Cost-of-living indexes for Germany

Claus Christian Breuer
University of Duisburg-Essen
Department of Economics
Universitätsstraße 12
45117 Essen
Germany
claus-breuer@uni-due.de

November 29, 2007

Abstract

The aim of this paper is to quantify the substitution bias in the German Laspeyres type price index for food commodities. Cross section micro data from the German income and expenditure survey has been used in order to estimate an Almost Ideal Demand System to derive expenditure functions. The cost-of-living indexes calculated by means of these expenditure functions for the years 1989 to 2006 are exceeding for almost every year the corresponding Laspeyres type price indexes. Using the price- and expenditure elasticities, theoretical explanations for these results are given.

1 Introduction

Recently a sharp increase in German milk and milk product prices aroused a discussion about the increasing cost-of-living especially for poor households. Politicians claimed to link the social welfare to the consumer price index (CPI).\footnote{At present the social welfare in Germany is linked to the development of the retirement pensions, which in turn is linked to the average wage development.} This provoked the known dispute about the reliability of the German Laspeyres type CPI as a measure of cost-of-living changes. Since the Boskin-Report in the mid-nineties of the last century the several sources of biases in the CPI
have been widely discussed in the empirical and theoretical literature (Boskin et al. (1996), Balk (1999), Unayama (2004)). Nevertheless empirical evidence for Germany is still rare.\(^2\)

The purpose of this paper is to quantify the substitution bias in the food commodity subgroup of the German CPI from 1989 till today. The prices in the food commodity group are especially important for the recipients of social welfare, as the relative household expenditures for food are negatively correlated with the household income. From the practical point of view, the food commodity group is favourable to analyze the substitution bias because the new good bias and the quality change bias were less dominant in this group compared to other commodity groups. To ensure a sufficiently large micro-database for the approximation of the cost-of-living index (COLI), a basket of nine staple foods that is consumed by most of the households was chosen. The COLI is approximated by estimating a demand system using micro-data from the German income and expenditure survey. To ensure homogeneity of the consumer behaviour, the demand system is estimated for the subgroup of couples with children. The quantity of the bias can be calculated by comparing the estimated COLI with a traditional Laspeyres price index for the same staple foods.

The structure of the paper is as follows. Section 2 takes a look at the COLI approximation used in this paper, and discusses its advantages and disadvantages with respect to other methods of approximating the COLI. Section 3 goes on to describe the data sources used for the empirical analyses. Limitations of the use of available German official statistics to approximate the COLI were also discussed. Section 4 provides the empirical results of the demand system estimation and the calculation of the COLI. The results are compared to the Laspeyres price index and possible explanations for the consumer behaviour are given. Section 5 concludes.

# 2 Approximating the cost-of-living index

The theory of the COLI for a single household was first developed by the Russian economist Konûs (1924).\(^3\) Under the assumption of neoclassical optimizing beh-

\(^2\)One exception is Hoffmann (1998) who analyzes several potential biases in the German CPI mainly by enriching data from official statistics with data from non-official sources and calculating superlative index formulas.

\(^3\)Diewert and Nakamura (1993) show in detail how to come from the representative household case to the many household case.
haviour, the household that faces a given price vector \( \mathbf{p}_t \) in period \( t \), tries to chose a commodity bundle \( \mathbf{q}_t \) in order to minimize its expenditures to reach a given utility level \( \bar{u} \). The COLI is defined as the ratio of the minimum expenditure at base period prices \( \mathbf{p}_0 \) to the minimum expenditure at comparison period prices \( \mathbf{p}_t \) while keeping the utility level \( \bar{u} \) constant:

\[
P_{Ct}^{COL} = \frac{c(\mathbf{p}_t, \bar{u})}{c(\mathbf{p}_0, \bar{u})},
\]

where \( c(\mathbf{p}_t, \bar{u}) \) is the cost or expenditure function generated by the following minimization problem:

\[
c(\mathbf{p}_t, \bar{u}) = \min_{\mathbf{q}} \left\{ \sum \mathbf{p}_t \mathbf{q}_t : u(\mathbf{q}_t) \geq \bar{u} \right\}
\]

with a specific utility function \( u() \). The quantities \( \mathbf{q}_t \) purchased in period \( t \) to achieve the reference utility level \( \bar{u} \) are not fixed and depend directly on the current price vector \( \mathbf{p}_t \). This is the main difference to the axiomatic approach to index numbers where prices and quantities are independent.

The problem of the COLI underlying economic approach to index numbers is the unobservability of the “true” COLI. The value of the COLI defined in (1) depends on four factors: The base period price vector \( \mathbf{p}_0 \), the comparison price vector \( \mathbf{p}_t \), the reference utility level \( \bar{u} \) and the utility function \( u() \). The two price vectors are observable, but as von Auer (2005) shows, only under the assumption of homothetic preferences the COLI is invariant to the choice of the unobservable reference utility level \( \bar{u} \). The major problem arises from the fourth factor, the unknown and not observable utility function. Without knowing the form of the utility function, it is impossible to derive the cost function and to calculate the COLI.\(^4\) Already Konüs (1924) founded the possibility to bound the COLI above by the Laspeyres price index and below by the Paasche price index. But, often overlooked, these boundaries are only valid under three assumptions: Optimization behaviour of the households, constant utility function over time and homothetic preferences.\(^5\)

In the following we want to distinguish three broad categories of approximation approaches to the COLI:

---

\(^4\)For an overview about the theory of consumer behaviour and especially the duality approaches see Deaton and Muellbauer (1980b).

\(^5\)See Pollak (1989) for an explicit proof.
whereas the focus lies on (b) which will be applied in the empirical analysis in section 4. Diewert (1976) shows that under the assumption of utility maximizing behaviour some price index numbers exactly equal the COLI for a particular functional form of the expenditure function, so that they can be called exact. A price index is defined as superlative, if it is exact for an expenditure function which provides a second-order differential approximation to an arbitrary twice continuously differentiable linearly homogeneous expenditure function.\footnote{The most widely used superlative index numbers are the Fisher- and Törnqvist index numbers. For a critical discussion of superlative index numbers see von der Lippe (2007).} Diewert (1978) also proofs that the superlative index numbers approximate each other to the second order and are often numerically very close.\footnote{Whereas Hill (2006) shows empirically that the spread between superlative index numbers is sometimes even bigger than that between Paasche and Laspeyres index numbers.}

The main advantage of the COLI approximation by superlative index numbers is their easy calculation. But for the practical use in the official CPI calculation the use of superlative index numbers is limited by the data availability, since most of the superlative index numbers require current quantity data which is in reality only available with a time delay.

The goal of the parametric approach is to estimate an expenditure function by means of a demand system. Knowing the parameters of the expenditure function, it is possible to calculate the COLI defined in (1) under different price regimes $\mathbf{p}$ and reference utility levels $\bar{u}$. The main drawback of the parametric approach is the necessary assumption of the specific form of the demand system estimated. For practical use the approach can become computationally burdensome, because, depending on the form of the demand system, the number of parameters can increase disproportionately to the number of commodities under consideration. Empirical studies (e.g. Unayama (2004)) were mostly limited to focus on a small selection or subgroups of commodities. The pivotal question when using the parametric approach is the choice of the specific form of the demand system estimated. Braithwait (1980) estimates a general linear and a linear expenditure system for 53 commodities. Christensen and Manser (1975) were searching for a less restrictive functional form of the demand system and so estimate the consumer demand with an underlying translog utility function. The translog does not require the homotheticity and additivity assumption and
is so more flexible than the other demand functions estimated by Christensen and Manser (1975).8

Deaton and Muellbauer (1980a) proposed a flexible demand system with several favourable properties called the almost ideal demand system (AIDS). The expenditure function of the model is defined as follows:

\[
\ln c(p_t, \bar{u}) = \alpha_0 + \sum_k \alpha_k \ln p_{kt} + \frac{1}{2} \sum_k \sum_j \gamma_{kj} \ln p_{kt} \ln p_{jt} + u_t \beta_0 \prod_k p_{kt}^{\beta_k}. \tag{3}
\]

Applying duality theory (Deaton and Muellbauer (1980b)), (3) can be minimized to yield the AIDS demand function that can be written in deterministic form as:

\[
w_{it} = \alpha_i + \sum_j \gamma_{ij} \ln p_{jt} + \beta_i \left(\ln x_t - \ln P_t\right), \tag{4}
\]

whereas \(w_{it}\) denotes the budget share for the \(i\)-th good in period \(t\) and the price vector \(P_t\) is defined as:

\[
\ln P_t = \alpha_0 + \sum_k \alpha_k \ln p_{kt} + \frac{1}{2} \sum_k \sum_j \gamma_{kj} \ln p_{kt} \ln p_{jt}. \tag{5}
\]

The three properties of consumer demand theory, the Slutsky symmetry, the homogeneity of degree zero in prices and total expenditure and the adding-up condition can now easily be implied.9

- The Slutsky symmetry, which requires that the compensated cross price derivatives of commodity \(i\) with respect to commodity \(j\) equals the compensated cross price derivative of commodity \(j\) with respect to commodity \(i\), is imposed by setting \(\gamma_{ij} = \gamma_{ji}\).

- The homogeneity of degree zero in prices and expenditure, which requires an unchanged demand if the prices and total expenditure all increase by the same percentage, is imposed by setting \(\sum \gamma_{ij} = 0\).

---

8 However, homothethicity and additivity can be achieved by imposing linear restrictions on the parameters of the translog function, so that their validity can be tested statistically.

9 As a translog demand system has a higher number of parameters to estimate compared to the linear expenditure system, the number of commodities in the Christensen and Manser (1975) analysis is restricted to six.

10 The following is based on Deaton and Muellbauer (1980a and 1980b), to which the reader is referred for additional properties and proofs of the AIDS model.
The adding-up condition, which requires equality of the sum of individual commodity expenditures and the total expenditures, is imposed by \( \sum \alpha_i = 1 \), \( \sum \beta_i = 0 \), and the homogeneity restriction \( \sum \gamma_{ij} = 0 \).

These parameter restrictions can be imposed either during estimation or after estimation of the demand system by using standard statistical methods. The AIDS expenditure function in (1) fulfills the flexible functional form property as it is a second order approximation to any expenditure function. This implies that the demand system derived from it, is a first order approximation to any demand system and even if the true underlying demand system is not the AIDS, an AIDS is for any price vector, not too far away from the point of approximation, a sufficient good approximation. The choice of AIDS as functional form can be seen as the less restrictive way concerning the necessary assumptions in applying the parametric approach to the COLI estimation. That is the reason, why AIDS is chosen as the appropriate demand system in this analysis. Another advantage of the presented household level AIDS model is the easy generalization to the aggregate level. Beyond all mentioned above, the AIDS is superior to other flexible demand systems in its convenient econometric estimation. The simplicity of estimation is improved, if we use the linear almost ideal demand system (LAIDS) suggested by Deaton and Muellbauer (1980a).

The reason is the nonlinearity of the price index parameters estimated in (4). A linear approximation to AIDS is obtained by replacing (5) by the Stone’s price index:

\[
\ln P_t = \sum_k w_k \ln p_k .
\]

The choice of the Stone’s price index is reasoned by the often collinear movement of price series. As this perfect collinearity of prices does rarely empirically appear, the Stone’s price index introduces a measurement error. Additionally the Stone’s price index violates the commensurability axiom, as it is not invariant in changes in the unit of measurement of the commodities. To overcome the potentially inconsistent parameter estimations, Moschini (1995) and Asche and Wessells (1997) discusses alternative price index numbers to the Stone's price index given in (6). They propose the Törnqvist price index, a “corrected” Stone price index where prices are normalized to one and a log-linear analogue to the Laspeyres price index. The later one is obtained by replacing \( w_k \) in (6) by the
mean budget share $\bar{w}_t$. This additionally solves the simultaneity problem that arises when $w_k$ appears on both sides of equation (4).

Finally the elasticity derivations for the AIDS and LAIDS should be discussed. Even if the elasticities are not essentially necessary to calculate the COLI, they can give us additional information about the own- and cross-price quantity reactions of the commodities under consideration.\footnote{The own price elasticity is defined as the percentage change of demand of the commodity that would result from a 1% increase in the commodity price. The cross price elasticity is defined as the percentage change of demand of a commodity A that would result from a 1% change in the price of a commodity B.} The uncompensated (Marshallian) price elasticity for AIDS derived from (4) is given as:

$$
\varepsilon_{ijt} = \left( \frac{\gamma_{ij}}{w_{it}} \right) - \left( \frac{\beta_i}{w_{it}} \right) \left( w_{jt} - \beta_j \ln \left( \frac{x_t}{P_t} \right) \right) - \delta_{ij},
$$

(7)

where Kronecker’s delta $\delta_{ij}$ equals one if $i = j$ and zero otherwise. The expenditure elasticity is given by:

$$
\eta_{it} = 1 + \frac{\beta_i}{w_{it}}.
$$

Green and Alston (1990) derive the exact elasticities for the LAIDS and found empirically no difference to the AIDS elasticities. Asche and Wessells (1997) show that when prices for all commodities are normalized to unity, the AIDS and LAIDS are equal when evaluated at the point of normalization and so also the price and expenditure elasticities from both systems are identical.

Banks et al. (1997) further developed the AIDS by incorporating the logarithm of total expenditure in quadratic terms in a model called quadratic almost ideal demand system (QAIDS). This specification allows for quadratic Engel curves, so that an increase in expenditure could change a luxury to a necessity. But in their empirical application Banks et al. (1997) reject the quadratic form for the food commodity group. As also Unayama (2004) finds in its COLI approximation via AIDS and QAIDS the same results up to the two decimal places, I will focus in Section 4 on the AIDS estimation using the dataset described in section 3.

As a promising outlook for future work, the third approximation approach to the COLI (c) is briefly presented here. Without making any assumptions of the functional form of the expenditure function, upper and lower bounds around the COLI can be derived from revealed preference restrictions. The revealed preference theory, based on the work of Samuelson (1938) and Houthakker (1950),
allows testing if the consumer preferences satisfy the regularity conditions (completeness, reflexivity and transitivity) by simply observing their demand behaviour. Continuous and monotonic preferences of a consumer who fulfils the regularity conditions can then be presented by a utility function. Based on the work of Afriat (1977), Varian (1982) develops algorithms that use only observed price and quantity consumption data to bound unknown indifference curves underlying the consumer behaviour. Knowing the bounding indifference curves, expenditure functions can be derived and so upper and lower bound values for the COLI are calculated. As Varian (1982) already mentioned, the bounds are often very wide and therefore not informative. By making different assumptions about the consumer behaviour or using tools like Engel curves, the bounds can be tightened.\textsuperscript{12}

3 Data

The analysis is based on four cross sections of the German income and expenditure survey (“Einkommens- und Verbrauchsstichprobe”, EVS) conducted in 1988, 1993, 1998 and 2003 and on price data from the German consumer price statistic. Both databases are provided by the Federal Statistical Office Germany.\textsuperscript{13}

Every five years, approximately 0.2\% of all households in Germany (at present about 75,000) are covered by the EVS, which provides household level statistical information on income, property, dept and final consumption expenditures of all social household groups.\textsuperscript{14} The EVS are stratified quota samples on the basis of a specified quota plan defined by household size, income and employment status. Participation is voluntary, which is criticised as a source of response bias, as it is assumed that the willingness to participate is higher for households behaving already rational and who are particularly price sensitive in consumption.

One component of the EVS, the so called “detailed log book” is the most important data source for the empirical part of this paper. Every fifth participating household enters in great detail its expenditures and quantities purchased

\textsuperscript{12}See Blundell et al. (2003) for a very auspicious approach to tighten revealed preference bounds on the COLI by non-parametric Engel curves.

\textsuperscript{13}I am very grateful to the Federal Statistical Office Germany and its employees for the provision of the database and many useful advices concerning the data sets used.

\textsuperscript{14}Households with a monthly net income of 18,000 € or above are not included.
on a variety of food-, beverage- and tobacco-commodities for a given month of the year in this book. By dividing the expenditures by the quantities, we know also the individual prices at which the household purchased the commodities. So I have a price variation in my cross section that enables me to estimate a demand system with data from only one year. In cases where the individual price data is not available, an econometric estimation of a demand system would only be possible by using time series of aggregated cross section data. The time series approach is only feasible if an income and expenditure survey is conducted every year, as it is the case in the US and the UK. The only yearly surveys available in Germany are the current household budget surveys ("Laufende Wirtschaftsrechnungen", LWR) of the Federal Statistical Office Germany, which are less detailed than the EVS and which are explicitly not recommended for any time-series analysis. However in the last years the Federal Statistical Office Germany tries to bring the sample designs of the LWR more in line with the one of the EVS to provide a better database for long-term household analysis.

Taking into account the above described data limitations, the AIDS is estimated for nine staple foods listed in the “detailed log book” of the EVS: Milk, cream, eggs, butter, margarine, apples, bananas, mineral water and beer. The choice of the specific commodities was driven by several considerations. First, the level of commodity detail available in the “detailed log book” of the EVS is very high. Second, the staple foods chosen are purchased by the majority of the households, so that a sufficiently large sample size is achieved without making assumptions about the households with zero consumption of an item. Third, the staple foods chosen are homogenous goods, without considerable quality changes, new types appearing or wide differences in product characteristics. Fourth, food is purchased frequently which is not the case for durables. Fifth, the food consumption is assumed to be separable from the non-food consumption in consumer’s utility function, so that the cross price reactions between food and non-food commodity groups should be rather small. And finally the availability of absolute price data from the consumer price statistic is also better for the food-, beverage- and tobacco-group than for other commodity groups.

---

15 The main reason is the high heterogeneity of the households surveyed over the years. Additionally it should be noticed neither the LWR nor the EVS are panel surveys.
16 See Kühnen (2005) for more details.
17 The other possibility would be to assume that households with zero consumption of an item are faced by an average market price of this item calculated from the other survey observations. For the estimation of the demand system this would imply that the reservation price of the household is lower than the averaged market price. If the zero consumption is taste and not price induced, biases can occur.
To ensure the homogeneity of the consumers analyzed, the socio-demographic variables of the EVS are used to include only households in the sample that are consisting of couples with one or more children younger than 18 years old. Because of obligation of secrecy, only a 98% sub sample of the 20% “detailed log book” of the entire EVS was as micro data available from the Federal Statistical Office Germany.

4 Estimation

The parameters of the AIDS model are estimated by applying the seemingly unrelated regression procedure (SUR) to the 2581 observations of the data set described in section 3. The SUR procedure developed by Zellner (1962) is recommended for estimation of multiple equation models with cross-equation parameter restrictions and correlated error terms, as it is the case here. The SUR procedure use generalized least square (GLS) estimation techniques and is implemented in standard statistical software packages like SAS.

After having estimated the parameters of the demand system, the matching expenditure function can be derived by using the duality approach (Deaton and Muellbauer (1980b)). A problem in defining the expenditure function (3) arises due to the $\beta_0$ parameter that cannot be identified by the demand system estimation procedure. Additionally a value for the reference utility level $\bar{u}$ can hardly be found empirically. For the numerical calculation of the expenditure we have to drop the last term of (3). Deaton and Muellbauer (1980a) show that the remaining part of (3) can be regarded as the cost of subsistence. The expenditure of the representative household to attain the cost of subsistence can then be calculated by inserting the price data for the nine commodities out of the official consumer price statistics from 1988 to 2006. For the same nine goods a traditional Laspeyres price index is calculated by using a weighting scheme obtained from the 2581 household observations of 1988s EVS that were used to estimate the AIDS. Table 1 presents the results of the COLI and the Laspeyres type CPI both with base year 1988 and their difference, the substitution bias.
Table 1: Laspeyres type CPI and COLI, base year 1988.

<table>
<thead>
<tr>
<th>Year</th>
<th>Laspeyres CPI</th>
<th>COLI</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>1989</td>
<td>1.0184</td>
<td>1.0109</td>
<td>0.0075</td>
</tr>
<tr>
<td>1990</td>
<td>1.0383</td>
<td>1.0576</td>
<td>-0.0193</td>
</tr>
<tr>
<td>1991</td>
<td>1.0606</td>
<td>1.0816</td>
<td>-0.0210</td>
</tr>
<tr>
<td>1992</td>
<td>1.0707</td>
<td>1.0803</td>
<td>-0.0096</td>
</tr>
<tr>
<td>1993</td>
<td>1.0506</td>
<td>1.0655</td>
<td>-0.0149</td>
</tr>
<tr>
<td>1994</td>
<td>1.1043</td>
<td>1.1343</td>
<td>-0.0300</td>
</tr>
<tr>
<td>1995</td>
<td>1.1308</td>
<td>1.1530</td>
<td>-0.0222</td>
</tr>
<tr>
<td>1996</td>
<td>1.1306</td>
<td>1.1599</td>
<td>-0.0293</td>
</tr>
<tr>
<td>1997</td>
<td>1.1524</td>
<td>1.1749</td>
<td>-0.0225</td>
</tr>
<tr>
<td>1998</td>
<td>1.2018</td>
<td>1.2131</td>
<td>-0.0113</td>
</tr>
<tr>
<td>1999</td>
<td>1.1558</td>
<td>1.1750</td>
<td>-0.0192</td>
</tr>
<tr>
<td>2000</td>
<td>1.2004</td>
<td>1.2239</td>
<td>-0.0235</td>
</tr>
<tr>
<td>2001</td>
<td>1.2414</td>
<td>1.2766</td>
<td>-0.0352</td>
</tr>
<tr>
<td>2002</td>
<td>1.2376</td>
<td>1.2725</td>
<td>-0.0349</td>
</tr>
<tr>
<td>2003</td>
<td>1.2298</td>
<td>1.2621</td>
<td>-0.0323</td>
</tr>
<tr>
<td>2004</td>
<td>1.2208</td>
<td>1.2552</td>
<td>-0.0344</td>
</tr>
<tr>
<td>2005</td>
<td>1.2448</td>
<td>1.2724</td>
<td>-0.0276</td>
</tr>
<tr>
<td>2006</td>
<td>1.2639</td>
<td>1.2821</td>
<td>-0.0182</td>
</tr>
</tbody>
</table>

Source: own calculations

On the first view the results are surprising. The predominant theory and also most of the empirical analyses on the substitution bias predict a positive bias of the Laspeyres type CPI. Here the opposite is the case: Apart from two years, the COLI is always higher than the CPI.

First, the estimation results of the AIDS are checked for errors, potential biases or misspecifications of the demand system. Most of the estimated coefficients are statistically significantly different from zero, some even highly significant with p-values below 0.01. The high number of 52 parameters to be estimated should not be problematic, as we used 2581 observations for the estimation. The LAIDS with the Stone price index described in section 2 is estimated for the same dataset.\textsuperscript{18} LAIDS parameter results are different in size.

\textsuperscript{18}The differences between AIDS and LAIDS estimation and the possibilities to improve
but not in their general tendency compared to the AIDS results. The COLI calculated through the LAIDS lies every year under consideration slightly below the COLI results of table 1, but as the results are the same up to the third decimal places, the negative substitution bias also occurs when applying LAIDS. Second, the choice of goods can be questioned. By only using nine out of the more than 700 commodities containing basket of the German consumer price index, it is definitely not possible to make any predictions about the overall level of the substitution bias of the general CPI. Section 3 describes in detail the reasons why the specific commodities are chosen and why their number is limited to nine.

Third and most promising is to have a closer look on theoretical considerations regarding COLI behaviour under the observable consumer behaviour. As a first step the own price-, cross price- and income elasticities calculated from the estimated AIDS are presented. Table 2 shows the uncompensated (Marshallian) price elasticity matrix calculated at the mean point of the data.

<table>
<thead>
<tr>
<th></th>
<th>milk</th>
<th>cream</th>
<th>eggs</th>
<th>butter</th>
<th>margarine</th>
<th>appels</th>
<th>bananas</th>
<th>water</th>
<th>beer</th>
</tr>
</thead>
<tbody>
<tr>
<td>milk</td>
<td>-0.4718</td>
<td>-0.0579</td>
<td>-0.1190</td>
<td>0.0355</td>
<td>-0.0712</td>
<td>-0.0053</td>
<td>-0.0253</td>
<td>-0.0280</td>
<td>-0.2353</td>
</tr>
<tr>
<td>cream</td>
<td>-0.1491</td>
<td>-0.8890</td>
<td>-0.0902</td>
<td>-0.0781</td>
<td>0.0361</td>
<td>0.0025</td>
<td>-0.0313</td>
<td>0.0163</td>
<td>0.3507</td>
</tr>
<tr>
<td>eggs</td>
<td>-0.2232</td>
<td>-0.0630</td>
<td>-0.2118</td>
<td>0.0245</td>
<td>-0.0940</td>
<td>-0.0955</td>
<td>-0.0221</td>
<td>-0.1082</td>
<td>-0.0977</td>
</tr>
<tr>
<td>butter</td>
<td>0.0679</td>
<td>-0.0503</td>
<td>0.0153</td>
<td>-1.0607</td>
<td>0.0449</td>
<td>-0.0969</td>
<td>0.0094</td>
<td>-0.0257</td>
<td>0.1535</td>
</tr>
<tr>
<td>margarine</td>
<td>-0.2900</td>
<td>0.0592</td>
<td>-0.2046</td>
<td>0.1454</td>
<td>-0.4237</td>
<td>-0.0159</td>
<td>-0.0330</td>
<td>0.0119</td>
<td>-0.0146</td>
</tr>
<tr>
<td>appels</td>
<td>0.0080</td>
<td>-0.0005</td>
<td>-0.1224</td>
<td>-0.1418</td>
<td>-0.0137</td>
<td>-0.7616</td>
<td>-0.0525</td>
<td>-0.0264</td>
<td>0.2387</td>
</tr>
<tr>
<td>bananas</td>
<td>-0.0322</td>
<td>-0.0265</td>
<td>-0.0197</td>
<td>0.0425</td>
<td>-0.0216</td>
<td>-0.0536</td>
<td>-0.6000</td>
<td>-0.0683</td>
<td>0.0390</td>
</tr>
<tr>
<td>water</td>
<td>-0.0214</td>
<td>0.0105</td>
<td>-0.0948</td>
<td>-0.0243</td>
<td>0.0022</td>
<td>-0.0152</td>
<td>-0.0457</td>
<td>-0.4641</td>
<td>-0.1814</td>
</tr>
<tr>
<td>beer</td>
<td>-0.3361</td>
<td>0.0744</td>
<td>-0.1055</td>
<td>0.0309</td>
<td>-0.0331</td>
<td>0.0472</td>
<td>-0.0329</td>
<td>-0.1644</td>
<td>-0.9226</td>
</tr>
</tbody>
</table>

Source: own calculations

The own price elasticities can be found on the diagonal of the matrix. All own price elasticities are negative, which means that changes in own prices have inverse effects on quantities demanded. Apart from butter, which has an own price elasticity slightly above unity, all own price elasticities are smaller than

 estimation results should not be discussed here. Henningsen (2003) gives a good overview about this topic.
one. That means that the demand for all nine goods, except butter, reacts inelastically to own price changes. As we are analyzing staple foods, this result is not surprising, especially for goods without real substitutes like milk, eggs or mineral-water. The sign of the cross price elasticities allows us to identify a commodity being a substitution (positive sign) or a complementary good (negative sign) with respect to the other commodities. For some goods, as for example for butter and margarine the assumed substitutional relationship can be observed, but for the majority of goods no substitutional relationship is founded in the analysis. This is one possible explanation why the substitution biases of the COLI have for most of the years a negative sign. If the majority of the goods under consideration are not substitutes to each other, the representative consumer is not able to reduce the positive expenditure effect of a price increase by simple substitution behaviour. It can be criticised that by the choice of goods possible substitutes are already eliminated, because the food commodities chosen are too different to each other. Indeed, already Boskin et al. (1996) stat that the lower-level substitution bias, on the lowest level of price data aggregation, is bigger than the upper-level substitution bias, occurring on higher aggregation steps of the index number calculation. Nevertheless, the results identify the food consumption as in average not very sensitive towards price changes, which can be explained by the necessity character of most of the food Commodities.

Table 3: Expenditure elasticities, 1988.

<table>
<thead>
<tr>
<th></th>
<th>$\eta_{it}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>milk</td>
<td>0.9606</td>
</tr>
<tr>
<td>cream</td>
<td>0.6929</td>
</tr>
<tr>
<td>eggs</td>
<td>0.8006</td>
</tr>
<tr>
<td>butter</td>
<td>0.8953</td>
</tr>
<tr>
<td>margarine</td>
<td>0.5709</td>
</tr>
<tr>
<td>appels</td>
<td>0.7663</td>
</tr>
<tr>
<td>bananas</td>
<td>0.5250</td>
</tr>
<tr>
<td>water</td>
<td>0.6787</td>
</tr>
<tr>
<td>beer</td>
<td>1.8089</td>
</tr>
</tbody>
</table>

Source: own calculations

The expenditure elasticities displayed in table 3 confirm the assumed necessity character of the commodities under consideration. A positive expenditure
elasticity below unity is associated with a necessity good, a negative expenditure elasticity is associated with an inferior good and a high positive expenditure elasticity above unity is associated with a luxury good. Except for beer, all commodities analyzed here are identified by positive expenditure elasticities below unity as necessities.

After knowing more about the demand characteristics of the commodities analyzed, some theoretical considerations about the possible range of values that a COLI can take under different forms of consumer behaviour should be made. The already in section 2 mentioned conditions necessary for the COLI boundary by the Laspeyres and Paasche price index

\[
P^P_{0t} \leq P^{COL}_{0t} \leq P^L_{0t} \tag{9}
\]

will be discussed in the following. As we have seen, the inequality (9) is violated in the present analyses for almost all years. The first assumption necessary for (9) to be held is the neoclassical optimization behaviour of the household. If the representative household, for which the COLI is calculated, is not trying to maximize his utility with a given expenditure amount or to minimize its expenditure to reach a given utility level, the COLI can lie above the Laspeyres price index. The household optimization hypotheses can be tested by revealed preference theory which is the first step in the nonparametric approach to COLI estimation described in section 2. The second assumption necessary for (9) to holds is a constant utility function underlying the consumer behaviour over time. As here the demand system is estimated with cross section data, the problem of changing utility function over time is negligible. More interesting is the aggregation over households since the COLI is by definition a household level concept and the estimated demand system can only be valid for a non observable representative consumer. This problem arises in all fields of index number calculation, as we are not able to calculate for every household its own personal CPI. Finally the underlying utility function has to be homothetic to make the COLI independent of the absolute reference utility level \( \bar{u} \). Von Auer (2005) shows, that if the utility function is not homothetic, the COLI can lay inside or outside the bounds defined by (9) depending only on the reference utility level \( \bar{u} \) chosen. The results of my demand system estimation reject the assumption of a homothetic utility function, being in line with most other empirical demand studies. Therefore, in my analysis the choice of reference utility level may play
a crucial role for the COLI calculation. But as described at the beginning of this section, no empirical evidence for the specification of a certain numerical value for the reference utility \( \bar{u} \) level exists.

5 Conclusion

The COLI approximation through the estimation of an AIDS with German micro data leads to negative substitution biases for most of the years analyzed. This unusual result of a COLI that exceeds the Laspeyres price index provoked a more detailed look on the implicit assumptions of the parametric approximation approach to the COLI. Especially the assumption regarding the specific form of the demand system estimated and the by assumption homothetic utility function, that is necessary to be independent of the reference utility level, should always be treated with care when applying the parametric approach. To overcome these restrictive assumptions, it might be worth to apply the nonparametric approach to the same micro data set used in the present analysis. This will be the objective of future empirical work. Till now, it can be stated that the price responsiveness and the willingness to substitute among staple foods is low for the household group analyzed. It would have been interesting to extend the group of commodities and to conduct a demand analysis on several stages, to see how the household budget is allocated on the broad commodity groups and if the separability condition holds. This would require a more detailed and more frequently income and expenditure survey than the present EVS. If a more detailed annually income and expenditure survey would be available for Germany, it would become possible to conduct not only cross section but also time series analysis. A further advantage of an annually conducted EVS would be the possibility to work with superlative index numbers, like the Fisher- and Törnqvist index numbers. As these price indexes require actual weighting schemes, they can nowadays only be calculated every five years.

Once again, this analysis shows the various difficulties in COLI approximation. If it is required by the policy, several improvements in the data provision are possible to facilitate the COLI approximation. With regard to the inflation compensation of social welfare, a special yearly survey on the consumption of goods especially important for poor households could be conducted among social welfare recipients. This would provide a more reliable database for calculation of a COLI as compensation benchmark. But the high costs of such a survey
would be only justifiable if the results of such a COLI would differ significantly from the common CPI figures.

References


