain commodities, and that we ought to take the average quantity purchased each year. This might be ascertained by taking the harmonic mean."

W. Stanley Jevons [1865; 295]

Dr. R. Zuckerkandl, for instance, in a paper contributed by him to the Handwörterbuch der Staatswissenschaften, reminds us of the well-known fact that variations of prices affect the quantities consumed. The demand diminishes for articles that have risen, it increases for articles that have fallen."

N.G. Pierson [1895; 332]

In this section, we assume that the Statistical Agency has outlet quantity information in addition to price information. Under these circumstances, it is possible to use an economic approach to index number theory in order to derive an appropriate functional form for the elementary price index. Assume that there are N outlets where the target population can buy a particular homogeneous commodity. Suppose that the price of the commodity in outlet n during period t^{28} is $p_n^t > 0$ and the corresponding quantity sold is $q_n^t > 0$ for $t = 0, 1$ and $n = 1, 2, \dots, N$. Denote the vectors of period t prices and quantities by p^t and q^t respectively. Suppose that each commodity in each outlet is regarded as a separate good in each purchaser's preference function (or production function). Further suppose (somewhat unrealistically) that each purchaser has the same linearly homogeneous aggregator function, $f(q)$, which aggregates combinations of the N outlet specific goods where $q = [q_1, \dots, q_N]$ and q_n denotes the quantity purchased from outlet n for $n = 1, \dots, N$. Finally, assume that each purchaser engages in cost minimizing behavior in each period. Then it can be shown that²⁹ assuming certain specific functional forms for the aggregator function f , or the

dual unit cost function c defined as

$$c(p) = \min_q \{p \cdot q : f(q) \geq 1\},$$

leads to specific functional forms for the price index $P^*(p^0, p^1, q^0, q^1)$ with³⁰

$$(20) \quad c(p^1)/c(p^0) = P^*(p^0, p^1, q^0, q^1).$$

If we follow the example of Reinsdorf and Moulton [1994] and assume that all purchasers have Leontief aggregator functions,³¹ then equation (20) becomes

$$(21) \quad c(p^1)/c(p^0) = p^1 \cdot q^0 / p^0 \cdot q^0 = p^1 \cdot q^1 / p^0 \cdot q^1$$

where the quantity vectors are proportional during the two periods; i.e., we have

$$(22) \quad q_n^1 = \lambda q_n^0 \quad \text{for } n = 1, \dots, N \text{ and some } \lambda > 0.$$

If we further assume that outlet quantities are equal in period 0 so that

$$(23) \quad q_n^0 = k \quad \text{for } n = 1, \dots, N,$$

then the price index on the right hand side of (21) becomes $P_{DU}(p^0, p^1)$, the Dutot elementary price index (3).

Instead of assuming as in (22) that quantities purchased in the outlets are proportional in the two periods, we could assume that outlet expenditures are proportional over the two periods; i.e., assume that³²

$$(24) \quad p_n^1 q_n^1 = \lambda p_n^0 q_n^0 \quad \text{for } n = 1, \dots, N \text{ and some } \lambda > 0.$$

The above expenditure proportionality assumption is implied under our cost minimization assumptions if the underlying aggregator function has the Cobb-Douglas functional form. Thus if we assume that

$$(25) \quad f(q_1, \dots, q_N) = \prod_{i=1}^N q_i^{\alpha_i}, \quad \alpha_i > 0, \quad \sum_{i=1}^N \alpha_i = 1,$$

for some constants α_i , then (24) will hold and (20) becomes³³

$$(26) \quad c(p^1)/c(p^0) = \prod_{i=1}^N (p_i^1/p_i^0)^{\alpha_i}.$$

If we further assume that expenditures are constant across outlets in period 0 so that

$$(27) \quad p_n^0 q_n^0 = k \quad \text{for } n = 1, 2, \dots, N$$

then the right hand side of (26) becomes $P_{JE}(p^0, p^1)$ defined by (2).

Index number theorists have been deliberating the relative merits of the constant (or proportional) quantities assumption (22) versus the constant (or proportional) expenditures assumption (24) for a long time. Authors who thought that the latter assumption was more likely empirically include Jevons [1865; 295], and Ferger [1931; 39] [1936; 271].³⁴ These early authors did not have the economic approach to index numbers at their disposal but they intuitively understood, along with Pierson [1895; 332], that substitution effects occurred and hence the proportional expenditures assumption was more plausible than the proportional quantities assumption.

In this section, we have provided economic justifications for the Dutot and Jevons elementary price indexes to augment the axiomatic justifications presented in the previous two sections. However, the above economic justifications are very weak for two reasons: (i) the equality assumptions (23) or (27) are unlikely to hold in practice (although the use of sampling techniques could make these assumptions approximately correct) and (ii) the Leontief and Cobb-Douglas assumptions for the underlying aggregator function are very restrictive.³⁵ However, the economic justification for the Jevons geometric elementary price index is much stronger than that for the Dutot index: cross shop elasticities of substitution are much more likely to be close to unity (the Cobb-Douglas case) than to zero (the Leontief case).

A less restrictive functional form for the aggregator function is $f(q) = (q \cdot Aq)^{1/2}$ where A is a symmetric matrix with one positive eigenvalue and $N-1$ zero or negative eigenvalues. This functional form is flexible³⁶; i.e., it can provide a second order approximation to an arbitrary linearly homogenous aggregator function. Similarly, a flexible functional form for a unit cost function is $c(p) = (p \cdot Bp)^{1/2}$ where B is a symmetric matrix with one positive eigenvalue and $N-1$ zero or negative eigenvalues. For either of these functional forms, equation (20) becomes³⁷

$$(28) \quad c(p^1)/c(p^0) = [p^1 \cdot q^0 \cdot p^1 \cdot q^1 / p^0 \cdot q^0 \cdot p^0 \cdot q^1]^{1/2} \\ = P_F^*(p^0, p^1, q^0, q^1)$$

where P_F^* is Fisher's [1927a] ideal price index.³⁸ Thus if price and quantity information is available at the elementary level, it seems preferable to use the Fisher ideal price index to aggregate the basic level price quotes rather than the Laspeyres, Paasche or geometric indexes which appear on the right hand side of (21) and (26).

6. ON THE USE OF UNIT VALUES AT THE BASIC LEVEL OF AGGREGATION

"Some nice questions arise as to whether only what is consumed in the country, or only what is produced in it, or both together are to be counted; and also there are difficulties as to the single price-quotation that is to be given at each period to each commodity, since this, too, must be an average. Throughout the country during the period a commodity is not sold at one price, nor even at one wholesale price in its principal market. Various quantities of it are sold at different prices, and the full value is obtained by adding all the sums spent (at the same stage in its advance towards the consumer), and the average price is found by dividing the total sum (or the full value) by the total quantities."

Correa Moylan Walsh [1921a; 88]

We now have to recognize a problem associated with the use of the Fisher ideal index number formula as an aggregator of elementary prices: how do we construct p_n^t and q_n^t in any time period t where outlet n sells the homogeneous commodity at different prices? We could attempt to regard the quantities sold at each price as separate quantity and price quotes but then the number of price quotes N over all of the outlets would not generally be equal for the two periods and thus a bilateral index number formula like Fisher's P_F^* could not be evaluated. Alternatively, we could attempt to make the time period so short that only one price for each outlet would apply in each period. However, this solution to the problem would lead to lots of zero quantities and hence purchasers would be at corner solutions to their optimization problems. Hence virtual prices or unobservable shadow prices would be required in place of the observed outlet prices in order to justify the use of exact index number formulae like (28).³⁹ Thus at some level of disaggregation, bilateral index number theory breaks down and it becomes necessary to define the average price and total quantity that pertain to an outlet using what might be called a "unilateral" index number formula.

What aggregation formula should we use in this unilateral context? Consider the following quotations:

"Of all the prices reported of the same kind of article the average to be drawn is the arithmetic; and the prices should be weighted according to the relative mass-quantities that were sold at them."

Correa Moylan Walsh [1901; 96]

"That price and quantity each requires a distinct type of formula is indicated by the simpler problem where only one commodity is involved, as in the case of the bushels of wheat previously discussed. In this case, the measure of quantities for each period is obviously obtained by merely summing up the number of units sold, while the measure of prices is obtained by dividing the aggregate value by the quantity units."

George Davies [1924; 187]

Thus Walsh and Davies simply took as an axiom that the appropriate period t measure of price at the elementary level is the unit value defined by

$$(29) \quad P^t = \frac{\sum_{n=1}^{N_t} p_n^t q_n^t}{\sum_{n=1}^{N_t} q_n^t}$$

and the corresponding period t measure of quantity at this elementary level is total quantity sold defined by

$$(30) \quad Q^t = \sum_{n=1}^{N_t} q_n^t$$

where N_t is the number of distinct sales prices in period t for the outlet under consideration.

From this unilateral point of view, the appropriate bilateral elementary price index between periods 0 and 1 for an outlet is the following ratio of unit values:

$$(31) \quad P_{DR}^*(p^0, p^1, q^0, q^1) = P^1/P^0$$

where the unit values P^0 and P^1 are defined by (29) for $t = 0$ and 1. The unit value price index P_{DR}^* was first proposed by Segnitz [1870; 184] for homogeneous commodities and for heterogeneous commodities by Drobisch [1871; 148]. As an aggregation formula at the first stage of aggregation over homogeneous commodities, it was proposed by Walsh [1901; 96] [1921a; 88] and Davies [1924; 183] [1932; 59] and many other modern writers,⁴⁰ assuming of course that information is available on both prices and quantities.

The Walsh-Davies approach to computing elementary price indexes seems very attractive. At the very lowest level of aggregation when price and quantity information is available, then appropriate period t price and quantity aggregates are unit values and total quantities, P^t and Q^t , defined by (29) and (30). At the next level of aggregation, a superlative index number formula like P_F^* defined by (28) can be used to aggregate up these lowest level prices and quantities.⁴¹

Thus the lowest level aggregates would normally be shop specific unit values. However, if individual outlet data on transactions were not available or were considered to be too detailed, then unit values for a homogeneous commodity over all outlets in a market area might form the lowest level of aggregation.

Some further discussion on the concept of a unit value for a homogeneous commodity seems warranted. Saglio [1994] noted that the unit value or average price of a homogeneous commodity could be distinguished by: (i) its point of purchase (outlet effect); (ii) the various competing brands or product lines of the commodity that are being sold at an outlet; e.g., Cadbury versus Hershey chocolate bars (brand effect) and (iii) the various package sizes at which the commodity is sold (packaging effect). Thus finely classifying unit values on the basis of outlets, brands and packages should in principle be done, if the requisite data were available. However, it may turn out that empirically, some of this fineness of classification is not required.⁴²

Another important characteristic of a unit value is the time period over which it is calculated. In principle, the time period should be the longest period which is short enough so that individual variations of price within the period can be regarded as unimportant. Thus our "ideal" time period appears to be the maximal Hicksian week (which is actually due to Fisher [1922; 318]):

"I shall define a week as that period of time during which variations in price can be neglected."

J.R. Hicks [1946; 122]

Thus the actual length of time over which unit values should be calculated will depend on the inflationary environment that the Statistical Agency faces: if the country has a rapidly changing

inflation rate, then the time period should be made shorter. In a situation of hyperinflation, the ideal time period will be very short indeed.⁴³

As a final comment on the problem of choosing the ideal time period, consider the problem of "time of day" commodities. If some commodity is sold at a lower price at a certain time of the day or week⁴⁴ and consumers shift their purchases over time to take advantage of these "time of day" prices, then in principle, time of day unit values should be constructed; i.e., the ideal time period should be subdivided to reflect these time of day purchases.

It should be noted that the approach to consumer price indexes that we are advocating here is a transactions based approach as opposed to the current Statistical Agency approach to the CPI⁴⁵ which can be viewed as an imputation based approach; i.e., the price quoted on a commodity that the Statistical Agency collects say once a month at a particular outlet is taken to be representative of the average price at which that commodity is sold at that outlet during the month. It should be evident that a unit value for the commodity provides a more accurate summary of an average transaction price than an isolated price quotation.

The feasibility of constructing unit values as basic level prices depends on the availability of detailed price and quantity information. Many index number practitioners have regarded traditional bilateral index number theory as being useless since detailed quantity information is not usually available to the Statistical Agency constructing price indexes.⁴⁶ The validity of this criticism is rapidly diminishing over time due to the computer (and price scanner) revolution. Most retail outlets in advanced market economies use scanners to generate electronic point of sale data, which generally include transaction prices and quantities, location, date and time of purchase and the product described by brand, make or model. The retail outlet can then compile this information or pass it on to private firms⁴⁷ who compile the data and then resell the results to product manufacturers or to the retailers. If Statistical Agencies had access to scanner data, it becomes quite feasible to calculate unit values for homogenous commodities.⁴⁸

In fact, recently Alain Saglio [1994], from the French Statistical Agency INSEE, has made use of Nielsen data for milk chocolate bars sold in France in 915 outlets for the years 1988-1990. Very detailed unit values, classified by outlet, brand, package size and time period (two months), were calculated and then aggregated up using the Laspeyres index number formula. He also used these data to construct an estimate of the outlet substitution effect, which will be discussed below in section 9.

In general, firms now process information on their costs and sales using computers so that summary information is available to managers on a monthly basis. Detailed information on prices and quantities could be extracted from this information base in many cases. In some cases, firms might be persuaded to provide information on prices and quantities to the Statistical Agency instead of filling out numerous forms.

The existence of private firms compiling detailed price and quantity information leads to an interesting dilemma for Statistical Agencies: (i) should the Agency buy the data from the information processing firm or (ii) should the Agency set up its own information processing subsidiary to compete against the private firm? Silver [1994; 5] points out that the first alternative will lead to a loss of control by the Statistical Agency in its data collection activities.⁴⁹ However, alternative (ii) may lead to charges that the Agency is providing unfair competition to the private sector. More public discussion on these issues seems to be required.⁵⁰ However, it is clear that eventually, Statistical Agencies will be forced to join the electronic highway in one form or another.

7. SAMPLING PROBLEMS

"Here it will appear that the probability is (1) that, even if we employ a perfectly correct method, the final errors which we shall inevitably commit in practice, being by the nature of the case relative, will decrease, and our accuracy increase, with the square root of the increase in the numbers of the commodities operated on; and (2) that as the measurements advance over a course of years, each being compared with the preceding in a new measurement, and the whole being strung out in one line, the errors to which even the perfectly correct method is exposed in practice will increase from the starting period (unless adjusted by direct comparison with it) with the square

root of the number of years traversed."

Correa Moylan Walsh [1921a; 113-114]

"Undoubtedly it is true that any index number is to be considered as made up of samples rather than as constituting a complete field. But I doubt if we shall ever improve greatly on the system now universally employed, of selecting and weighting samples on the basis of value-importance."

Irving Fisher [1922; 380].

Even with the availability of scanner data, coverage of outlets will not be complete. More generally, it will usually be necessary to sample outlets when collecting price information in order to reduce costs. In the previous section, we indicated that at the individual outlet level of disaggregation, the best estimate for the representative price of a homogeneous commodity is its unit value defined by (29). However, in order to compute a unit value in an environment of changing outlet prices within the sample period, price information alone will not suffice: information on the total value of transactions as well as on the total quantities transacted will be required. Thus from this point of view, most of the literature dealing with price sampling problems is off the mark: rather than sampling just prices, values and quantities transacted should be sampled.

To summarize: at the individual outlet level, we recommend using the unit value (29) and the total quantity transacted (30) to form price and quantity estimates for the homogenous commodity for the two periods under consideration. To form an elementary price index across N outlets for two periods, the arithmetic mean price formula P_{DU} defined by (3) or the geometric mean price formula P_{JE} defined by (2) could be used, using the outlet unit values as the price quotes p_n^t that appear in (2) or (3). However, since outlet quantities sold are necessarily calculated as a by-product of the calculation of unit values, it would be preferable from the viewpoint of economic theory to use the Fisher ideal price index P_F^* defined by (28) in order to calculate an aggregate of outlet price changes for the "homogeneous" commodity under consideration.⁵¹ This approach to the calculation of elementary price indexes seems to be broadly consistent with the sampling procedures recommended by Pigou [1924; 66-67] and Fisher [1922; 380] [1927b].⁵²

8. EMPIRICAL DIFFERENCES BETWEEN ELEMENTARY PRICE INDEXES

"Other methods, many of them in use today, are absolutely bad and pernicious. In many of them the errors are so great and so cumulative, that they cannot be used in the proper way, in the 'chain' system; but must be measured from a common base, and then they give rise to all sorts of haphazard weighting, involving unknowable errors, when later periods (the more recent periods) are compared with one another. All these should be thrown on the scrap heap."

Correa Moylan Walsh [1921a; 104-105]

"So long as the same weights are used forward and backward, the product of the arithmetic forward and backward will exceed unity.... By similar reasoning, it may be shown that the harmonic index number, with or without any given weighting, has an inherent bias downward. That is, its forward and backward forms, multiplied together, give a result always and necessarily less than unity."

Irving Fisher [1922; 87]

In addition to the elementary index number formulae (1) - (3), Statistical Agencies have considered the use of the following two formulae:

$$(32) \quad P_H(p^0, p^1) = [\sum_{i=1}^N (1/N)(p_i^1/p_i^0)^{-1}]^{-1};$$

$$(33) \quad P_{AH}(p^0, p^1) = [P_{CA}(p^0, p^1)P_H(p^0, p^1)]^{1/2}.$$

P_H is the harmonic mean of the price ratios p_i^1/p_i^0 and it was first suggested in passing as an index number formula by Jevons [1884; 121] and Coggeshall [1887]. P_{AH} is the geometric mean of the arithmetic mean P_{CA} and the harmonic mean P_H of the price ratios. It was first suggested by Fisher [1922; 472] as his formula 101. Fisher [1922; 211] observed that P_{AH} was empirically very close to the geometric mean index P_{JE} and these two indexes were the best unweighted index number formulae.⁵³ In more recent times, Carruthers, Sellwood and Ward [1980; 25] and Dalén [1992; 140] also proposed P_{AH} as an elementary price index.

Several investigators have derived various inequalities or approximate relationships between the elementary indexes (1) - (3) and (32) - (33). We shall indicate how these relationships can be derived below.

The first relationship we consider is between the ratio of averages index P_{DU} and the average of ratios index P_{CA} . As usual, let p^0 and p^1 be the N dimensional price vectors pertaining to periods 0 and 1. Let $r_n = p_n^1/p_n^0$ and denote the vector of price relatives as $r = [r_1, \dots, r_N]$. Define the

mean and variance of the N dimensional vector x as $\bar{x} = x \cdot 1_N / N$ and $\text{var}(x) = (x - \bar{x} 1_N)(x - \bar{x} 1_N) / N$ where 1_N denotes a vector of ones of dimension N . Finally, define the correlation coefficient between the vectors x and y by $\rho(x, y) = (x - \bar{x} 1_N) \cdot (y - \bar{y} 1_N) / N [\text{var}(x) \text{var}(y)]^{1/2}$. Armed with the above definitions, we can derive the following relations:⁵⁴

$$\begin{aligned}
 P_{DU}(p^0, p^1) &= \sum_{n=1}^N (p_n^1 / p_n^0) p_n^0 / p^0 \cdot 1_N \\
 &= r \cdot p^0 / p^0 \cdot 1_N \\
 &= r \cdot 1_N (1/N) + r \cdot [(p^0 / p^0 \cdot 1_N) - (1/N) 1_N] \\
 &= P_{CA}(p^0, p^1) + r \cdot [(p^0 / p^0 \cdot 1_N) - (\bar{p}^0 / p^0 \cdot 1_N) 1_N] \\
 &= P_{CA}(p^0, p^1) + [r - \bar{r} 1_N] \cdot [(p^0 / p^0 \cdot 1_N) - (\bar{p}^0 / p^0 \cdot 1_N) 1_N] \\
 (34) \quad &= P_{CA}(p^0, p^1) + N[\text{var}(r) \text{var}(p^0 / p^0 \cdot 1_N)]^{1/2} \rho(r, p^0 / p^0 \cdot 1_N).
 \end{aligned}$$

The correlation between the normalized base period prices, $p^0 / p^0 \cdot 1_N$, and the vector of price relatives, r , will usually be negative and if so, $P_{DU}(p^0, p^1) < P_{CA}(p^0, p^1)$. We will have $P_{DU} = P_{CA}$ if: (i) $\text{var}(r) = 0$ (so all price ratios $r_n = p_n^1 / p_n^0$ are equal); (ii) $\text{var}(p^0 / p^0 \cdot 1_N) = 0$ (so all base period prices p_n^0 are equal) or (iii) $\rho(r, p^0 / p^0 \cdot 1_N) = 0$ (so that the price ratios r_n are uncorrelated with the base period prices p_n^0).

The second relationship that we will derive is an approximate one between P_{DU} and P_{JE} . We first rewrite the price vectors p^0 and p^1 in terms of their means and deviations from their means as follows:

$$(35) \quad p_n^t = \bar{p}^t (1 + \epsilon_n^t); \quad t = 0, 1; \quad n = 1, 2, \dots, N;$$

$$(36) \quad \sum_{n=1}^N \epsilon_n^t = 0; \quad t = 0, 1$$

where $\bar{p}^t = p^t \cdot 1_N / N$ is the (arithmetic) mean price for period t . Note that (36) implies that the deviation vector $\epsilon^t = [\epsilon_1^t, \dots, \epsilon_N^t]$ satisfies $\epsilon^t \cdot 1_N = 0$ for $t = 0, 1$. Note also that

$$(37) \quad P_{DU}(p^0, p^1) = \bar{p}^1 / \bar{p}^0.$$

Upon substituting the relations (35) into definition (2) for the geometric elementary price index P_{JE} , we obtain the following equalities:

$$\begin{aligned}
 P_{JE}(p^0, p^1) &= \Pi_{n=1}^N [\bar{p}^1(1+\epsilon_n^1)/\bar{p}^0(1+\epsilon_n^0)]^{1/N} \\
 (38) \quad &= P_{DU}(p^0, p^1) f(\epsilon^0, \epsilon^1)
 \end{aligned}$$

where we have used (37) and defined $f(\epsilon^0, \epsilon^1)$ as $\Pi_{n=1}^N [(1+\epsilon_n^1)/(1+\epsilon_n^0)]^{1/N}$. Expanding $f(\epsilon^0, \epsilon^1)$ by a second order Taylor series around $\epsilon^0 = 0_N$ and $\epsilon^1 = 0_N$ and using (36), we obtain the following second order approximation:

$$\begin{aligned}
 P_{JE}(p^0, p^1) &\cong P_{DU}(p^0, p^1) [1 + (1/2N)\epsilon^0 \cdot \epsilon^0 - (1/2N)\epsilon^1 \cdot \epsilon^1] \\
 (39) \quad &= P_{DU}(p^0, p^1) [1 + (1/2)\text{var}(\epsilon^0) - (1/2)\text{var}(\epsilon^1)].
 \end{aligned}$$

Since the variance of the deviations of the prices from their means in each period is likely to be constant⁵⁵; i.e., $\text{var}(\epsilon^0) = \text{var}(\epsilon^1)$, under these conditions, the geometric elementary price index P_{JE} will approximate the ratio of mean prices index P_{DU} to the second order.⁵⁶ The approximate equality (39) was first derived by Carruthers, Sellwood and Ward [1980; 25].

We turn now to a comparison of P_{CA} , P_{JE} , P_H and P_{AH} . Since P_{CA} , P_{JE} and P_H are the arithmetic, geometric and harmonic means of the price ratios $r_n = p_n^1/p_n^0$, it is well known that⁵⁷

$$(40) \quad P_H(p^0, p^1) \leq P_{JE}(p^0, p^1) \leq P_{CA}(p^0, p^1)$$

with equalities only if the price vectors p^0 and p^1 are proportional.

The relations (40) do not indicate how big the inequalities will be if prices are not proportional. Thus we follow the example of Dalén [1992; 146-147] and calculate second order approximations to each of the indexes in (40) as well as P_{AH} . We first rewrite r , the vector of price relatives, in terms of its mean \bar{r} and some deviations ϵ_n as follows:

$$(41) \quad r_n = \bar{r}(1+\epsilon_n); \quad n = 1, \dots, N;$$

$$(42) \quad \sum_{n=1}^N \epsilon_n = 0.$$

Define the vector $\epsilon = [\epsilon_1, \dots, \epsilon_N]$. Now substitute (41) into the definitions of P_{CA} , P_{JE} , P_H and P_{AH} to obtain the following equalities:

$$(43) \quad P_{CA}(p^0, p^1) = \sum_{n=1}^N (1/N)r_n = \bar{r} \sum_{n=1}^N (1/N)(1+\epsilon_n) = \bar{r} f_A(\epsilon);$$

$$(44) \quad P_{JE}(p^0, p^1) = \Pi_{n=1}^N r_n^{1/N} = \bar{r} \Pi_{n=1}^N (1+\epsilon_n)^{1/N} = \bar{r} f_G(\epsilon);$$

$$(45) \quad P_H(p^0, p^1) = [\sum_{n=1}^N (1/N) r_n^1]^{-1} = \bar{r} [\sum_{n=1}^N (1/N) (1 + \epsilon_n)^{-1}]^{-1} = \bar{r} f_H(\epsilon);$$

$$(46) \quad P_{AH}(p^0, p^1) = [P_{CA}(p^0, p^1) P_H(p^0, p^1)]^{1/2} = \bar{r} [f_A(\epsilon) f_H(\epsilon)]^{1/2} = \bar{r} f_{HA}(\epsilon)$$

where the last equality in (43) - (46) serves to define the deviation functions f_A , f_G , f_H and f_{AH} . The second order Taylor series approximations to each of these functions around the point $\epsilon = 0_N$ are:

$$(47) \quad f_A(\epsilon) \cong 1;$$

$$(48) \quad f_G(\epsilon) \cong 1 - (1/2N)\epsilon \cdot \epsilon = 1 - (1/2)\text{var}(\epsilon);$$

$$(49) \quad f_H(\epsilon) \cong 1 - (1/N)\epsilon \cdot \epsilon = 1 - \text{var}(\epsilon);$$

$$(50) \quad f_{AH}(\epsilon) \cong 1 - (1/2N)\epsilon \cdot \epsilon = 1 - (1/2)\text{var}(\epsilon)$$

where we have made repeated use of (42) in deriving (47) - (50). Thus to the second order, the arithmetic mean of price relatives index P_{CA} will exceed the geometric mean index P_{JE} by $(1/2)\bar{r}\text{var}(\epsilon)$; the geometric mean index P_{JE} will exceed the harmonic mean index P_H by $(1/2)\bar{r}\text{var}(\epsilon)$ and finally, the geometric mean of the arithmetic and harmonic mean indexes P_{AH} will equal the geometric mean index P_{JE} . The second order approximations (47) - (50) are due to Dalén [1992; 143].⁵⁸

Thus empirically, we expect P_{JE} and P_{AH} to be very close to each other. Using (39), we expect P_{JE} to be reasonably close to P_{DU} , with some fluctuations over time due to changing variances of the absolute deviations ϵ^t . Finally, we expect P_{CA} to be substantially above P_{JE} and P_{JE} to be above P_H by a similar substantial amount. We turn now to the available empirical evidence on these expectations.

Carruthers, Sellwood and Ward [1980; 26] compared January 1973 with January 1974 prices for 8 food categories in the UK. They found that P_{DU} averaged .07% below P_{JE} ; P_{AH} averaged .005% above P_{JE} ; P_{CA} averaged .57% above P_{JE} and P_H averaged .57% below P_{JE} .

Schultz [1994] calculated 61 month to month price indexes for a few components of the CPI for the Canadian Province of Ontario over the years 1988-1993 for matched samples. For soft drinks, P_{DU} averaged .005% below P_{JE} ; P_{CA} averaged 3.5% above P_{JE} ; P_H averaged 3.3% below P_{JE} .

For butter, P_{DU} averaged .008% below P_{JE} ; P_{CA} averaged .23% above P_{JE} and P_H averaged .24% below P_{JE} . These are per month averages.

Dalén [1994] calculated alternative elementary price indexes over the years 1990-1993 using Swedish data. Averaging his results over years and commodities, P_{DU} averaged approximately .03% above P_{JE} and P_{CA} averaged approximately 1.6% per year higher than P_{JE} .

Finally, Woolford [1994] calculated some elementary price indexes using Australian data on fresh fruit and vegetables over the year running from June 1993 to June 1994. He found that P_{DU} exceeded P_{JE} by .17% and P_{CA} exceeded P_{JE} by 4.7%.

It can be seen that the empirical results are in line with our expectations based on the theoretical second order approximations derived above.

We turn now to a general discussion of possible sources of bias in computing consumer price indexes.

9. SOURCES OF BIAS IN CONSUMER PRICE INDEXES

"Retail markets furnish many examples of the Schumpeterian process of "creative destruction" in which more efficient producers enter and displace less efficient incumbents. The displacement of various classes of small, independent retailers by large mail order supply houses, department stores and chain grocery stores furnish historical examples of this. Recent times have seen phenomenal growth of a variety of large discount chains such as Wal-mart, Home Depot, Staples and Food Lion, as well as various "warehouse" style food stores and wholesale clubs."

Marshall Reinsdorf [1994c; 18]

"Numerical computation of alternative methods based on detailed firm data on individual prices and quantities where new goods are carefully distinguished would cast light on the size of the new good bias."

W. Erwin Diewert [1993a; 63]

Before we can discuss sources of bias in the computation of consumer price indexes, it is necessary to note that "bias" is a relative concept. Thus when we speak of bias, we have in mind some specific conceptual framework or purpose for the price index and if we had complete information, this underlying "truth" could be measured and "bias" would be relative to this "true

Economists and statisticians have been debating the question of the appropriate conceptual basis for a price index for over a hundred years.⁵⁹ The conceptual framework that we shall adopt in order to discuss bias is the cost of living framework due originally to Konüs [1924]. More specifically, we adopt Pollak's [1981; 328] social cost of living index as the underlying "correct" concept.⁶⁰ This concept assumes utility maximizing (or expenditure minimizing) behavior on the part of consumers and thus is open to the criticism that it is unrealistic. However, as Pierson [1895; 332] observed 100 years ago, consumers do purchase less in response to higher prices; i.e., substitution effects do exist. The existing economic theory of cost of living indexes can be viewed as a way of incorporating these substitution effects into the measurement of price change (as opposed to the traditional Statistical Agency fixed basket approach which holds quantities fixed as prices change⁶¹).

Instead of using the economic theory of the consumer as the theoretical basis for the construction of price indexes, it is possible to use instead a producer theory approach to the measurement of price change; see Court and Lewis [1942-3], Fisher and Shell [1972], Samuelson and Swamy [1974], Archibald [1977] and Diewert [1983b; 1054-1077].⁶² We will not pursue this approach here.

Once a theoretically ideal price index has been chosen, bias can be defined as a systematic difference between an actual Statistical Agency index and the theoretically ideal index. Instead of the term "bias," Fixler [1993; 7] and other BLS economists use the term "effect." Since most academic economists use the term "bias," we will follow in this tradition.⁶³

In addition to the elementary index functional form bias considered in the previous section, we shall follow the example of Gordon [1993] and Fixler [1993] and consider commodity substitution bias, outlet substitution bias, linking bias and new goods bias.

The Laspeyres fixed basket price index suffers from commodity substitution bias; i.e., it is biased upward compared to a cost of living index because it ignores changes in quantities demanded

that are induced by changes in relative prices. Estimates of the size of this bias (at levels of aggregation above the elementary level) can be obtained by comparing Statistical Agency Laspeyres type indexes with superlative index numbers such as Fisher's ideal index P_F^* defined by (28). Superlative indexes provide good approximations to the unobservable cost of living indexes.⁶⁴ Using this methodology, Manser and McDonald [1988] (using 101 categories of goods and services) and Aizcorbe and Jackman [1993] (using 207 categories in 44 U.S. locations) found an average substitution bias in the U.S. CPI of about .2% per year. Using the same methodology, Génereux [1983] found the same substitution bias in the Canadian CPI over the years 1957-1978. Using a different methodology, Balk [1990; 82] obtained estimates for the substitution bias in the Dutch CPI in the .2% to .3% per year range using 106 commodity groups over the years 1952-1981.⁶⁵

In section 1, we defined outlet substitution bias in the context of disappearing high cost outlets. We now want to broaden the above preliminary definition to encompass the possibility that consumers may shift their purchases from high cost to low cost outlets over time. Thus instead of calculating outlet specific unit values for a commodity, a unit value could be calculated over all outlets in the market area. The difference between this market area unit value price relative and the corresponding Laspeyres component for the commodity in the official CPI can be defined as outlet substitution bias.⁶⁶ This definition of outlet substitution bias assumes that commodities should not be distinguished by their point of purchase; i.e., a particular make of a video camera yields the same utility to a consumer whether it is bought in Dan's Discount Den or Regal Imports Boutique. This assumption may not be appropriate in other situations.⁶⁷ Turning to empirical evidence on the size of the outlet substitution bias, in his direct statistical method, Reinsdorf [1993; 239-240] found that the outlet substitution bias in the food at home and motor fuel components of the U.S. CPI was about .25% per year during the 1980's (although he regarded this as an upper bound due to possible quality differences). Saglio [1994], using Nielsen data for 915 French outlets over 2 years 1988-1990, found that the outlet substitution bias for milk chocolate bars averaged .8% per year; i.e., the market

unit value for chocolate bars of the same size and brand averaged .8% per year lower than the corresponding Laspeyres index which treated chocolate bars of the same size and brand in each outlet as separate commodities. Saglio [1994], using INSEE data on 29 food groups over 12 years, also found an outlet substitution bias of approximately .4% per year below the corresponding Laspeyres price index.

The outlet substitution bias is formally identical to what might be termed the linking bias; i.e., a new good appears which is more efficient in some dimension than an existing good. After two or more periods, the Statistical Agency places a price relative for the new good into the relevant elementary price index, but the absolute decline in price going from the old to new variety is never reflected in the relevant elementary price index. This source of bias was recognized by Griliches [1979; 97] and Gordon [1981; 130-133] [1990] as the following quotations indicate:

"By and large they [Statistical Agencies] do not make such quality adjustments. Instead, the new product is 'linked in' at its introductory (or subsequent) price with the price indices left unchanged."

Zvi Griliches [1979; 97]

"An even more dramatic case largely involving a producer durable involved the supplanting of the old rotary electric calculator by the electronic calculator; all of us can purchase for \$10 or so a calculator that can perform all the functions (in a fraction of the elapsed time) of an old 1970-vintage \$1000 rotary electric calculator. Yet in the U.S. the electronic calculator was treated as a new product, and the decline in price from the obsolete rotary electric model to the early models of the electronic calculator was "linked out" in the official indexes."

Robert J. Gordon [1992; 9-10]

A more appropriate treatment of the above situation would be to calculate an average price or unit value per the relevant characteristic over the old and repackaged goods. A similar bias was recognized by Griliches and Cockburn [1994] in the context of generic drugs which are chemically identical to brand name drugs (it should be noted that the BLS changed its procedures in January 1995 to fix this problem). An analogous bias in the Statistical Agency treatment of illumination was pointed out by Nordhaus [1994]. These last two papers obtain very large linking biases.⁶⁸

The new goods bias results from the inability of bilateral price indexes to take into account the fact that the number of commodities that consumers can choose from is growing rapidly over time.⁶⁹ Hill makes the following comment on this situation:

"In general, it may be concluded that in the real world, price indices which are inevitably restricted to commodities found in both situations will fail to capture the improvement of welfare associated with an enlargement of the set of consumption possibilities. The benefits brought by the introduction of new goods are not generally taken into account in price indices in the period in which the goods first make their appearance."

Peter Hill [1988; 138]

Diewert [1980; 498-505] [1987; 779] [1993a; 59-63], following Marshall [1887; 373] and Hicks [1940; 114], discussed the new goods bias and suggested along with Griliches [1979; 97] and Gordon [1981; 130] that this bias could be substantially reduced by simply introducing new goods into the pricing basket in a timely fashion; (this would not eliminate the bias in the period when the good makes its first appearance). Triplett [1993; 200] termed the subset of the new goods bias caused by delays in introducing new products into an index as the new introductions bias.⁷⁰ Turning now to empirical estimates of the new goods bias, Gordon [1990] estimated that the U.S. consumer durables price index had a new goods or quality change bias of 1.5% per year over the period 1947-83. Berndt, Griliches and Rosett [1993] provided evidence that the BLS did not sample the prices of new drug products in a sufficiently timely fashion. They found that from January 1984 through December 1989, the BLS producer price index for prescription pharmaceutical preparations (drugs) grew at a rate of 3% per year higher than a superlative price index that used the monthly price and quantity sales data for 2,090 drug products sold by 4 major pharmaceutical manufacturers in the U.S., accounting for about 29% of total domestic industry sales in 1989. Thus they found a combined drug substitution and new introductions bias of about 3% per year. Hausman [1994] used Nielsen scanner data from January 1990 to August 1992 on cereal consumption for 7 major metropolitan areas in the U.S. He used econometric techniques to estimate consumer preferences over cereals and thus he was able to estimate the Hicksian [1940; 114] reservation prices that would

cause consumers to demand zero units of a new cereal. His conclusion was that an overall price index for cereals, which excluded the effects of new brands, would overstate the true cost of living subindex for cereals by about 25% over a ten year period.⁷¹ Finally, Trajtenberg [1990] attempted to measure reservation prices for Computer Assisted Tomography (CAT) scanners over the decade 1973-1982. His nominal price index went from 100 to 259 but his quality adjusted price index went from 100 to 7, implying a 55% drop in prices per year on average.

Summarizing the empirical evidence reviewed in this section and the previous one, we see that it is likely that in recent years, a typical official CPI has a .2% per year commodity substitution bias, a .25% per year outlet substitution bias, a linking bias of perhaps .1% per year and a new goods bias of at least .25% per year; i.e., an upward bias of at least .8% per year. If the Statistical Agency is also making use of a biased elementary price index formula, this will add an additional upward bias to the official index. The reader will note that all of the 5 above sources of bias were regarded as being additive, an assumption which is probably approximately correct.⁷²

We conclude this section with a detailed discussion of the possible biases in the U.S. CPI.

Marshall Reinsdorf and Brent Moulton [1994] have provided important empirical evidence of upward bias in the U.S. consumer price index due to an inappropriate choice of functional form used to aggregate price quotations at the lowest level of aggregation. Reinsdorf and Moulton found that their geometric mean index (which used the elementary price index P_{JE} defined by (2) at the lowest level of aggregation) grew by 2.48% from June 1992 to June 1993, compared to a simulated U.S. consumer price index growth rate of 2.95%. Their simulations excluded housing and hence covered 70.3% of the U.S. CPI universe. Thus their simulated U.S. CPI (which largely uses the Carli-Sauerbeck price index P_{CA} defined by (1) at the elementary level) appears to have an upward bias of about .5% per year. Furthermore, Armknecht, Moulton and Stewart [1994] noted that since 1987, the owner's implicit rent component of the CPI used a Carli elementary price index, which led to a .5% per year upward bias in that component since 1987. Thus the choice of index number

formula at the elementary level is not a trivial matter.

Reinsdorf [1993; 242-247] earlier compared the behavior of official U.S. rates of inflation for food and gasoline with corresponding rates obtained using average prices; i.e., he compared CPI rates of inflation for food and gas with those obtained by using the elementary price index P_{DU} defined by (3). Over the 1980's, he found that means of the U.S. CPI food indexes weighted according to their importance in the CPI showed an average annual increase of 4.2%, while the corresponding weighted mean of the average prices grew at a rate of 2.1% per year. For gasoline, he found that average prices fell faster than the corresponding CPI prices at about 1% per year during the 1980's. Reinsdorf [1993; 242] attributed these results to outlet substitution bias but it now seems clear that some of this upward bias in food and gas was due to the inappropriate method used by the Bureau of Labour Statistics to aggregate price quotes at the elementary level. However, it is also clear that not all of Reinsdorf's results can be explained away as being elementary level functional form bias: a substantial portion of the bias that he found must be outlet substitution bias.

The results of Reinsdorf and Reinsdorf and Moulton suggest that outlet substitution bias in the U.S. CPI as a whole was somewhere between .1 to .5% per year in the 1980's and the elementary functional form bias was somewhere between .35 and .5% per year in the 1990's. In addition to the above two sources of bias, we have commodity substitution bias at levels above the elementary level, linking bias and new goods bias. These three sources of bias probably add an additional .3 to .7% per year upward bias to traditional fixed basket type indexes. Adding up all of these sources of bias for the U.S. consumer price index leads to a total upward bias in the region of .75 to 1.7% per year in the 1980's. This is a substantial bias.⁷³

10. RECOMMENDATIONS AND CONCLUSIONS

"... every person in the room would have realized after hearing his Paper that the measurement of the cost of living was by no means a simple conception. Nobody would expect that a difficult question of engineering or a nice point of art could be put in the Press and explained in words of one syllable and in a single sentence."

A.L. Bowley [1919; 371] commenting on his own paper

"Would it not be well if statisticians and economists should again come together and decide authoritatively on the proper method of constructing index-numbers?"

Correa Moylan Walsh [1921a; 138]

A number of recommendations seem to follow from the empirical work of Reinsdorf and Moulton:

- (i) Statistical Agencies should follow the emphatic advice of Irving Fisher [1922; 29-30] and avoid the use of the Carli arithmetic mean of price relatives formula (1) to form elementary price aggregates.
- (ii) If information on quantities is not available at the elementary or basic level, either the geometric price index (2) advocated by Jevons or the average price index (3) suggested by Dutot should be used. Axiomatic justifications for these two indexes were provided in sections 3 and 4 and (weak) economic justifications were presented in section 5.
- (iii) At the level of the individual outlet, the best elementary average price for a homogeneous commodity would seem to be its unit value: the value of units sold during the sample period divided by the total quantity sold. If outlet unit values are available, then in aggregating over outlets, there is no need to restrict ourselves to using the Jevons or Dutot formulae to construct elementary prices. From the viewpoint of economic theory, it seems preferable to use the Fisher ideal price index in this second stage of elementary aggregation.
- (iv) Values and quantities should be sampled rather than just prices. Sampling values and quantities will greatly reduce the new introductions bias.

(v) Statistical Agencies should consider either purchasing electronic point of sale data from firms currently processing these data or the Agencies should set up Divisions which would compete in this area.

(vi) Recent economic history will have to be rewritten in view of the substantial outlet substitution and elementary price index biases that Reinsdorf and Moulton have uncovered in U.S. price indexes. Since the U.S. is so large in the world economy, world inflation was lower in the 1980's than was officially recorded and world output growth (and hence productivity growth) was higher. It is very likely that many of the sources of bias in price indexes documented for the U.S. economy are also applicable to other economies.

NOTES

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1. When an outlet supplying a price quote disappears and is replaced by a new outlet, the new outlet price quote does not immediately replace the missing price quote. Usually, price quotes are obtained from the new outlet for at least two periods, and then a price ratio using only new outlet prices is linked into the index at the end of the second period. Thus any absolute change in prices going from the old outlet to the new outlet is ignored.
2. See also Moulton [1993], Reinsdorf [1994] and Armknecht, Moulton and Stewart [1994].
3. Turvey [1989; ch. 3] and Dalén [1992] refer to this situation as computing elementary aggregates while Szulc [1987] refers to it as constructing a price index below the basic aggregation level. Additional references which deal with this situation are Forsyth [1978; 352-355], Carruthers, Sellwood and Ward [1980], Forsyth and Fowler [1981; 241] and Balk [1994].
4. Unweighted price indexes of the form (1) - (3) were among the first to appear in the index number literature; see Walsh [1901; 553-558], Fisher [1922; 458-520] and Diewert [1993a] for references to the early history of price indexes. Pigou [1924; 59], Frisch [1936], Szulc [1987; 13] and Dalén [1992; 139] refer to (1) as the Sauerbeck [1895] index.

5. Reinsdorf and Moulton [1994] note that (1) is called the unbiased and efficient Horvitz-Thompson estimator in the statistical literature, provided that the outlets in the Statistical Agency's sample were selected with probabilities proportional to their sales to consumers in the base period (period 0).
6. See Reinsdorf and Moulton [1994] or Armknecht, Moulton and Stewart [1994] for the details.
7. Armknecht, Moulton and Stewart [1994] found that the U.S. owners' equivalent rent component of the U.S. CPI exceeded the corresponding geometric mean CPI by about .5% per year over the period March 1992 to June 1994. They attributed this difference to the use of (1) as the elementary price index formula rather than (2). This upward "bias" in the owners' equivalent rent component of the CPI is likely to be present since the current implicit rent formula was introduced in January 1987.
8. Price index theorists who have used or derived the inequality (4) include Walsh [1901; 517], Fisher [1922; 375-376], Szulc [1987; 12] and Dalén [1992; 142].
9. See Fisher [1911] [1922] and Eichhorn and Voeller [1976].
10. Fisher [1922; 82] credited the time reversal test to the Dutch economist Pierson [1896; 128]. Letting P denote the index number formula, Pierson's test on page 128 was $P(1_N, p_1, p_2, \dots, p_N) = P(p_1^1, p_2^1, \dots, p_N^1, 1_N)$ where 1_N is a vector of ones. This can be interpreted as an invariance to changes in the units of measurement test. However, Pierson [1896; 130] later gave a simple example which showed that the Carli price index did not satisfy the time reversal property. Walsh [1901; 389] and Fisher [1911; 401] gave the first formal statements of the time reversal test.
11. Note that $1/P_{CA}(p^1, p^0)$ is the harmonic mean of the price ratios $p_1^1/p_1^0, \dots, p_N^1/p_N^0$. The inequality (6) now follows from the fact that the arithmetic mean of N positive numbers is always equal to or greater than the corresponding harmonic mean; see Walsh [1901; 517] and Fisher [1922; 383-384].

12. See also Pierson [1896; 130], Pigou [1924; 59 and 70], Szulc [1987; 12] and Dalén [1992; 139].
13. This bias problem is probably much more widespread; e.g., Allen [1975; 92] and Carruthers, Sellwood and Ward [1980; 15] mentioned that the U.K. retail price index used the Carli formula at the elementary level, as well as the Dutot formula. Woolford [1994] reported that the Australian Bureau of Statistics also uses the Carli formula. Flux [1907; 619] reported that the early U.S. Bureau of Labor price index was a Sauerback (or Carli) index.
14. The fathers of the axiomatic or test approach to bilateral index number theory were Walsh [1901] [1921a] [1921b] [1924] and Fisher [1911] [1921] [1922] [1923] [1927a] [1927b]. For references to the more recent literature, see Eichhorn and Voeller [1976] and Diewert [1992; 214-223].
15. Initially, Dalén regarded his price index $I(w, p^0, p^1)$ as a function not only of prices in the two periods but also as a function of a quantity weight vector $w = (w_1, \dots, w_N)$ which he held constant for the two periods under consideration. Later, Dalén [1992; 138] explicitly assumed that each component of the weight vector w_n was equal to $1/N$. In this case, we can ignore the weight vector.
16. Notation: 1_N and 0_N are N dimensional vectors of ones and zeros respectively; $p >> 0_N$ means each component of the vector p is positive; $p \geq 0_N$ means each component is nonnegative and $p > 0_N$ means $p \geq 0_N$ but $p \neq 0_N$; $p \cdot q = \sum_{n=1}^N p_n q_n$.
17. Eichhorn attributed the idea of the proof to Helmut Funke.
18. The term "bouncing" is due to Szulc [1983; 548]. Forsyth [1978; 357] used the term "noise effect," Carruthers, Sellwood and Ward [1980; 20] used the term "hunting" to describe the ebb and flow of prices around a trend and Forsyth and Fowler [1981; 236] used the term "oscillating prices."
19. See Diewert [1992; 219-220].

20. See Diewert [1992; 215] for the history of this test.
21. Diewert [1993b; 361] defined a symmetric mean $m(p)$ as an increasing, continuous and symmetric function which has the mean property, $m(\lambda 1_N) = \lambda$. For a homogeneous symmetric mean, add the property $m(\lambda p) = \lambda m(p)$ for all $\lambda > 0$.
22. This property follows from parts (iii) and (v) of the Corollary to Proposition 8.
23. Schimmack [1910; 128] was the first author to use a consistency in aggregation (or separability) axiom similar to T14; he established the first rigorous axiomatic characterization of the arithmetic mean. Beetle [1915] later showed that Schimmack's axioms were independent. Finally, Huntington [1927; 2] used a consistency in two stage aggregation property to establish axiomatic characterizations for the arithmetic, geometric, harmonic and root-mean-square means.
24. For the properties of means of order r , see Hardy, Littlewood and Polya [1934; 12-15].
25. Even for physically homogeneous commodities, distinct outlet locations or sales at different times of the day or week will serve to differentiate the commodities.
26. Balk [1994] also provides an axiomatic characterization of the geometric mean elementary price index. He assumes that $P(p^0, p^1)$ can be written as a function of the price relatives, $z_n = p_n^1/p_n^0$, $n = 1, \dots, N$; i.e., $P(p^0, p^1) = m(z_1, \dots, z_N)$ where m is: (i) separable; (ii) $m(z, \dots, z) = z$; (iii) $m(\lambda z_1, \dots, \lambda z_N) = \lambda m(z_1, \dots, z_N)$ for $\lambda > 0$ and (iv) $m(1/z_1, \dots, 1/z_N) = 1/m(z_1, \dots, z_N)$. Balk's property (iv) seems to have been first used by Huntington [1927; 3].
27. See Diewert [1993b; 365]. The term is due to Blackorby and Donaldson [1980; 109] but the concept appeared in the mathematics literature much earlier; e.g., see Schimmack [1910; 126] and Nagumo [1930; 77].
28. The average price of a commodity sold in an outlet during a period is taken to be the outlet's unit value for that commodity. This is an appropriate concept of price if we are constructing a producer price index but it may not be appropriate in the consumer price context, where each

individual's unit value for the commodity in each shop should be constructed -- an impossible task using today's technology.

29. See Afriat [1972], Samuelson and Swamy [1974], Diewert [1976] or Pollak [1989].
30. Equation (20) can be generalized to situations where there is preference diversity on the part of demanders of the product; see Diewert [1983a; 181] [1993c; 294-299]. Diewert draws on the work of Pollak [1980] [1981].
31. Reinsdorf and Moulton [1994] and Reinsdorf [1994c] initially assumed Laspeyres price indexes over goods and outlets which is consistent with Leontief aggregator functions but they later assumed Leontief preferences over goods and perfect substitutability of sellers. However, they recognized that their latter assumptions imply that rational consumers should make all of their purchases of a homogeneous commodity at the lowest cost outlet -- behavior which does not occur. Our functional form assumptions which justify (28) allow for perfect substitutability across outlets or for Leontief behavior across outlets; see Diewert [1976; 134].
32. This assumption was made by Ferger [1931; 39] and it was criticized by Lewis [1937; 341]. Walsh [1901; Ch. IV] made assumptions (22) and (24) with $\lambda = 1$.
33. See Pollak [1989; 22-23]. The demand functions that correspond to the aggregator function defined by (25) have unitary own elasticities of demand, which is consistent with Reinsdorf and Moulton's [1994] analysis that assumed unitary demand elasticities.
34. Ferger was later criticized by Lewis [1937; 341]. Walsh [1901; 100] [1921a; 86 and 91] considered assumptions (22) and (24) with $\lambda = 1$.
35. The unit cost functions that correspond to these functional forms can provide only a first order approximation to an arbitrary once differentiable unit cost function and hence these functional forms are not flexible.
36. Diewert [1974; 113] introduced this term to the economics literature.
37. See Diewert [1976; 116 and 133-134].

38. Bowley [1899; 641] suggested the use of either the arithmetic or geometric mean of the Paasche and Laspeyres indexes; see Fisher [1927a; xv].
39. This is perhaps the point that Dalén [1992; 135] made as the following quotation indicates: "First, it does not take care of varying prices for the same item in the same outlet during the reference period--you would still have to define a mean price within that period in some way (or consider 0 and t to be fixed points in time rather than periods which would give you a less relevant index)."
40. See for example Szulc [1987; 13], Dalén [1992; 135], Reinsdorf [1994c] and Reinsdorf and Moulton [1994].
41. It seems clear that Davies [1932; 59] had this two stage aggregation procedure in mind as the following quotation indicates: "In the first place there is involved an averaging of prices as ratios of values to quantities, and a comparison of the averages, or else a direct averaging of double ratios in the form of relatives. In the second place there is the problem of aggregating in commensurable physical units which is implied in even a price index, since an unequivocal value index is always theoretically obtainable. Since both of these problems are involved in the making of general index numbers, it may prove advantageous to separate them for purposes of analysis." Note that Fisher [1927a; 529] attacked Davies' [1924] first paper, since Davies argued against Fisher's factor reversal test. However, elsewhere, Fisher [1922; 318] half heartedly endorsed the use of unit values at the first stage of aggregation.
42. In his empirical work, Saglio [1994] found the packaging effect to be negligible.
43. Thus countries which have more variable inflation rates (typically high inflation countries) will have to allocate more resources to the calculation of a CPI than low inflation countries to achieve the same level of accuracy.
44. Consider "happy hour" drinks at a bar or reduced power or telephone rates at off peak hours.
45. A very clear and comprehensive statement of the current Statistical Agency approach to the CPI

- may be found in Turvey [1989].
46. Consider the following quotation from Turvey [1989; 1]: "The manual deals with the practice of consumer price index numbers and does not attempt to survey the academic literature on the subject. Much of that extensive and fascinating literature is irrelevant for the purposes of this manual. One reason is that no compiler of a consumer price index, whether it be monthly or quarterly, can hope to obtain new weights more than once a year at most, and the data used to compute new weights always refer to the past rather than to the present, whereas much of the literature deals with other types of index."
 47. Examples of such firms are A.C. Nielsen Co. and Information Resources Inc. in the U.S. and GFK Marketing Services in the U.K.
 48. Silver [1994] compiled unit values for colour television sets sold in the U.K. at the lowest level of aggregation. He then aggregated up these unit values using the Laspeyres and Törnqvist bilateral index number formulae. Thus Silver actually implemented the approach to constructing components of the CPI which was outlined at the end of section 5, except that he used the Törnqvist formula rather than the Fisher formula at the second stage. Silver had access to the scanner sales data compiled by GFK Marketing Services for 1993. The data covered over 2.8 million transactions with a sales value of 830 million pounds.
 49. Haworth [1994] reports on the UK experience in contracting out some of the Central Statistical Office's data collection activities to the private sector.
 50. Astin [1994; 93] makes a strong case for the government to act as a monopolist in collecting data for three reasons: (i) the government has access to the most extensive sampling frames; (ii) only the government can ensure compliance to its data inquiries through the use of statutory powers and (iii) the Statistical Agency can credibly guarantee confidentiality.
 51. Thus we are implicitly assuming that the goods sold in different outlets are not perfectly substitutable, even though the physical characteristics of the good are the same across outlets.

In many cases, it may be preferable to assume that the outlet specific goods are perfectly substitutable across outlets, in which case unit values should be constructed across outlets. This would eliminate outlet substitution bias. Unfortunately, the Statistical Agency will have to use its judgment to determine whether an outlet specific good should be aggregated across outlets or not.

52. These authors both suggested sampling values for the two periods under consideration and then using the prices and quantities associated with the sampled values to construct a Fisher ideal price index. Pigou [1924; 67] went on to suggest that the sample based Fisher ideal price index be used to deflate the population value ratio in order to obtain an estimate of the population real quantity ratio for the two periods under consideration. Neither of these authors specifically recommended the use of unit values as prices at the individual outlet level (although Fisher [1922; 318] gave a lukewarm endorsement for the use of unit values). The average price that Fisher [1927b; 420] used at his lowest level of aggregation was the monthly mean of the high, low, first and last price quotes.
53. However, Fisher [1922; 245] still classified P_{AH} and P_{IE} as being "poor" index number formulae compared to his "superlative" ideal index number formula P_F , which of course uses quantity weights. Fisher [1922; 244-245] also classified P_H as the worst "poor" and P_{CA} as the second best "worthless" index number formulae.
54. The specific equality (34) seems to have been first derived by Forsyth [1978; 356] and repeated by Carruthers, Sellwood and Ward [1980; 20]. However, the general technique dates back to Bortkiewicz [1923; 374-375].
55. Actually, the evidence for the UK presented in Leser [1983; 178] suggests that the variance increases if there is a dramatic change in the inflation rate. For additional evidence and further references to the literature on the relationship between inflation and price dispersion, see Golub [1993] and Reinsdorf [1994b].

56. The idea of using second order approximations to compare various index number formulae dates back to Edgeworth [1901; 410-411] [1923; 346-347]. The technique has been used more recently by Diewert [1978; 893-898] [1993a; 49-50], Carruthers, Sellwood and Ward [1980; 24-25] and Dalén [1992; 146-147].
57. For example, see Walsh [1901; 517], Fisher [1922; 375] or Hardy, Littlewood and Polya [1934; 26].
58. We have extended Dalén's analysis slightly, since he assumed $\bar{r} = 1$.
59. The debate started with Edgeworth [1888; 347] as the following quotation indicates: "The answer to the question what is the Mean of a given set of magnitudes cannot in general be found, unless there is given also the object for the sake of which a mean value is required." Other papers discussing different purposes and alternative conceptual frameworks include Edgeworth [1901; 409] [1923; 343-345] [1925], Flux [1907; 620], Bowley [1919; 345-353], March [1921], Mudgett [1929; 249], Ferger [1936], Mills [1943; 398], Triplett [1983], Turvey [1989; 9-27] and Sellwood [1994].
60. This concept excludes the newer economic approaches to cost of living indexes that incorporate consumer search; see Anglin and Baye [1987] and Reinsdorf [1993] [1994a].
61. This traditional Laspeyres approach to measuring price change is comprehensively discussed in Turvey [1989]; for earlier discussions, see Flux [1907; 621], Bowley [1919; 347] and Mills [1943].
62. Diewert [1983b; 1051-1052] also compared the consumer and producer theory approaches.
63. Fisher [1922; 86] called an index number formula "erratic" if it did not satisfy the time reversal test and "biased" if it were "subject to a foreseeable tendency to err in one particular direction." Thus using Fisher's terminology, the arithmetic and harmonic elementary price indexes, P_{CA} and P_H , are biased while the Laspeyres price index, $P_L^*(p^0, p^1, q^0, q^1) = p^1 \cdot q^0 / p^0 \cdot q^1$, is merely erratic. Note that Lovitt [1928; 11] seems to have been the first to show that P_L^* was "erratic"

and not "biased" in the sense of Fisher.

64. See Diewert [1976] [1978]. Hill [1988; 134] assumed that superlative price indexes are essentially weighted averages of price relatives which have quantity or expenditure weights that treat the two periods under consideration in a symmetric manner.
65. A topic closely related to substitution bias is the sensitivity of the Laspeyres index to the choice of the base year or to the choice of expenditure weights for the price relatives; see Hogg [1931; 56], Mudgett [1933; 30], Saulnier [1990], Schmidt [1993] and Dalén [1994].
66. This definition of outlet substitution bias coincides with Reinsdorf's [1993; 228] original definition and includes both of Fixler's [1993; 7] sellers and outlet substitution biases. It also corresponds to Saglio's [1994] point of purchase effect.
67. This ambiguity creates difficulties for Statistical Agencies; i.e., the decision whether to aggregate over outlets in a market area or not is clearly a matter of judgment.
68. Again, this source of bias creates problems for Statistical Agencies; i.e., when should a new product be treated as a genuinely new good or a superficially repackaged old product? It should also be noted that linking bias could go in the opposite direction if firms simply repackage their products to disguise price increases.
69. Actually, what is relevant is the number of commodities that are available in the consumer's market area. Thus the growth of cities and urbanization leads to more specialized goods and services to be offered by producers and hence will lead to a growth in the number of commodities that are effectively available to the consumer. Transportation and communication improvements also lead to larger choice sets, a point already noticed by Marshall [1887; 373-374].
70. Mudgett [1933; 32] noted that in 1930, the BLS had not yet added such important items of expenditure in its basket as automobile expenditures, meals outside the home and life insurance. Gordon [1993] noted that autos entered the U.S. CPI in 1940, penicillin in 1951 after it had

experienced a 99% decline from its initial price, and the pocket calculator in 1978 after it had declined in price about 90% since 1970. Mudgett [1929; 250] also noted that only 40 commodities were comparable between 1870 and 1920 out of 500 commodities whose prices were collected by the BLS in 1920.

71. This bias is the "pure" new goods bias (the bias that occurs in the period when the new good is introduced) as opposed to the new introduction's bias (the bias that occurs in the second and subsequent periods after the good is introduced). Hausman found that his estimated reservation prices were approximately double the first appearance prices of the new cereals.
72. Sellwood [1994] discussed the question of additivity. He also noted that estimates of bias have standard errors attached to them.
73. Similar sources of bias apply to the producer price index; see Gordon [1990] [1993] and Triplett [1993]. Recent surveys of sources of bias in the CPI are Gordon [1993], Crawford [1993] and Wynne and Sigalla [1994].

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