Reconciliation of Consumer Price Index Trends with Corresponding Trends in Average Prices for Quasi-Homogeneous Goods using Scanning Data

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Abstract: The advent of scanning and other point-of-sale data in most industrialized countries in recent years has forced (or enabled, depending on one's point of view) Price Index theorists and practitioners to confront a number of research issues that have long been viewed as hypothetical, settled, or irrelevant. The availability of current information on quantities sold, as well as prices paid, each week, for every individual item sold in many kinds of retail outlets, has made the construction of "Ideal" and "Superlative" price indexes a practical opportunity rather than a theoretical possibility. This paper shows how scanning data can themselves be used to provide answers to these critical research issues, especially those questions involving the aggregation of data across outlets, items, and weeks.

"After such knowledge, what forgiveness?" T. S. Eliot (1920)

1. Introduction

The tendency of CPI Food-at-Home and Energy Item Strata in the U. S., to show higher year-to-year price changes than the average prices of their constituent items has been noted by Reinsdorf (1992, 1993, 1994), MacDonald (1995), and others.[2] This phenomenon is particularly evident in the years since the 1978 CPI revision, and has been estimated to be in the range of 1% to 1.5% per year, but with considerable variation by individual year and item-stratum around the overall mean.

These differences have been attributed to a combination of factors, including the functional form of CPI basic component indexes, seller substitution, the effects of new outlets and items, and quality changes. Research carried out by the U. S. Bureau of Labor Statistics has helped to quantify the likely effect of some of these factors, and to make substantial improvements in CPI procedures as a result. However, exact delineation of the contribution of individual factors to the observed differences has generally not been possible because of sample size constraints and other limitations in the data.

This paper will illustrate how a detailed partitioning and, consequently, an improved understanding of these differences can be achieved using scanning data. It will also show how weekly scanning data can address corresponding issues across the temporal dimension. Finally, it will consider how scanning data can be used to reconcile time-series and cross-sectional (geographic) price comparisons.

[1] I thank Ralph Bradley, Marshall Reinsdorf, Ralph Turvey, and Dick Wittink for useful discussions on this topic.

[2] For example, the A.C. Nielsen Company in the U. S. has observed a similar pattern in comparing brand, sub-category, and total category price trends for certain fast-moving consumer goods. Saglio (1994) has described the same phenomenon in France, drawing upon data from A. C. Nielsen and other sources.

2. The conflict between "unit values" and binary price index ratios in constructing elementary aggregates

As a first approximation, an indexed CPI "average price" series can be viewed as a "unit value" price index at the level of individual homogeneous commodities. This is especially true in comparing price changes over the full 5-year PSU outlet rotation cycle (see Moulton (1995) for a description of this process). Because of the PPS design used for the selection of both individual outlets and items within outlet, the linear "per-pound" average of individual price quotes provides, in effect, a unit value (i.e. quantity-weighted average) across both outlet and item dimensions (although not across the temporal dimension).

Some notation may be useful. Let:

P_{ijk} = price for i^{th} item in the j^{th} outlet during the k^{th} week

Q_{ijk} = quantity sold at P_{ijk}

P_{mnt} = elementary aggregate price for the m^{th} item stratum in the n^{th} geographic stratum for the t^{th} month.

A variable-quantity (unit value) elementary aggregate can be constructed across all 3 dimensions:

$$P_{mnt} = \Sigma\Sigma\Sigma W_{ijk} P_{ijk} \text{, where } W_{ijk} = Q_{ijk} / \Sigma\Sigma\Sigma Q_{ijk} .$$

In this formulation, an "elementary aggregate" price index can be obtained by computing the ratio of unit values for successive months for some aggregation of outlets, items, and/or weeks. The outlets and items to be included need not be identical from one month to the next, but it is assumed that the metrics used to denote individual item quantities are commensurable across the entire item stratum.

Alternatively, an "elementary aggregate" price index can be constructed using "fixed-basket"[3] weights across a constant set of items and/or outlets in successive months. Fixed-weight aggregation can also be carried out across the weekly dimension by taking a simple average of weekly prices at the item-within-outlet level, or by assigning ordinal correspondence between individual weeks in successive months.

The choice as to which dimensions - items, outlets, or weeks - should be summed using variable quantity weights (i.e. unit values), and which using fixed quantity weights (e.g. Laspeyres, Paasche, Fisher, Törnqvist, Edgeworth, Sato-Vartia, etc.), is not intuitively obvious. In general, the literature on this topic [e.g. Balk (1995), Dalén (1992), Davies (1924), Diewert (1987 and 1995), Fisher (1922), Hill (1993), Reinsdorf (1994)] favors construction of unit values at the basic component level if price dispersion - whether across time, items or outlets, and especially when observed for different classes of customer simultaneously within a particular item and outlet - can be regarded as "real" and not reflective of differences in quality. However, such distinctions are, in practice, very difficult to make.[4] Moreover, unit values can be distorted when price dispersion varies across time or when changes in demand are triggered by exogenous events such as changes in income, or weather, rather than by changes in relative price. The theoretical complications of such income-driven variable demand changes can be held at bay by the requirement of homotheticity, but even the wall of homotheticity collapses under the pressure of other real-life exogenous events, such as weather and promotional activities like retailer displays and feature advertising.

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[3] Reinsdorf (1996) has shown how price indexes that satisfy the "identity" axiom (including those in log-index form) can be given a "fixed" basket interpretation.

[4] Pollock (1995) discusses these difficulties at some length. A useful characterization of this issue is also given by Dalen (1992).

Thus, use of unit values can result in price index changes even if individual prices do not change, as the following hypothetical example illustrates for an individual item:

Example 1

	First Month				**Second Month**		
	p	q	pq		p	q	pq
Outlet l	1.00	100	100	Outlet 1	1.00	50	50
Outlet 2	1.00	100	100	Outlet 2	1.00	50	50
Outlet 3	.50	200	100	Outlet 3	.50	250	125
Outlet 4	.50	200	100	Outlet 4	.50	250	125
		600	400			600	350
	unit value = .667				unit value = .583		

In this example, all fixed weight indexes are unchanged between the first and second month, while the unit value price shows a 12.5% decrease. This violates Diewert's (l994) "mean value test" requirement[5] that the price index ratio for a particular commodity, when aggregated across outlets, should fall within the range of the individual outlet price ratios. Diewert (1987) also applies the same test to aggregation across commodities.

Note that the same phenomenon exists if the word "outlet" in the above example is replaced with the word "week". In either instance, the observed shift in quantities might reflect income-driven (or weather driven) rather than price-driven choices.

Hill (1993) recommends against aggregating across (or within) the temporal dimension if the price variation results from "peak period" or "seasonal" factors causing products offered at different times to be viewed as being of different quality[6]. Similar arguments can be made against aggregating across both inter-outlet and inter-item price variation. Viewing "quality" as existing across all three dimensions provides a framework consistent with the notion that the manufacturing sector generates varying quality levels in "form utility", while the distribution sector generates varying quality levels in "place and time utility".

Unit values also provide the most convenient way to accommodate the effect of new items and outlets in the "average price" formulation. However, they do so at the expense of intermingling quality with non-quality-related effects, as well as intermingling price with quantity effects, and new outlet and item effects with price changes for matched items and outlets. In regard to quality effects, both hedonic and "reservation price" quality adjustment procedures are possible but these are difficult to administer, especially given the relatively sparse data sets associated with CPI “item stratum” or “Entry-level item”

[5] Attributed, by Diewert, to Eichhorn and Voeller (1976). The “identity” axiom is a particular case of the “mean value” axiom. Saglio (1994) states this axiom succinctly as a first principle: “if none of the basic prices moves, the index does not move”. Others are not so sure. A central question in this regard is whether the “identity” axiom must hold in each actual realization or simply in expectation; that is, should $E(\hat{I} - I) = 0$ or should $E|\hat{I} - I| = 0$? Is “identity” required, in an arithmetical sense, or is “statistical” identity close enough? The “price index” choice for elementary aggregates imposes arithmetical exactness; the “unit value” choice accepts “statistical” approximation to zero difference as a trade-off for accommodating other, conflicting requirements. As will be shown later in this paper, achieving consistency between geographic and temporal price comparisons is an example of such conflicting requirements.

[6] See also Diewert (1995 b) for some incisive comments on seasonal issues in price index construction. An important early paper on this subject is by Zarnowitz (1961).

(ELI) prices, although both procedures do provide a theoretical means to identify and adjust for differences in quality.

In an important series of papers Wittink and his colleagues [Wittink, Addona, Hawkes, and Porter (1988); van Heerde, Leeflang, and Wittink (1996); Gupta, Chintagunta, Kaul, and Wittink (1996) and Christen, Gupta, Porter, Staelin and Wittink (1996)] have shown what happens to estimates of price elasticity as data are aggregated across the "spatial" dimension from the individual household level successively to the outlet, the organization and, finally, to the geographic market level. Generally, these results show that:

A. Aggregation presents no difficulties in estimating elasticities, providing that the aggregation is done across identical "price points" for the item.

B. The foregoing condition is almost always met when households are aggregated to the individual outlet level, is generally met when outlets are aggregated to the organization level, and is almost always violated at the geographic market aggregation level. It is not clear in what circumstances the last restriction (due to multicollinearity and Jensen's inequality) can be overcome by statistical "corrections".

The "item" dimension is much more problematic. Accepting unit values constructed across homogeneous commodities is an attractive generalization but, to some extent, begs the question since it is not clear what is meant by a "homogeneous" commodity. There are, in the U. S. , around 800,000 separate food items (UPC's) sold in supermarkets, with a typical individual supermarket outlet handling around 15,000 items. The U. S. Consumer Price Index compresses these down to 53 item strata (with 66 entry-level line items), so the "right" number of commodities is somewhere between 53 and 800,000[7]. There are several possible agglutinative criteria, such as ingredients, usage, brand name, flavor, cross-elasticities of demand, and perceptual or competitive maps; but the most reliable criteria are also likely to include homogeneity of prices or, perhaps, price changes. Szulc (1994) suggests (but seems to reject as unworkable) defining homogeneity in terms of price levels.

Scanner data will be critical to an understanding of this important issue. The 800,000 items cited above represent more than 150 billion individual purchases transactions per year. As Balk (1994) has noted, each of these transactions is, in some way, different from every other with respect to conditions of sale or product attributes.

[7] The A. C. Nielsen Company, in the U. S., divides all 800,000 food-at-home items into around 1,000 mutually exclusive and exhaustive product categories, called "modules". This is almost certainly a lower bound.

2. Description of the data analyzed

Differences between "average price" and "fixed weight" indexes involve the joint contribution of 7 item and outlet price and quantity components:

1. The difference between fixed and variable quantities for items common to both periods in outlets that are in business in both periods (called “identical outlets").
2. Prices and quantities of items common to both periods at the geographic market level but disappearing from individual “identical outlets”.
3. Prices and quantities of items disappearing from the geographic market.
4. Prices and quantities of items in outlets that go out of business.
5. Prices and quantities of items common to the geographic market but new to individual “identical outlets”.
6. Prices and quantities of items new to the geographic market.
7. Prices and quantities of items in newly constructed outlets.

In this paper I examine the effect of each factor on the coffee scanning data set described by Reinsdorf (1995). In that data set, A. C. Nielsen Scantrack scanning data for Roasted and Instant Coffee (quantities and prices) from 150 supermarkets in the Chicago and Washington DC metropolitan areas for 4 weeks in December 1993, and for the corresponding 4 weeks in December 1994, were aggregated using unit values across weeks to generate December "monthly" prices and quantities for each of the years. Laspeyres and Fisher Ideal price indexes were computed separately for Roasted and Instant Coffee and separately for each market based on common items in identical stores. These resulting price index trends are "direct" December year-to-year measures, and have not been chained through the intervening months.

For the present analysis, a "unit value" index was also computed for each of these four data sets, enabling the effect of "unit value" weighting to be measured directly. Similarly, each factor contributing to the difference between the full data set of items and outlets, on the one hand, and the "Fisher" data set of identical items and outlets, on the other, was identified, with the effect for each factor tabulated separately.

3. What do the data show?

The results of this analysis are summarized on the attachments. While the results are largely self-explanatory, a few comments seem appropriate.

First, the most surprising (to this author) result is the difference in importance between items "new to the market" (around 3% of dollar sales) and items "common to the market but new to the outlet” (around 6% of dollar sales), with a similar difference in importance for discontinued items. Most discussion of "new items" in the literature tend to ignore this distinction. The deeper question raised is how to define a "competitive set" of outlets such that "new to the set of outlets" is viewed by consumers as equivalent to "new to the market". In this context, other definitions of item "newness" are also possible (e.g. new to the neighborhood, new to the city, new to the U. S.) It is not evident which of these definitions is to be preferred. Finally, new and discontinued outlets contribute 7% to 8% of total sales, with the result that more than 16% of dollar sales, in each year, is absent from the "direct" Fisher Ideal Price Index year to year trend (Table 1).

Second, the differences between Fisher and unit value weights average less than 1% per year (Table 2). The effect of changing from Fisher to unit value weights is larger across items than it is across stores. This is not surprising, since most marketing research studies[8] have shown that brand switching is far more important than store switching in this industry.

Third, from Tables 2 through 5 it is apparent that no one source of difference is dominant across each of the two markets and product types examined. Nor - except, as expected, for the Laspeyres upward bias - is any source of difference persistently in one direction or the other. The inclusion of new items and stores serves to reduce price increases, as expected.

Fourth, the large number of items, and the large turnover of items at the individual store level, suggest that some unit value aggregation may be needed to stabilize the data, such as:

A. Aggregation across outlets to the organization (or at least the price zone within organization) level.
B. Aggregation across items with identical prices and price-determining characteristics (e.g., flavors of yogurt or gelatin desserts, or grinds of coffee, within brand).
C. Aggregation across "parent - child" item combinations (e.g. "bonus pack", or "cents off" packs bearing separate Universal Product Codes). Reinsdorf (1996) has shown that failure to take account of "bonus packs" can create a significant bias in a price index for coffee.

Fifth, I have disregarded, for purposes of this analysis, some of the "functional form" issues mentioned earlier, such as the "Sauerbeck Index" overstatement and the difference between POPS survey and "price initialization month" item weights. These issues have been, or will be, addressed through other CPI data improvement programs.[9]

Sixth, the major source of difference between BLS "average price" and "CPI" methods, in this data set, reflected the inadequacy of "Roasted Coffee" as a proxy measure for "Total Coffee". There was a roughly 30% - 40% difference in year-to-year price trends between Roasted and Instant Coffee in 1994, and this resulted in a + 7% difference in year to year trends between "Roasted" and "Total Coffee". This points out the definitional problems of comparing item stratum CPI trends with constituent item average prices, and it leads to the following point.

Seventh, "average price" computations require a common size metric across items. Because of non-linear pricing across sizes within brand[10], this requires caution even when all other product characteristics are in common. Other equivalizations across different product forms are more difficult, and require common industry standards, as in trying to equivalize across Roasted and Instant Coffee varieties.

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[8] Described, for example, by Adamek (1991), Blattberg, Briesch, and Fox (1995), Chiang (1991), Gupta (1988), and van Heerde, Leefland, and Wittink (1996)

[9] For example, those discussed by Moulton (1993), Reinsdorf and Moulton (1994), and U.S. Bureau of Labor Statistics (1995).

[10] See attached example of how non-linear prices are often non-linear in non-obvious ways (Table 7). Saglio, however.(1994) found little brand-size effect in his analysis of candy prices trends in France.

Nonetheless, in examining average prices across components, we note the following patterns (Table 3 and 4):

A. New stores have average prices per pound, in 1994, around 10% lower than continuing stores. As discussed above, this is in line with expectations.

B. In contrast, both new and old items (with respect to the market) are around 25% higher in average price per pound than continuing items. This indicates that the higher-priced coffee segments (in this instance, "gourmet" or specialty coffee items) have a higher rate of "churn" (i.e., year-to-year turnover)than the lower priced items. Again, this is consistent with expectations based on commodity-specific knowledge. It may not be a generalizable result.

C. On an unweighted basis, the average price of all items in all stores shows a 1.0% lower year-to-year trend than the universe of common items in unified stores. New and discontinued outlets contribute about half of this difference, with new and discontinued items contributing the other half.

Eighth, these findings should be viewed as indicative rather than definitive. The decompositions shown are arithmetically exact but, because the data are based on a sample of stores in a purposive selection of markets, the results are not exact in a statistical sense. The price increases shown for coffee items in 1994, driven primarily by a severe frost in Brazilian coffee-growing regions, were exceptional. However, the methods used in the analysis should be applicable to other goods, other markets, and other time periods.

Ninth and last, the effects of excluding the cross-sectional effect of new items and outlets from a "Fisher Ideal" framework can be reduced in several (or a combination of) ways in a scanning environment:

A. Hedonic measures, such as those described by Kokoski, Cardiff and Moulton (1994), Primont and Kokoski (l990, 1991), Liegey (1994), and Kokowski (1994). These could be greatly improved by using scanning data to increase the amount of information available for estimation of individual hedonic factor effects.[11]

B. "Reservation price at zero prior sales" estimation methods[12] (although these are likely to be impractical in a large-scale processing environment).

C. A blend of "Fisher Ideal" (or Edgeworth or Tornqvist) index trends, for identical items and outlets, with the quantity-weighted contribution of new and discontinued items and outlets adjusted for quality change (Table 6).

D. The outlet and item aggregation procedures described on page 11.

[11] In an early paper, Grilliches (l961) attributes the initial use of hedonic techniques to A. T. Court (1939). The literature on the subject is extensive.

[12] Described, for example, by Hausman (1995).

4. The question of temporal aggregation

For purposes of the initial discussion, we will be making several simplifying assumptions, viz.

1. The price of an individual item in an individual outlet changes no more frequently than once per week, which is the same assumption that Hicks (1946) proposed. For the U. S. Supermarket industry, it is a reasonable assumption.

2. Such price changes as do occur take place precisely at the beginning of the week.

3. Each month is neatly divided into 4 calendar weeks.

4. Seasonal effects (either climate-driven or event-driven, in Diewert's (1996) characterization) are not strong enough to require separate coding of individual items seasonal peaks.

5. Simultaneous price variation for an individual item in an individual outlet can, and should, be accommodated by unit values, as Hill (1993) suggests.

6. "Chaining" or "transitivity" issues, across the temporal dimension, can be ignored, and a consequent focus on "direct" measurement of year-to-year price changes can be regarded as instructive.

7. Individual weekly price and quantity information, updated through the "current period", is available through scanning data for every item, for at least a representative sample of outlets.

These assumptions inform the conclusions that follow. Assumption 6 is the most problematic, as will be seen in footnote 18 and Table 9. In this study, the monthly data sets described above are divided into 4 constituent data weeks within each December month.[13] Price Indexes were computed comparing each week with the year-ago week, and are presented in Table 8. In addition, a "monthly" price index was prepared by pooling all weekly binary year-to-year item-by-outlet-by-week price ratios multiplied by their associated expenditures, and summing to a "monthly" total. Finally, price indexes were estimated by multiplying December 1993 and December 1994 expenditures by each of the 4 paired weeks price ratios. The results are significant. They show that:

1. Weekly Laspeyres indexes are biased upward relative to monthly indexes. This is true even when the weekly observations are pooled across the month.

2. In contrast, weekly Fisher indexes, particularly when pooled to the monthly level, are remarkably consistent with monthly indexes. This means that, at the monthly level, data pooling and data aggregation both lead to the same outcome.

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[13] For the analysis in section 4, "monthly" aggregates have, in all cases, been taken as unit value aggregates across the dimension of time; i.e., the sum of 4 weekly dollar sales at the item-within-outlet level, constitute "expenditures", while the corresponding sum for unit sales constitutes "quantities". It should be recognized that the resulting implied "price" values themselves make use of current period weekly quantities in their unit value weighting. Strictly speaking, this results in a oddly constructed "Laspeyres" index to the extent that current period quantities are taken into account in the current price variable but are disregarded as quantities in the Laspeyres form. Note, however, that this is not an issue in dealing with individual weekly price indexes, since weekly prices are generally measurable without the need to measure weekly quantities.
This implies that it would be inappropriate, in evaluating a "base-period-weighted geometric mean" Index formula, to utilize "current period" prices that are weighted, within the month, by current week quantities. For such an evaluation, a linear average of weekly prices across each month would seem a more appropriate price measure.

3. The difference between the preceding two conditions is a function of the base periods used and not the price ratios themselves. To see this, note in table 8 that the 4 possible "pseudo-monthly" Laspeyres price indexes, generated by multiplying December 1993 expenditures by each of the 4 paired weeks' price ratios, resemble the true "monthly" calculation. Evidently the correlation between price ratios and base period expenditures is sharply reduced by use of the longer base period.

4. There is strong intra-month week-to-week variation in prices and in price index year-to-year trends, pointing out the need to represent prices through the entire month in a price index, rather than "sampling" individual weekly prices. This has both variance and bias implications.

The net of the foregoing is to suggest that, if these results are borne out by further studies on other product categories, weekly data can should be aggregated to the monthly level using "unit value" procedures as input to a Fisher Index. This has the virtue of reducing the data set by three-fourths, while also avoiding the need to estimate prices for items that are handled by the outlet but with zero sales for a particular week. Such instances are still possible in a monthly environment, but to the extent that these zero sales conditions are driven by random[14] variations in demand rather than out-of-stocks, the probability of their occurrence in a month of data is only the fourth power of their probability of occurrence in a single week of data. This result should have signal consequences for the size and complexity of the data bases that would need to be handled in a scanning environment.

Beyond the issues discussed above, temporal aggregation also raises difficult questions of between-week substitutability, carryover effects, "forward buying", household inventory accumulation, opportunistic and "predictive" household purchasing patterns, "flow of services" from purchases, and related topics. Researchers have speculated at length about the existence of carryover effects associated with short-term retailer price promotions, but these have been difficult to demonstrate empirically amid the general noise of outlet-level data. Household-level analyses have fared a little better, with two studies describing very slight "stockpiling" effects. These studies are cited in van Heede, Leeflang, and Wittink (1996); but a more important result brought out in their paper is that when outlet-level data are aggregated to the chain or organizational level, purchase acceleration then becomes discernible, and accounts for no more than 11% of the price-driven sales increase for canned seafood, a category highly susceptible to stockpiling. The comparative absence of stockpiling effects may help explain the consistency of Fisher monthly and weekly Indexes cited above.

Bradley and Verdon (1996) have also discussed some of these issues in a COL context, as did Pollock (1975).

[14] The randomness here can be viewed as Poisson, negative binomial, gamma, or log-series, depending on the consumer behavior model assumed. Szulc (1994) characterizes the variability across time in outlet-level sales as "Brownian movements."

4. Reconciling cross-sectional and time-series price index measurements

It is interesting to note that for geographic cross-sectional analysis (i.e., comparison of price levels between individual cities or regions,) unit values are clearly required across the outlet dimension, since there is no natural sequence order for outlets, no correspondence in number of outlet observations, and hence no "binarity" on which to compute price ratios. Whether appropriate "matching" criteria can be found even at the item level is an open question. If every item found in region "A" were also to be found in Region "B", one could weave these regions together using a Fisher or some other "binary" item linkage procedure and compare overall price levels accordingly. However, certain items and even brands found in one city will often not be found in another (in fact, in comparing individual country price levels, exact item matches will be even less frequent). Thus one may be forced into unit value aggregation (or acceptance of less-than-exact matches) across items as well as across outlets.

Note, however, that the temporal dimension is exactly the same across all geographic units. Should annual price level comparisons between cities, then, be based on aggregation of 12 individual monthly indexes; or should an annual price first be determined for each commodity using unit values, with the annual index then computed from these annual commodity indexes? The same question will arise in geographic comparisons of monthly totals derived from weekly data.
These aggregation issues are encountered in trying to answer, for example, the seemingly simple question of how rapidly coffee prices increased between 1993 and 1994 in Chicago compared with Washington (Table 10). Ideally, one would like to see the property of "rectangularity", i.e., dimensional consistency[15], between a geographic comparison of individual city year-to-year price ratios, and year-to-year changes in the comparative price levels of two cities. Such "rectangularity" or "dimensional consistency", however, is difficult to achieve, as Kokoski, Moulton, and Zieschang (1996) have discussed. In the Coffee data set examined above, for example, 97% of expenditures take place among items matched across time within each city; but only 70% of expenditures in December 1994 are for items found in both cities. It is evident that the concepts of "item" and "commodity" will need to be refined before meaningful price level comparisons can be made across geographic units. Note, however, that even for a group of items common both markets and both periods, dimensional consistency is not assured.

In the preceding example the Sato-Vartia price index formulation does improve dimensional consistency to some extent. One may note (see table 11 for an example) that under constant but non-unity elasticity of substitution the Sato-Vartia achieves "price and quantity permutation indifference" when neither Fisher nor Tornqvist indexes do so. "Price and quantity permutation indifference" means that if outlets simply exchange prices and quantities in successive periods, the index does not change. This suggests that the Sato-Vartia index may be less sensitive to the usual requirement that binary price ratios be computed from the same outlets. Research into this question is now underway.

It is clear that the "law of one price" fits the price of, say, Coca-Cola in the stock market better than it fits the price of Coca-Cola in supermarkets. While it is not fully understood how search costs and other frictions affect price variability, Anglin and Baye (1987) have made a good start. The availability of detailed item and outlet characteristics from scanning data should permit the development of models that will separate "real" price differences, at both item and outlet levels, from "quality-based" price differences, both across time and markets.

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[15] I have chosen "rectangularity" to denote dimensional consistency between temporal and geographic comparisons. The term "circularity" has, unfortunately, already been appropriated for use in describing transitivity across time, even though "circularity" implies an eventual return to a starting point and thus makes more sense in a geographic context. Except in relativity, only in a Viconian or Joycean world does the arrow of time traverse a circle.

5. Summary

"Average Price" or "Unit value" price indexes, at the basic component or elementary aggregate level, have the attractive property of being able to accommodate changes, over time, in both store and item universes. However, they do so at the expense of intermingling price with quantity effects for existing stores and items, and of requiring adjustments for any quality changes represented by the new and disappearing items and stores.

In contrast, fixed-base or even "superlative" indexes perform well for measuring price changes across a constant universe of items and stores, but cannot easily accommodate new varieties and outlets in the bilateral context that these indexes require.[16]

To resolve this dilemma, a cross-sectional adjustment of Fisher ratios to accommodate the effect of new and discontinued items and stores after quality adjustments, together with some partial unit value aggregation procedures across similar stores and items, may provide an attractive compromise.
Finally, it is encouraging to note that there seems to be little conflict between Fisher indexes at the weekly and monthly levels, although these very preliminary findings need to be replicated on more product categories.

As Dalén (1992) has pointed out, there is still little theoretical, let alone practical, consensus on how to handle new items and outlets in a Consumer Price Index framework; nor is there full agreement on how Price Indexes should best be constructed within the temporal dimension or how best to achieve temporal and spatial consistency.

However, scanning data[17] offer a uniquely rich research environment to foster the development of such procedures[18] as may lead to consensus in the future. [19]

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16 As Armknecht, Lane, and Stewart (1994) describe, new items are accounted for in the Consumer Price Index, between point-of-purchase survey revisions, only to the extent that they are required to replace discontinued items in specific outlets. Because of the linkage procedures used, differences in price levels between continuing and new items and outlets are generally not reflected in the CPI.

17 At the present time (April 1997), scanning and other point-of-sale recording accounts for about half of all U. S. consumer expenditures for goods, and 20-25% of total consumer expenditures. Silver (1994) has also shown how scanning data can be used to good effect for consumer durables in the U. K.

18 The U. S. Consumer Price Index collects prices for each sampled food outlet on one day of each month. Pricing is carried out over the first 18 working days of the calendar month. It is an open question how monthly price indexes would actually be constructed if weekly quantities and prices were available on a current basis. Coffee scanning data for the months between December 1993 and 1994, although outside the scope of the study, show evidence of drift between chained and direct Fisher Indexes (Table 9). More empirical work is needed to determine what Index form would best optimize transitivity.

19 Even if government statistical agencies cannot yet, for technical, budgeting or political reasons, utilize scanning data directly in a full production environment, these data can still be used for three key purposes: to help keep expenditures weights up-to-date; to select individual items for pricing, and to incorporate new items into the Consumer Price Index on a timely basis; and to make better decisions on the appropriate functional form of the Price Index even in the absence of current period quantities. For example, there is considerable speculation as to whether a “geometric mean” compilation of individual price quotes up to the basic aggregation level would or would not provide good approximation of a true cost-of-living measure. This debate hinges on assumptions about between-item cross-elasticities and the stationarity of item price dispersion across time. Yet these assumptions could easily be checked, using scanning data, for many food-at-home item-strata. This would enable better decisions to be made regarding the appropriateness of the geometric mean formula at the basic aggregation level.

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Table 1. Percent of total scantrack coffee dollar sales in combined metro Chicago and Washington markets (December 1993 and 1994, by status of item and outlet)

Item/store universe	Percent of sales	
	1993	**1994**
Identical Items in Identical Outlets	83.7%	83.4%
Items in Discontinued Outlets	7.8%	-------
Items Disappearing from the Market 1994	2.2%	-------
Items Disappearing from the Outlet but Retained at the Market Level in 1994	6.3%	-------
Items in New Outlets	-------	7.3%
Items New to the Market in 1994	-------	3.5%
Items New to the Outlet in 1994 but in Existence at the Market Level in 1993	-------	5.8%
Total All Items	100.0%	100.0%

Table 2. Summary of year-to-year price index ratios and differences between Laspeyres and unit value index for coffee in metro Chicago and Washington supermarkets (based on scantrack data for December 1993 and December 1994)

	Coffee Type (ELI) and Market				Weighted** Average of Markets and Types
	Chicago		Washington		
Index Type and Composition	**Roasted**	**Instant**	**Roasted**	**Instant**	
Laspeyres Direct*	184.6	131.9	157.9	130.7	161.9
Fisher Direct*	175.3	130.3	156.9	128.8	157.1
Fisher across Items, Unit Value across Outlets*	172.0	130.8	157.8	128.2	155.9
Unit Value across Matched Items in Identical Outlets	180.1	133.0	154.2	131.2	159.4
Unit Value across All Items and Outlets	176.7	131.6	156.5	130.1	158.0

** Törnqvist weights were used for aggregation across markets and types in all cases, since the intent here is to focus on lower-level rather than higher-level aggregation issues.

* These numbers differ slightly from the results reported by Reinsdorf. His December l993 quantities and prices included 5 weeks of data; this table includes 4 weeks.

Table 3. Components of percentage difference between Laspeyres and unit value price index year-to-year ratios (December 1993 to December 1994; for coffee in Chicago and Washington metro area)

	Coffee type (ELI) and market						
	Chicago		Washington		Both markets combined		
Sources of difference	**Roasted**	**Instant**	**Roasted**	**Instant**	**Roasted**	**Instant**	**Total**
Difference in formula (Laspeyres vs. Fisher)	+5.3%	+1.2%	+0.6%	+1.4%	+3.9%	+1.3%	+3.1%
Difference in weights (Fisher vs. Unit value)	-2.6%	-2.1%	+1.7%	-1.8%	-1.4%	-1.9%	-1.5%
Difference in composition ("matched" items and outlets vs. all items and outlets)	+1.9%	+1.1%	-1.5%	+0.8%	+1.0%	+1.0%	+1.0%
Total difference	**+4.5%**	**+0.2%**	**+0.9%**	**+0.4%**	**+3.5%**	**-0.4%**	**+2.5%**

Table 4. Effect of new and discontinued items and outlets on year-to-year trends in average price per pound (for scantrack total coffee data, metro Chicago and Washington combined)

	Price per pound			
Universe component	**December 1993**	**December 1994**	**Year-to Year index**	**Index vs. Total universe**
All items and outlets	$2.862	$4.434	1.550	-----
Identical outlets	$2.869	$4.469	1.558	+0.5%
New outlets	--------	$4.037	------	-------
Discontinued outlets	$2.782	-------	------	-------
Matched items in identical outlets	$2.842	$4.405	1.546	-0.2%
New outlets and outlet level items	--------	$5.429	------	-------
Discontinued outlets and outlet-level items	$3.624	------	------	-------
Items common to unified outlets (fisher universe)	$2.841	$4.447	1.565	+1.0%
New outlets and outlet-level items	-------	$4.370	------	-------
Discontinued outlets and outlet-level items	$2.973	-------	------	------

Table 5. Effect of new and discontinued items and outlets on year-to-year trends in average price per pound (for scantrack coffee data, December 1993-December 1994)

	Price per pound											
	Chicago						Washington					
	Roasted			Instant			Roasted			Instant		
	1993	1994	Index Trend	1993	1994	Index Trend	1993	1994	Index Trend	1993	1994	Index Trend
All items/outlets	$2.132	$3.767	176.7	$8.589	$11.303	131.6	$2.354	$3.684	156.5	$8.272	$10.763	130.1
Matched items in matched outlets	$2.082	$3.749	180.1	$8.814	$11.772	133.0	$2.404	$3.707	154.2	$8.317	$10.910	131.2
“Discontinued” items or outlets	$2.455	------	------	$7.745	-------	------	$2.153	-------	------	$7.971	--------	------
New items or outlets	------	$3.852	-----	------	$ 9.892	-----	------	$3.564	-----	------	$ 9.712	-----

Table 6. Adjustment of "direct" Fisher price index year-to-year ratios, December 1993 to December 1994 (for effect of new and discontinued items and outlets assuming no quality differences from such items and outlets.)

	Unit value			Index	
Scantrack data components	**All items/outlets**	**Common items/outlets**	**All item adjustments**	**Direct Fisher**	**Adjusted Fisher**
Chicago roasted	176.7	180.1	0.9811	174.7	171.4
Chicago instant	131.6	133.0	0.9895	130.3	128.9
Washington roasted	156.5	154.2	1.0149	157.8	160.2
Washington instant	130.1	131.2	0.9916	128.6	127.5

Table 7. Examples of non-linear pricing organization "y"

Brand "a" corn oil

Size	Price	Price per quart
16oz.	$1.53	$3.06
24oz.	$1.95	$2.60
32oz.	$2.45	$2.45
48oz.	$3.15	$2.10
64oz.	$4.13	$2.07
128oz	$7.87	$1.97

Brand "b" coffee

Size	Price	Price per pound
13oz.	$2.71	$3.34
26oz.	$5.23	$3.22
39oz.	$7.59	$3.11

Brand "c" rice

Size	Price	Price per pound
1 lb.	$0.63	$0.63
2 lb.	$1.19	$0.59
5 lb.	$2.75	$0.55
10 lb	$4.79	$0.48

Brand "d" peanut butter

Size	Price	Price per pound
12oz.	$1.65	$2.20
18oz.	$1.89	$1.68
28oz.	$3.09	$1.77
40oz.	$4.35	$1.74
64oz.	$6.85	$1.71

Brand "e" sugar

Size	Price	Price per pound
1 lb.	$0.63	$0.63
2 lb.	$1.01	$0.51
5 lb.	$1.99	$0.40
10 lb.	$3.95	$0.39

Brand "f" large eggs

Size	Price	Price per egg
6	$0.79	$0.131
12	$1.29	$0.107
18	$1.95	$0.108

Brand "g" 2 % milk

Size	Price	Price per quart
1 qt.	$0.95	$0.95
1/2gal	$1.69	$0.84
1 gal	$2.69	$0.67

Brand "h" catsup

Size	Price	Price per quart
14oz.	$0.91	$2.08
20oz.	$1.12	$1.79
28oz.	$1.95	$2.23
40oz.	$2.59	$2.07
60oz	$3.59	$1.80

Table 8. Year-ago "direct" price index % change (December 1993 to December 1994)

	Chicago		Washington		Both markets types
	Roasted	**Instant**	**Roasted**	**Instant**	
Laspeyres indexes					
Monthly prices and weights	+84.6	+31.9	+57.9	+30.7	+61.9
Weekly prices and weights					
first week	+98.8	+35.1	+63.2	+30.0	+69.5
second week	+90.9	+33.4	+57.8	+38.8	+65.4
third week	+94.9	+31.3	+55.5	+28.6	+66.7
fourth week	+84.0	+29.5	+57.4	+24.3	+60.9
all weeks pooled	+92.3	+32.4	+58.5	+30.9	+65.7
Weekly prices, monthly weights					
first week	+87.3	+33.3	+60.1	+27.7	+63.4
second week	+83.3	+32.7	+57.6	+33.0	+62.0
third week	+78.7	+30.8	+56.3	+27.8	+58.2
fourth week	+78.8	+29.7	+56.6	+24.3	+57.6
all weeks pooled	+82.1	+31.9	+57.6	+28.2	+60.3
Fisher indexes					
Monthly prices and weights	+75.3	+30.3	+56.9	+28.8	+57.3
Weekly prices and weights					
first week	+84.3	+31.8	+59.9	+28.5	+62.4
second week	+76.5	+31.1	+55.9	+32.9	+57.7
third week	+72.6	+29.3	+54.4	+26.7	+55.8
fourth week	+72.2	+28.8	+56.4	+23.6	+55.4
all weeks pooled	+76.4	+30.3	+56.7	+28.3	+57.8
Weekly prices, monthly weights					
first week	+80.0	+30.8	+59.1	+26.6	+59.6
second week	+76.6	+30.8	+56.3	+30.2	+58.2
third week	+69.2	+29.3	+55.5	+26.7	+53.8
fourth week	+72.6	+28.7	+56.5	+24.1	+55.0
all weeks pooled	+74.6	+29.9	+56.9	+26.9	+56.6
Unit value indexes*					
Monthly	+80.1	+33.0	+54.2	+31.2	N. A.
Weekly					
first week	+82.3	+35.0	+61.5	+32.4	N. A.
second week	+76.8	+32.6	+52.1	+33.2	N. A.
third week	+86.1	+33.3	+49.0	+31.7	N. A.
fourth week	+74.6	+27.1	+54.6	+26.6	N. A.
all weeks pooled	+80.1	+33.0	+54.2	+31.2	N. A.

* Based on common (matched) items in identical stores.

Table 9. Comparison of "chained" vs. "direct Fisher price indexes for December 1994 (December 1993 = 100; based on scantrack coffee data for metro Chicago and metro Washington)

	Metro Chicago		differences		Metro Washington		differences	
	Chained	Direct	Points		Chained	Direct	Points	
Roasted	168.2	175.3	-7.1	-4.1	154.4	156.9	-2.5	-1.6
Instant	135.5	131.9	+3.6	+2.7	130.6	129.1	+1.5	+1.2
Total	159.4	164.3	-4.9	-3.0	147.1	146.2	+0.9	+0.6

Table 10. Rectangular intransitivity between Chicago and Washington year-to-year price changes. (December 1993 - December 1994, based on scantrack data for 33 major roasted coffee items)

	1994 vs. 1993 price trends			Chicago vs. Washington price levels			Discrepancy between 2 ratios
Index	Washington	Chicago	Ratio	1993	1994	Ratio	
Fisher	1.5479	1.8820	1.2158	0.8949	0.9972	1.1143	+9.11
Törnqvist	1.5476	1.8666	1.2061	0.8939	0.9984	1.1169	+7.99
Sato-Vartia	1.5503	1.8659	1.2036	0.8913	1.0009	1.1230	+7.08

Table 11. Sensitivity to price and quantity permutations

Example 1

Outlet	**P_0**	**P_1**	**Q_0**	**Q_1**			**I_1/I_0**	**I_0/I_1**
a	$4.00	$2.00	1	4	Laspeyres	=	3.3333	2.3333
b	$2.00	$1.00	4	16	Paasche	=	0.4286	0.3000
c	$1.00	$4.00	16	1	Fisher	=	1.1952	0.8367
					Törnqvist	=	1.0506	0.9518
					Sato-Vartia II	=	1.0000	1.0000

Example 2

Outlet	**P_0**	**P_1**	**Q_0**	**Q_1**			**I_1/I_0**	**I_0/I_1**
a	$1.00	$2.00	64	16	Laspeyres	=	1.8750	4.5000
b	$2.00	$4.00	16	4	Paasche	=	0.2222	0.5333
c	$4.00	$8.00	4	1	Fisher	=	0.6455	1.5491
d	$8.00	$1.00	1	64	Törnqvist	=	0.8705	1.1488
					Sato-Vartia II	=	1.0000	1.0000

Summary for examples

Index	**Permutation indifferent**	**Time-reversible**
Laspeyres	No	No
Paasche	No	No
Fisher	No	Yes
Törnqvist	No	Yes
Sato-Vartia II	Yes	Yes