# Measuring price change in mobile-telephony services: an arduous task 

Françoise Le Gallo and François Magnien

INSEE, France
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The recent but prodigious growth of mobile telephony has made it the emblematic product of the new information and communication technologies (NICTs). In France, its rise has been spectacular: introduced in 1995, it now boasts a combined total of nearly 40 million individual and corporate users. Three operators share the French market: Orange ( $50 \%$ of the customer base at end-2002), SFR (35\%), and Bouygues Telecom (15\%). Individual users account for somewhat over two-thirds of their customer base.

There are two reasons for the relatively high prices of mobile-telephony services. First, they are the counterpart of the massive investments by operators for their network implementation and expansion. Second, they reflect subscriber-acquisition costs in the form of bonuses paid to service-marketing companies and subsidies for handsets.

Today, cards and packages are equally popular with the public. Cards are now by far the more common solution among individual users. This has not always been the case, as chart 1 shows.

## 1. A "constant-utility" or rather "constant-usage" index

The price/volume decomposition of mobile-telephony services is a maddeningly difficult puzzle for statisticians. Measuring price change in individual countries is tricky. The methods are diverse and lack transparency. However, a suitable approach may gradually win acceptance. It is already being applied in the German consumer price index (CPI) and, since January 2003, in the French CPI. The method has also been used-admittedly on a trial basis-in the United States, as part of a study by Hausman (1999), as well as in the United Kingdom and France by their telecoms regulatory bodies, respectively OFTEL and ART. The principle is simple: it involves tracking the minimum expenditure required to satisfy individual or corporate consumption patterns as summarized in selected "profiles." The minimum expenditure makes allowance for the abundant supply of pricing plans from mobiletelephony operators. The principle can subsequently be tailored to factor in the time taken by consumers to adjust to the choice of optimal offerings.

It would be more appropriate to describe this as a constant-usage index rather than a constantutility index (CUI). A true CUI for prices would require a particularly delicate exercise: the explicit or implicit estimation of a utility function (Magnien and Pougnard 2000) or demand function (Hausman 1999). That would involve an unreasonable effort within the framework of current CPI production. A "constant-usage" index is a far easier proxy to develop. With the

Chart 1: Explosion of "consumer" base' and growing share of pre-paid


1. Residents of metropolitan France (mainland + Corsica), users of a SIM card or pre-paid card who have made at least one call and have not exceeded the time limit for receiving incoming calls with the card.

Sources: ART; authors' computations; INSEE
aid of surveys, we can describe consumption patterns: number of calls, call duration, callperiod distribution, number of SMSs (text messages) sent, etc. For each product offered, we can then compute the expenditure entailed by the usage defined on these criteria and determine the minimum expenditure.

To go a step further in presenting the methodology of the price index for mobile-telephony services, some formalization is needed. In each month $m$, consumers can choose from the products $p$-packages or cards-put on the market by operators. Assuming that consumers are rational, fully informed, and have no constraints (these hypotheses will be discussed in the final section of this paper, where we propose an alternative model), they will opt for product $\hat{p}$, which minimizes their expenditure $D^{p, m}$ :

$$
\begin{equation*}
D^{\hat{p}, m}=\operatorname{Min}_{p} D^{p, m} \tag{1}
\end{equation*}
$$

(such a product, not always unique, will be called "optimal"). With $\hat{D}^{m}$ as this minimal expenditure, the index using month 0 as base 100 is $\hat{D}^{m} / \hat{D}^{0}$.

## 2. A detailed segmentation into consumption profiles

It is assumed that consumers change products "instantly" as soon as a more economically attractive one appears. But reality is very different. For example, switching operators has a cost-in particular, paperwork and changing phone numbers. We therefore had to restrict the application of the proposed methodology to consumer sub-categories, identifying product ranges with a very low short-term substitutability. We accordingly separated the profiles for the three operators.

Switching from a package to a card entails costs. Therefore, we also had to classify consumers according to their contract: subscription (package) or pre-paid (card).

But a classification restricted to six consumer types or "consumption profiles" would have been very inadequate. The product that minimizes a consumer's expenditure basically depends on his or her call volume. If we amalgamate consumers whose "sizes" are too diverse in the same profile, we will summarize their consumptions by an average monthly duration. In model (1), the optimal product will diverge from the actual optimal products for the consumers in that profile. Accordingly, we have specified three consumption levels (high, medium, and low) and three call time distributions (mostly daytime, mostly evenings and weekends, mixed) for the two categories of products selected (packages and cards).

These criteria yield 54 profiles, 18 per operator. We aggregated the indices of the 54 profiles using a Laspeyres procedure so as to obtain the index for all consumers in a month $m$ measured against a base month 0 :

$$
\begin{equation*}
I^{m / 0}=\sum_{T} w_{T}^{0} \hat{D}_{T}^{m} / \hat{D}_{T}^{0} \tag{2}
\end{equation*}
$$

In this formula, $w_{T}^{0}$ denotes the expenditure by all consumers in profile $T$ for the use of the optimal product in month 0 :

$$
\begin{equation*}
w_{T}^{0}=S_{T}^{0} \hat{D}_{T}^{0} / \sum_{T=1}^{54} S_{T^{\prime}}^{0} \hat{D}_{T^{\prime}}^{0} \tag{3}
\end{equation*}
$$

where $S_{T}^{0}$ is the number of consumers in profile $T$ in period 0 . We chained the resulting indices (on a monthly, quarterly or annual basis) to allow for the changing structure of the population of consumers of mobile-telephony services.

## 3. The survey of mobile-telephony operators on consumption profiles

To calculate the price index for mobile-telephony services, INSEE has launched an "annual survey of operators on consumption profiles." The purpose is to construct the "profiles" (or typical consumers) mentioned earlier. The first survey, in early 2002, covered consumption patterns for mobile-telephony services in 2001. The questionnaire was prepared in consultation with the three operators.

Each operator divided its customer base into consumption "profiles" defined by the successive application of three criteria: contract type (package or pre-paid), total monthly call duration (short, medium or long), and the call-time distribution (daytime, evenings and weekends, mixed).

Table 1, compiled from the survey results, shows that call volume is much larger for package users. The survey allows a finer analysis of these figures at profile level-an analysis that does not appear in the table. In particular, the number of calls and monthly call duration increase with the consumption level, irrespective of whether packages or cards are used.

After classifying their customer bases by profile, the operators described the profiles by computing mean values, listed in table 2. Averages were calculated for all available months in 2001 (or at least for the latest six) for the following: monthly duration of national calls; breakdown of monthly call duration by call period; breakdown of monthly call duration by
destination; number of calls per month; monthly duration of calls to "favorite number." The survey also asks operators for the breakdown of customers in each profile by product.

Table 1: Number of calls and average monthly call duration ${ }^{1}$ per user ${ }^{2}$

| Contract type | Number of calls | Actual duration of <br> one call | Actual monthly call <br> duration |
| :--- | :---: | :---: | :---: |
| Packages | 87 | 2 | 175 |
| Cards | 21 | 0.9 | 20 |
| Total | $\mathbf{5 5}$ | $\mathbf{1 . 8}$ | $\mathbf{1 0 0}$ |

1. Average data, in minutes for call duration, computed over at least the last six months of 2001 for all operators.
2. Individual users.

Source: INSEE (National Accounts Department)

Table 2: Profile descriptions

| VARIABLE | MEASURE- <br> MENT UNIT | NOTATION |  |
| :--- | :---: | :---: | :---: |
| Total monthly duration of outgoing <br> national calls | mean | Minutes | $\Lambda$ |
| Percentage of total monthly call <br> duration consisting of calls made in | daytime | Minutes | $t_{1}, t_{2}$ |
| Percentage of total monthly call <br> duration consisting of calls to | fixed telephone |  | $\alpha(t)$ |

1. Includes short numbers and WAP.

Source: INSEE (National Accounts Department)

## 4. A detailed description of call duration and volume, with the aid of distributions

The existence of thresholds in product pricing (indivisible minutes, higher charge for calls exceeding package allowances, time limit on card use, etc.) requires a description of typical consumers using distributions rather than relying exclusively on call duration and volume.

We assume call duration obeys an exponential law with an $\mu$ mean, i.e., of density $\varphi(\Delta)=1 / \mu$ $e^{-\Delta \mu}$. In other words, when the consumer has been using the service for some time, the likelihood of further consumption depends only on the further duration and not on the time already elapsed. We estimated the parameter $\mu$ as the ratio of the mean monthly call duration and the mean monthly number of calls, two information items supplied by operators in the survey on consumption profiles. ${ }^{1}$

Rather than estimating the distribution of the monthly number of calls, we estimated the distribution $\lambda$ of the total monthly call duration. This was possible because the operator survey gave us the limit durations $t_{1}$ and $t_{2}$ that allowed us to classify consumers into three quantiles according to their monthly call duration.

## 5. Computing expenditure

The randomness of call duration and volume makes the expenditure for each profile random too:

$$
\begin{equation*}
D^{p, m}=E\left(M^{p, m}\right) \tag{4}
\end{equation*}
$$

where $E$ stands for the mathematical expectation. The random expenditure $M^{p, m}$ is the sum of several terms. The first is the "basic" expenditure $A^{p, m}$, whether the product is a package (optionally adjustable) or a card. This is the expenditure strictly generated by actual telephone calls. It is highly complex to calculate, given the large number of variables involved: call periods; surcharge for minutes exceeding the packages; roll-over minutes, free of charge or not; time limit on card use; favorite number; etc. Computing this basic expenditure is so complex that we devote the entire following section of this paper to it. The second is a series of expenditures that are not generated by telephone calls in the strict sense. They include charges for SMSs and services such as itemized billing, call waiting, and caller ID.

Packages and cards offer a varying number of SMSs (10, 20 or more) for a given monthly charge. Users who send more SMSs than the chosen number pay each additional SMS at a unit price listed in the package plan. A consumer who sends an average 12 SMSs a month will get a better deal by choosing a stock of 10 SMSs and paying for two extra; a consumer who sends an average of 18 SMSs should choose a stock of 20 SMSs-even if this means "losing" two-rather than paying for eight extra. Our model specifies this rational-consumer logic.

Itemized billing, call waiting, and caller ID entail a fixed monthly charge of the form $\tau(s) \pi$ ${ }^{p}(s)$, where $\tau(s)$ is the proportion of customers in a profile who have subscribed to option $s$ and $\pi^{p}(s)$ the monthly cost of the option if it comes with product $p$.

[^0]The final form of the monthly-expenditure expression is thus:

$$
\begin{equation*}
M^{p, m}=A^{p, m}+S M S^{p}+\Sigma_{s} \tau(s) \pi^{p}(s) \tag{5}
\end{equation*}
$$

where $s$ denotes any one of the three options chosen: itemized billing, call waiting, and caller ID. We now analyze basic expenditure $A^{p, m}$ and its mathematical expectation $E\left(\mathrm{~A}^{p, m}\right)$, which is used in computing the expenditure $D^{p, m}$.

## 6. The complexity of call pricing is unequaled

In the field of consumer products, mobile-telephony services are characterized by a pricing system of unprecedented complexity, mainly determined by the monthly call duration $\Lambda$.

To take the changes in pricing procedures into account when tracking price change, we need a minimum amount of information on the consumption behavior of individuals, namely (1) the proportion $\alpha(t)$ of call minutes spent in each of the call periods $t$ defined by the operator for a given product, and (2) the proportion $\beta(d)$ of call minutes to the "destination" $d$ : fixed network, mobile network of the same operator or mobile network of another operator. In fact, we distinguish another destination: the "favorite" number, which, for a flat charge, can be called at a cheaper rate (Magnien 2003). With $\Lambda^{m}$ as the combined call duration in month $m$, $C^{m}(t, d)$, the number of call minutes in call period $t$ to destination $d$ is:

$$
\begin{equation*}
C^{m}(t, d)=\alpha(t) \beta(d) \Lambda^{m} \tag{6}
\end{equation*}
$$

In summer 2002, the French press talked about a "battle of the second." Here are the facts: while all operators had long espoused the principle of an indivisible first minute, followed by 30 -second increments, SFR announced in July 2002 that it was extending per-second charges beyond the indivisible first minute. In August, Orange "responded" by offering straightforward per-second charges in its packages, albeit with offsetting provisions: for minutes in excess of the "one hour" package, the indivisible first minute would continue to apply. Also, calls to its competitors' cell phones would be charged extra, an arrangement that Orange soon rescinded. In September, it was Bouygues Telecom's turn to introduce persecond charging, to which SFR immediately responded by offering per-second charging as well.

How have these pricing changes been incorporated into the index? For each profile and product, we have established a billing coefficient that gives the billed call duration by simply multiplying the price by the actual call duration. Each month $m$, the user makes $N^{m} \varphi(\Delta) d \Delta$ calls lasting between $\Delta$ and $\Delta+d \Delta\left(N^{m}\right.$ is the total monthly number of calls and $\varphi$ their distribution by duration). The monthly call duration is therefore proportional to the monthly number of calls: $\Lambda^{m}=N^{m} \int_{0}^{+\infty} \Delta \varphi(\Delta) d \Delta$. Operators bill $f^{p}(\Delta)$ call minutes when the actual call duration is $\Delta$. The monthly call duration billed by the operator is thus $\Lambda^{p, m}=N^{m} \int_{0}^{+\infty} f^{\rho}(\Delta) \varphi(\Delta) d \Delta$. It is therefore proportional to the actual duration:

$$
\begin{equation*}
\Lambda^{p, m} / \Lambda^{m}=\int_{0}^{+\infty} f^{p}(\Delta) \varphi(\Delta) d \Delta / \int_{0}^{+\infty} \Delta \varphi(\Delta) d \Delta \tag{7}
\end{equation*}
$$

This ratio, which exceeds 1 , is the "billing coefficient." Its computation, under the hypothesis of a Poisson distribution of the mean call duration, is described in box 1 .

## Box 1: Billing coefficient

Let us consider a product $p$. The call duration $f^{p}(\Delta)$ used by an operator differs from its actual duration $\Delta$ :

$$
f^{p}(\Delta)=\left\{\begin{array}{c}
\Delta_{1}^{p} \text { if } \Delta \leq \Delta_{1}^{p} \\
\Delta_{1}^{p}+n \Delta_{2}^{p} \quad \text { if }(n-1) \Delta_{2}^{p} \leq \Delta-\Delta_{1}^{p} \leq n \Delta_{2}^{p} \quad(n \geq 1)
\end{array}\right.
$$

where $\Delta_{1}^{p}$ denotes the duration of the first call segment billed and $\Delta_{2}^{p}$ the duration of the following segments. The likelihood that the billed call duration will equal $\Delta_{1}^{p}+n \Delta_{2}^{p}$ is:

$$
p_{n}=\int_{\Delta_{1}^{p}+\int_{(n-1) \Delta_{2}^{p}}^{\Delta_{1}^{p}+} \varphi(\Delta) d \Delta}^{n \Delta_{2}^{p}}=\int_{\Delta_{1}^{p}+}^{\Delta_{1}^{p}+\mu^{-1}} e^{-\Delta / \mu \Delta^{p}} d \Delta=\left(1-e^{-\Delta_{2}^{p} / \mu}\right) e^{-\left(\Delta_{1}^{p}-\Delta_{2}^{p}\right) / \mu} e^{-n \Delta_{2}^{p} / \mu}
$$

for $n \geq 1$ and: $p_{0}=1-e^{-\Delta_{1}^{p} / \mu}$ for $n=0$. From this we deduce the mean billed duration:

$$
\sum_{n \geq 0}\left(\Delta_{1}^{p}+n \Delta_{2}^{p}\right) p_{n}=\Delta_{1}^{p} p_{0}+\Delta_{1}^{p}\left(1-p_{0}\right)+\Delta_{2}^{p}\left(1-e^{-\Delta_{2}^{p} / \mu}\right) e^{-\left(\Delta_{1}^{p}-\Delta_{2}^{p}\right) / \mu} \sum_{n \geq 1} n e^{-n \Delta_{2}^{p} / \mu}
$$

or, since the sum of the series is equal to $e^{-\Delta_{2}^{p} / \mu} /\left(1-e^{-\Delta_{2}^{p} / \mu}\right)^{2}$ :

$$
\Delta_{1}^{p}+\Delta_{2}^{p} \frac{e^{-\Delta_{1}^{p} / \mu}}{\left(1-e^{-\Delta_{2}^{p} / \mu}\right)}
$$

This gives us the equation for the billing coefficient:

$$
\frac{\int_{0}^{+\infty} f^{p}(\Delta) \varphi(\Delta) d \Delta}{\int_{0}^{+\infty} \Delta \varphi(\Delta) d \Delta}=\frac{1}{\mu}\left\{\Delta_{1}^{p}+\Delta_{2}^{p} \frac{e^{-\Delta_{1}^{p} / \mu}}{\left(1-e^{-\Delta_{2}^{p} / \mu}\right)}\right\}
$$

The monthly call duration billed in period $t$ to a destination $d$ depends on the product $p$ used:
$C^{p, m}(t, d)=\alpha(t) \beta(t) \Lambda^{p, m}$
(using equation (6)), hence:
$C^{p, m}(t, d)=\alpha(t) \beta(d) \frac{1}{\mu}\left\{\Delta_{1}^{p}+\Delta_{2}^{p} \frac{e^{-\Delta_{1}^{p} / \mu}}{\left(1-e^{-\Delta_{2}^{p} / \mu}\right)}\right\} \Lambda^{m}$

The following table gives an estimate of the mean billing coefficient for packages and cards:

Table 3: Average call duration, actual and billed ${ }^{1}$

| Contract type | Number of <br> calls | Actual <br> duration of <br> one call | Actual <br> monthly call <br> duration | Billed <br> monthly call <br> duration | Billing <br> coefficient, <br> December <br> $\mathbf{2 0 0 1}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Packages | 87 | 2 | 175 | 207 | 1.19 |
| Cards | 21 | 0.9 | 20 | 30 | 1.52 |
| Total | $\mathbf{5 5}$ | $\mathbf{1 . 8}$ | $\mathbf{1 0 0}$ | $\mathbf{1 2 1}$ | $\mathbf{1 . 2 2}$ |

1. Average data, in minutes for call durations, computed over at least the last six months of 2001. Source: INSEE (National Accounts Department)

Other difficulties must be resolved to calculate basic expenditure. They are specific to the type of contract: package or card. For cards, see (Magnien 2003).

The inclusion of roll-over minutes and excess call duration-an issue specific to packages-is a complex step in the determination of the price index. In principle, the excess in a given month $m$ depends on the consumption in that month and on any minutes rolled over from the previous month. But the roll-over from $m-1$ depends, in turn, on the number of actual call minutes in month $m-1$ measured against the "inclusive" minutes allowed in the package, and therefore on the roll-over of unused minutes from month $m-2$. Using a recursive procedure, ${ }^{2}$ from the consumption of minutes $\Lambda^{l}(l=m-k, \ldots, m)$ in successive months since the acquisition of the package in month $m-k$, we can determine the roll-over for month $m$ and thus the (random) number of call minutes exceeding the package. We assume that the unused minutes for a given month were only available in the following month. There are two reasons for this: first, in practice, operators restrict roll-over arrangements; second, we wanted to simplify the calculations. The formula for excess consumption becomes $D E P^{p, m}=\Theta^{p}\left(\Lambda^{m}, \Lambda^{m-l}\right)^{3}$.

The expectation of excess consumption is what we take into account when computing the expenditure that the consumer will seek to minimize. The expectation is written:

$$
\begin{equation*}
E\left(D E P^{p, m}\right)=\int_{0}^{+\infty} \Theta^{p}(x, y) \lambda(x) \lambda(y) d x d y \tag{8}
\end{equation*}
$$

where $\lambda$ is the distribution density of the monthly call duration estimated previously from the information gathered in the operator survey.

In the index computation, call-related expenditure for a package is therefore written:

$$
\begin{equation*}
\mathrm{E}\left(\mathrm{~A}^{\mathrm{p}, \mathrm{~m}}\right)=\mathrm{F}^{\mathrm{p}}+\tau \Delta \mathrm{F}^{\mathrm{p}}+\mathrm{R}^{\mathrm{p}}+\mathrm{P}^{\mathrm{p}}+\Sigma_{\mathrm{d}} \delta^{\mathrm{p}}(\mathrm{~d}) \beta(\mathrm{d}) \mathrm{E}\left(\mathrm{DEP}^{\mathrm{p}, \mathrm{~m}}\right) \tag{9}
\end{equation*}
$$

where $\mathrm{F}^{p}$ is the price of the monthly subscription, $\Delta \mathrm{F}^{\mathrm{p}}$ the change in that price due to a nonstandard contract period, and $\tau$ the proportion of customers (specific to each profile) with a contract of that duration, $\mathrm{R}^{\mathrm{p}}$ and $\mathrm{P}^{\mathrm{p}}$ the monthly subscription to "roll-over minutes" and "favorite number" options, $\mathrm{DEP}^{\mathrm{p}, \mathrm{m}}$ the monthly excess call duration (the only random

[^1]component of package-related expenditure (4)), $\beta(\mathrm{d})$ the proportion of call minutes to destination d , and $\delta^{\mathrm{p}}(\mathrm{d})$ the price of an excess minute for calls to that destination.

Operators offer highly original pricing plans that go far beyond adding options to a standard product. We have incorporated these plans into the index calculation. Examples include "adjustable" packages and low-cost "zero-use" monthly packages to receive incoming calls only.

## 7. Prices stopped falling three years ago

Using the methodology described earlier, we computed the price index for mobile-telephony services from January 1999 to December 2002. The main lesson of this exercise is the overall stability of prices of mobile-telephony services in the past three years, after the sharp decline in 1999 and, no doubt, in previous years (chart 2). ${ }^{4}$

Chart 2: Prices of packages and cards have moved in tandem


Source: INSEE
This break from the previous pattern is likely due to operators' heavy investments in developing the present GSM networks and the prospects for deployment of the thirdgeneration UMTS (Universal Mobile Telecommunications System) networks.

The stabilization of prices for mobile-telephony services since 2000 is corroborated by that of the average prices of call minutes. These prices were computed from ART data by national accountants until 2002, absent a tracking of prices of mobile-telephony services in the CPI. While average prices fell $20 \%$ between 1999 and 2000, the "constant usage" price index eased $7 \%$ in annual-average terms. Prices then effectively stabilized according to both indicators.

[^2]The steepest fall in average prices between 1999 and 2000 (also observed between 1998 and 1999 using the price index based on the ART study cited earlier) is recorded by the constantusage index. Operators offer price cuts that are often substantial but targeted, for example, at calls to the same network or calls made in specific time periods. Consumers inevitably take advantage of these price reductions by changing their calling patterns in ways we can imagine, thereby lowering average prices. But these reductions will not be incorporated into the index since it is based on a constant usage of mobile-telephony services: the index computation will be based on a fixed distribution of calls by destination or by call periods, as provided by the operator survey.

Cards and packages do not exhibit significantly different price changes over the long run (chart 2).

The prices of mobile-telephony services measured by ART in 1998 (ART 1999) already showed a sharper downtrend for the "high-volume" consumers (i.e., individuals). The finding still held true in 1999 (chart 3a). Prices stabilized in 2000; in 2001, their movements were far more beneficial to "extreme" users (low- and high-volume). However, while operators seem to promote a diversity in consumption patterns, they also seem intent on lessening the difference between contract types. Charts 3 b and 3 c show greater swings in card prices for low-volume consumers and package prices for high-volume consumers: cards are more widespread among low-volume users, packages among high-volume users.

Chart 3a: Price change has favored low-volume and high-volume users


Source: INSEE

Chart 3b: Since 2002, card prices have been more attractive for high-volume consumers...


Source: INSEE

Chart 3c: ... and package prices for low-volume consumers


Source: INSEE

## 8. Consumers have imperfect information

Consumers' real expenditure exceeds their minimal expenditure calculated with (1). We estimated the excess from the operator survey, which gives the consumption breakdown by product in December 2001. At that date, for $70 \%$ of package subscribers, the cost excess was lower than $30 \%$. We do not think it is appropriate to give a more detailed distribution of consumers by excess-cost level, as the latter is tricky to evaluate and probably overestimates
the proportion of subscribers with high cost excess. This proportion is indeed artificially increased when the computation of data relies on only one month - as here-, due to month to month fluctuations in subscribers consumption. Moreover, the computed excess cost differs from the actual excess cost, since the calculation replaces the actual requirements of consumers in a given profile (monthly call duration, distribution by call period, by destination, etc.) by their mean values. The fewer the profiles, the wider the scatter of customers in each profile around their mean; as a result, the greater the bias in the calculation of a consumer's expenditure on a product and hence of the consumer's minimal expenditure. However, by including many profiles, our study reduces the bias on the excess-cost estimation.

The supply of mobile-telephony services and, above all, their pricing are so complex that consumer choices - even if rational - can be based only on a partial knowledge of product advantages and drawbacks. Consumers are also constrained by their past choices: a product purchase involves a commitment on the part of the consumer, for example, the duration of a package contract or the total cost of a pre-paid card. The financial benefits of the product change must also outweigh its non-monetary drawbacks (paperwork, new number, etc.).

## 9. A model with "frictions"

The preceding analysis suggests an alternative approach to tracking prices of mobiletelephony services. It consists in assuming that consumers optimize their choices but with limited, imperfect information on product supply. The construction of a constant-usage price index with "frictions" requires a re-examination of equation (1), used until now to describe the dynamics of consumer mobility between products. Each month, all consumers migrated toward the optimal product (the one that minimizes expenditure in a friction-free setting); in the "with frictions" model, only some of the consumers whose expenditure exceeds the minimal expenditure will switch to the optimal product. These dynamics are formalized in box 2 .

To implement this approach, we need additional information: the user distribution by product in the base month and in each profile. This distribution is the starting point for the new dynamics of consumer mobility. It is known for December 2001 thanks to the operator survey on consumption profiles.

The aggregation of the profile indices resembles the perfect-information procedure: in relationships (2) and (3), we simply replace minimal expenditure $\hat{D}^{m}$ with mean expenditure

$$
\begin{equation*}
\sum_{p} f_{T}^{p, m} D_{T}^{p, m} \tag{10}
\end{equation*}
$$

where

$$
\begin{equation*}
f_{T}^{p, m}=S_{T}^{p, m} / \sum_{p} S_{T}^{p, m} \tag{11}
\end{equation*}
$$

is the proportion of profile- $T$ consumers in month $m$ for product $p .{ }^{5}$

[^3]
## Box 2: Dynamics of consumer mobility with imperfect information

The consumer-mobility dynamics provide an explicit formula, in each profile, for the transition between its successive distributions $\left(S^{p, m-l}\right)_{p}$ and $\left(S^{p, m}\right)_{p}$ between the products $p$ available in each period ${ }^{6}$. In an imperfect-information model, the formalization of these dynamics requires the introduction of a "mobility coefficient" $\pi^{p, m}$, equal to the proportion of consumers using product $p$ in month $m-1$ who migrate in month $m$ to an optimal product on the market ${ }^{7}$.

The imperfect-information dynamics are as follows:

- if a product $p$ is not optimal in month $m\left(D^{p, m}>\hat{D}^{m}\right)$ then: $\quad S^{p, m}=\left(1-\pi^{p, m}\right) S^{p, m-1}$
- if a product $p$ is optimal in month $m\left(D^{p, m} \leq \hat{D}^{m}\right)$, then:

$$
\begin{aligned}
& S^{p, m}=S^{p, m-1}+\sum_{p^{\prime}, D^{p^{\prime}, m}>\hat{D}^{m}} \pi^{p^{\prime}, m} S^{p^{\prime}, m-1} \text { if } p \text { is still on the market in month } m \\
& S^{p, m}=S^{p, m-1} \quad \text { if } p \text { is no longer on the market in month } m
\end{aligned}
$$

These dynamics were initialized with the "actual" distribution for December 2001. If we admit that parameter $\pi^{p, m}$ does not depend on product $p$, then we can easily show that:

$$
\begin{equation*}
\left(1-\pi^{m}\right) \sum_{p, D^{p, m}>\hat{D}^{m}} f^{p, m-1}=1-\rho^{m} \tag{*}
\end{equation*}
$$

where $\rho^{m}$ is the fraction of consumers optimally positioned in each month $m$ and $f^{p, m-1}$ is the proportion $S^{p, m-1} / \sum_{p e n m} S^{p, m}$ of consumers of product $p$ in month $m-1$. The sum:
$\sum_{p, D^{p, m}>\hat{D}^{m}} f^{p, m-1}$ is therefore the proportion of consumers who, in month $m$, should switch to the
new product. The $\left(^{*}\right)$ relationship is a natural one: if we take the number of consumers who should switch and subtract the number of consumers who do not (left-hand member), we obtain the number of "poorly positioned" consumers (right-hand member).

Computing the with-frictions index for months prior to December 2001 was a problem because we did not have the consumer distributions by product to initiate the dynamics. These were therefore "reversed" as explained in box 3 .

Thus, there exists in $m-1$ a state older than the state in month $m$ if and only if the proportion of optimal customers in $m$ exceeds the mobility coefficient. Note that the relationship becomes trivial when there are no frictions, as $\rho^{m}$ and $\pi^{m}$ both equal unity. With frictions, the relationship does not always obtain, which has led us to make a slight change in the indexcalculation model (Magnien 2003).

## 10. Results depend heavily on the mobility coefficient

We conducted the calculation of the index in 2002 and its backward extrapolation to January 1999 using different values for the mobility coefficient, which we assume to be identical for

[^4]
## Box 3: Consumer optimality, mobility coefficients, and backward extrapolation

Assuming the mobility coefficient is product-independent, the inversion of the mobility dynamics (box 2) within the profiles gives:

$$
\begin{aligned}
& S^{p, m-1}=S^{p, m} /\left(1-\pi^{m}\right) \quad \text { if } p \text { is non-optimal in } m \\
& S^{p, m-1}=S^{p, m}-\frac{\pi^{m}}{1-\pi^{m}} \sum_{\substack{p^{\prime} \text { non-optimal } \\
\text { in } m}} S^{p^{\prime}, m} \quad \text { if } p \text { is optimal and on the market in } m \\
& S^{p, m-1}=S^{p, m}
\end{aligned}
$$

Thus, there exists in month $m-1$ a state $\left(S^{p, m-1}\right)_{p}$ preceding the state $\left(S^{p, m}\right)_{p}$ of month $m$ if and only if the following condition is satisfied:
for all optimal products on the market in $m$, i.e.:

$$
\rho^{m} \geq \pi^{m} \quad\left({ }^{*}\right)
$$

where $\rho^{\mathrm{m}}$ is the proportion of optimal consumers in a profile and $\pi^{n}$ the mobility coefficient within that profile.
all profiles, all products, and the entire period of study. The coefficient may be expressed either as a rate or a duration, namely, the average period in which a consumer keeps the same product. Chart 4 gives the results obtained with periods of one, two, three, and four years, i.e., with mobility coefficients of $1 / 12,1 / 24,1 / 36$, and $1 / 48$.

A higher mobility seems to entail a sharper fall in the index. This finding is not all that selfevident. Indeed, we observe that the minimal-expenditure index (i.e., with "absolute" mobility) ultimately falls less than the with-frictions indices! This seemingly paradoxical result actually has a simple explanation. Despite the high rate of change in product offerings and prices, a product within a given profile often remains cheaper than the others (and its price unchanged) for what can be a fairly long period. The frictionless profile index now depends only on that product and therefore disregards changes in the prices of the other products. It remains stable during the period. By contrast, the gradual transition of users of the other, non-optimal products toward the cheaper, optimal product causes a gradual decline in the with-frictions indices.

Estimating the mobility coefficient is crucial but extremely difficult. Operators reckon that about $25 \%$ of customers change products each year, which means total mobility in four years (mobility coefficient of $1 / 4$, or $2 \%$ a month). However, they do not necessarily switch from a non-optimal product to an optimal one: it therefore seems difficult to infer a value for the mobility coefficient as formalized above. OFTEL has adopted a far higher coefficient of $10 \%$ a month, i.e., a mobility of less than a year. Relationship (*) in box 2 yields an order of magnitude-albeit very fragile-for the mobility coefficient, also of about four years. This

Chart 4: Price-change tracking is highly sensitive to consumer mobility


Source: INSEE
assumes that the proportion of "poorly positioned" consumers in December 2001 (month $m$ $1)^{8}$ stays the same in January 2002 (month $m$ ). The index computed with the highest mobility (one year) is the one that falls most sharply over the entire period: the proportion of consumers using optimal products rises significantly over time. This does not seem realistic and is not corroborated by the data. One possible approach would consist in using the results of the next operator survey, which will provide the December 2002 user distribution by products in each profile. Such a solution would, however, be very difficult to implement. An iterative procedure would be applied to the mobility coefficients starting with the December 2001 distribution in order to arrive at the December 2002 distribution (as yet unknown), or at least to get as close to it as possible.

## 11. What index should be selected for the current production of the price index?

The complexity and fragility of the with-frictions model and its lack of robustness for the chosen value of the mobility coefficient argue in favor of selecting the frictionless model for the current (monthly) production of the CPI. This is in fact what has been decided. The CPI field of coverage has therefore been extended, since January 2003, to mobile-telephony services. The frictionless model is clear, relatively simple, and neutral toward changes in the rationality of behaviors. It also allows the inclusion of a mobility factor, in a radical manner since it is dichotomous: adjustments are instantaneous within each profile, whereas the shifts between profiles-such as shifts between operators or between packages and cards-are excluded. However, a chaining procedure, characteristic of the French CPI, does allow an adjustment of profile weights. The relatively detailed segmentation of consumers into profiles therefore enables us to modulate the inclusion of their mobility between products.

[^5]
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[^0]:    ${ }^{1}$ We were able to "test" the validity of the assumption made concerning the call-duration distribution (Magnien 2003).

[^1]:    ${ }^{2}$ See (Magnien 2003) for details on the procedure.
    ${ }^{3}$ See (Magnien 2003) for details on the calculations.

[^2]:    ${ }^{4}$ A study by ART (1999) establishes this fact for the year 1998.

[^3]:    ${ }^{5} S_{T}^{p, m}$ is the number of $T$-type consumers consuming product $p$ in month $m$ as determined by the dynamics described in box 2 .

[^4]:    ${ }^{6} S_{T}^{p, m}$ (or, more simply, $S^{p, m}$ ) denotes the number of consumers in profile $T$ in month $m$ for product $p$.
    ${ }^{7}$ When several optimal products are available on the market (an uncommon situation, but one that does occur), we assume that the migrating consumers will be evenly distributed among them.

[^5]:    ${ }^{8}$ Thanks to the survey, we know the proportion of "poorly positioned" customers in December 2001.

