Breaking Down the Differences between the CPI-U and C-CPI-U: Weights vs. Formula

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Overview

- A major question the price index literature is the appropriate formula to use for a price index
- Public concerns over what is being assumed
- For a major purpose of price indexes - the upper level aggregation of the U.S. CPI, – the formula has little relevance if current weights are used
- The Laspeyres-type CPI-U formula vs. the Tornqvist chained CPI-U (C-CPI-U) formula
- Using currently monthly weights in the CPI-U makes it a chained Laspeyres
- Theoretically, as chaining becomes more frequent, the formulas should approach each other and the current inflation rate

- Spurious chain drift must be removed by using long term price relatives
- When this is done, the monthly chaining explains the large majority of the difference between the CPI-U and the C-CPI-U
- Using currently monthly weights holds quantities constant for only one month at a time instead of 36 months on average
- Other differences between the formulas make little difference over one month periods
- Therefore, monthly frequency is sufficient

The CPI-U vs. C-CPI-U

- CPI-U is a Laspeyres-type index, cost of a fixed basket:
- $CPIU_{t-1,t} = \frac{\sum_i q_{iB} I_{it}}{\sum_i q_{iB} I_{i,t-1}} = \sum_i \frac{q_{iB} I_{i,t-1}}{\sum_i q_{iB} I_{i,t-1}} R_{it}$
  - q_{iB} denotes the implicit quantity for base period B for item-area i
  - I_{it} is the index level, R_{it} is the index relative
- An arithmetic mean with shares updated by the index level
- C-CPI-U, Tornqvist formula, estimates a cost-of-living index (COLI)
- It's a geometric mean with mean shares as weights:
- $T_{t-1,t} = \prod_i R_{it}^{\frac{s_{it}+s_{i,t-1}}{2}}$
  - s_{it} is the expenditure share

- In general, a COLI lower than cost of fixed basket due to consumer substitution
  - Obtain a higher standard of living when relative prices change
- Tornqvist incorporates this with:
- Assumed substitution: a geometric mean is lower than arithmetic mean
- Current weights have direct information on substitution
  - Purchase less of higher inflation goods, reducing weight
- C-CPI-U weights vs. CPI-U weights
  - From two months at a time instead of two years at a time
  - Updated every month instead of every two years
  - No lag instead of a year processing lag

Convergence of Indexes with Chaining

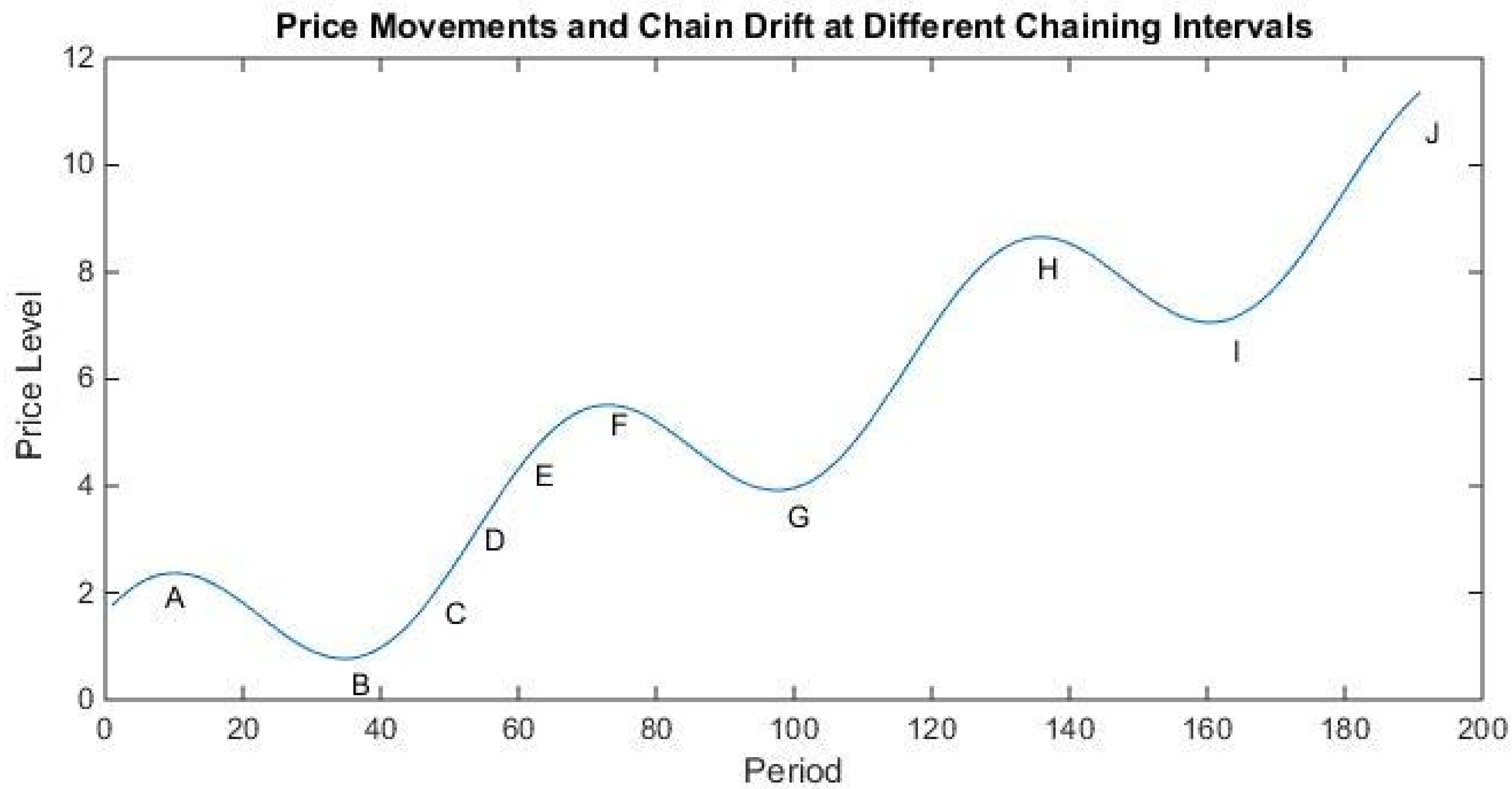
- Chained Laspeyres and chained Tornqvist are discrete approximations to continuous Divisia index:
- $P_{t',t}^{Div} = exp\left(\int_{t'}^{t} \sum_{ia} s_{ia}(\tau)\frac{dlnp_{ia}(\tau)}{d\tau}d\tau\right)$
- The change in Divisia index = instantaneous, or 'current' COLI
- Therefore as chaining becomes more frequent, consumer substitution is incorporated by weights so that indexes converge to each other, the Divisia index, and yield the current COLI

Method

- Make intermediate indexes between CPI-U and C-CPI-U
- Change one part at a time that moves a cost of a fixed basket to a COLI
- Done in different orders to check robustness
- A. change weights first
- B. change formula first, to geometric mean
- C. change formula first, to Constant Quantity Tornqvist (CQTQ)
  - A first order approximation to a Laspeyres
  - Tornqvist formula that uses shares that hold the implicit quantities constant

Changing Weights First

- Spurious Chain drift: Can't just use monthly updated weights in CPI-U
- Correlation between implicit quantity weights and relatives at monthly frequency
- Chained Laspeyres would fly up before falling toward Tornqvist
- Smooth relatives by using Long Term relatives:
- $d_{it} = \prod_{\tau=1}^{t} R_{i\tau}^{\frac{1}{t}} = \left(\frac{I_{it}}{I_{i1}}\right)^{\frac{1}{t}}$
- Works for this data because shares can be treated as constant for index construction
  - Geomeans was a good initial estimator of final C-CPI-U
  - No bias from smoothing high and low inflation periods


Price Movements and Chain Drift at Different Chaining Intervals
Price Level
Period
0
2
4
6
8
10
12
0
20
40
60
80
100
120
140
160
180
200
A
B
C
D
E
F
G
H
I
J

A. Change Weights First

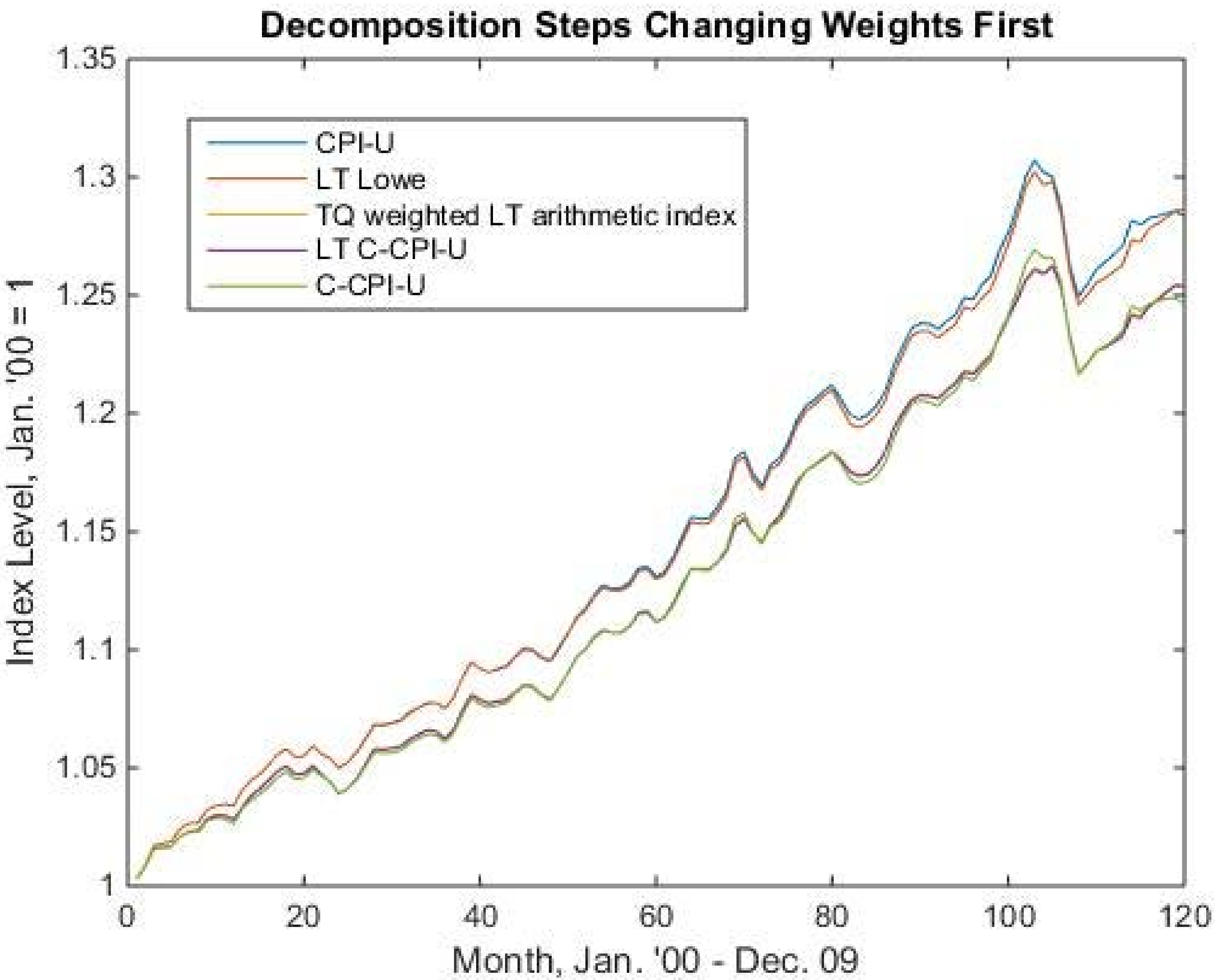
- 1. Use LT relatives
- From $\sum_i \frac{q_{iB} I_{i,t-1}}{\sum_i q_{iB} I_{i,t-1}} R_{it}$ to $\sum_{ia} \frac{q_{iB} d_{it}^{\tau}}{\sum_{ia} q_{iB} d_{it}^{\tau-1}} d_{it}$
- 2. Use Tornqvist share weights
- From $\sum_{ia} \frac{q_{iB} d_{it}^{\tau}}{\sum_{ia} q_{iB} d_{it}^{\tau-1}} d_{it}$ to $\sum_{ia} s_{i\tau}^{TQ} d_{i\tau}$
- 3. Use geometric mean instead of arithmetic mean
- From $\sum_i s_{i\tau}^{TQ} d_{i\tau}$ to $\prod_i d_{i\tau}^{s_{i\tau}^{TQ}}$
- 4. Switch back to short term relatives
- From $\prod_i d_{i\tau}^{s_{i\tau}^{TQ}}$ to $\prod_i R_{i\tau}^{s_{i\tau}^{TQ}}$
- 5. Use non-interpolated relatives

B. Change Formula First, to Geomeans

- Chain drift also in geomean index
- 1. Use LT relatives
- 2. Use geometric mean instead of arithmetic mean
- From $\sum_{ia} \frac{q_{iB} d_{it}^{\tau}}{\sum_{ia} q_{iB} d_{it}^{\tau-1}} d_{it}$ to $\prod_i d_{i\tau}^{\frac{q_{iB} d_{it}^{\tau}}{\sum_{ia} q_{iB} d_{it}^{\tau-1}}}$
- 3. Use Tornqvist share weights
- From $\prod_i d_{i\tau}^{\frac{q_{iB} d_{it}^{\tau}}{\sum_{ia} q_{iB} d_{it}^{\tau-1}}}$ to $\prod_i d_{i\tau}^{s_{i\tau}^{TQ}}$
- 4. Switch back to short term relatives
- 5. Use non-interpolated relatives

C. Change Formula First, to CQTQ

- No spurious drift problems or LT relatives needed, checks robustness of LT relatives
- 1. Use Constant Quantity Tornqvist
- From $\sum_i \frac{q_{iB} I_{i,t-1}}{\sum_i q_{iB} I_{i,t-1}} R_{it}$ to $\prod_i R_{it}^{\frac{1}{2} s_{i\,B}\left(\frac{R_{i,B,t-1}}{L_{B,t-1}} + \frac{R_{i,B,t}}{L_{B,t}}\right)}$
  - $L_{B,t} = \frac{\sum_i q_{iB} I_{it}}{\sum_i q_{iB} I_{i,B}}$
- 2. Use Tornqvist weights
- From $\prod_i R_{it}^{\frac{1}{2} s_{i\,B}\left(\frac{R_{i,B,t-1}}{L_{B,t-1}} + \frac{R_{i,B,t}}{L_{B,t}}\right)}$ to $\prod_i R_{i\tau}^{s_{i\tau}^{TQ}}$
- 3. Use interpolated weights



Decomposition Steps Changing Weights First
CPI-U
LT Lowe
TQ weighted LT arithmetic index
LT C-CPI-U
C-CPI-U
Index Level, Jan. '00 = 1
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
0
20
40
60
80
100
120
Month, Jan. '00 - Dec. 09

Results: Jan. ‘00 – Dec. ’09

Differences between index levels over entire period as fraction of the total CPI-U – C-CPI-U difference

	Method		
	A. Weights First	B. Formula First: Geomeans	C. Formula First: CQTQ
Replace ST Relatives with LT Relatives	8.38%	Same	NA
Weight Effect: Replace Constant Quantity Shares with Actual Shares	85.98%	86.25%	96.22%
Formula Effect: Replace Arithmetic Mean with Geometric Mean	2.05%	1.79%	1.45%
Replace LT Relatives with ST Relatives	1.25%	Same	NA
Replace Non-interpolated Relatives with Interpolated	2.34%	Same	Same

Different Base Periods

- Different start months used for time range studied
- Months after Dec. 05 not used since sample too small

Results:

Mean Effects Across Initial Periods of each Step by Method of Breakdown as % of Total CPI-U vs. C-CPI-U Difference

	Method		
	A. Weights First	B. Formula First: Geomeans	C. Formula First: CQTQ
Replace ST Relatives with LT Relatives	6.29%	Same	NA
Weight Effect: Replace Constant Quantity Shares with Actual Shares	88.10%	88.33%	94.04%
Formula Effect: Replace Arithmetic Mean with Geometric Mean	3.64%	3.41%	2.82%
Replace LT Relatives with ST Relatives	-1.18%	Same	NA
Replace Non-interpolated Relatives with Interpolated	3.14%	Same	Same

Summary/Conclusions

- Using current weights and LT relatives is sufficient for CPI-U to approximate a COLI
- Formula is not relevant if chaining is frequent enough
- Monthly frequency is sufficient on the scale of the CPI-U vs. the C-CPI-U
- C-CPI-U does not depend on formula assumptions it is often criticized for

Contact Information

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