How to Better Measure Hedonic Residential Property Price Indexes

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Macroeconomists and central banks need to identify house price bubbles. Timely, proper measurement.

Other purposes include requirement of separation of land prices from structures – Diewert, Huang, and Burnett-Isaacs, last session. Not the concern this paper.


G20 Data Gaps Initiative-2, IMF’s SDSS plus, and Financial Soundness Indicators

Literature: huge on hedonics; emerging property price indexes; practice. Many here.
The hard problem: requires a constant quality property price index

- Indexes of average prices tainted by changes in the quality-mix of properties transacted
- Matched models breaks down: infrequent transactions of heterogeneous items. Secondary source data

- Three approaches:
  - Repeat sales
  - Sales price appraisal ratio (SPAR)
  - Hedonic regression

- Commercial property price indexes even harder
- Erwin Diewert and Chihiro Shimizu; Inês Gonçalves Raposo and Rui Evangelista; and Barra Casey - later session.
Three main ways to compile a hedonic property price index: a practical paper

■ Time dummy method:

■ Imputation method

■ Characteristics method
  - Many variants of each method: includes:
  - which period the characteristics held constant, superlative
  - which functional form/aggregators/average of characteristics) linear or semi-logarithmic and arithmetic or geometric for characteristics; and
  - single or double imputation.
A semi-logarithmic form is usually appropriate for a hedonic price index, with reference to the constant, $\beta_0$, given as:

$$\ln p_{i,t}^0 = \beta_0 + \sum_{k=1}^{K} z_{k,i}^{0,t} \ln \beta_k + \sum_{t=1}^{T} \delta_t D_{i,t} + \epsilon_{i,t}$$

Rolling window advantageous if thin market, but effectively smooths and lags.

Weights can be introduced by WLS (Diewert (2005) but the paper warns of leverage effects.)
Hedonic characteristics approach

- Constant period 0 average characteristics

\[
\prod_{k=0}^{K} \left( Z_k^0 \right)^{\hat{\beta}_k^0} = \exp \left( \sum_{k=0}^{K} Z_k^0 \ln \hat{\beta}_k^0 \right)
\]

- Constant period t average characteristics

\[
\prod_{k=0}^{K} \left( Z_k^t \right)^{\hat{\beta}_k^t} = \exp \left( \sum_{k=0}^{K} Z_k^t \ln \hat{\beta}_k^t \right)
\]
Hedonic imputation indexes: geomeans; double imputation

- Constant period 0 characteristics

\[
\prod_{i \in N^0} \left( \hat{p}^t_{i|z_i^0} \right)^{\frac{1}{N^0}} = \exp \left( \sum_{i \in N^0} \ln \hat{p}^t_{i|z_i^0} \right)
\]

- Constant period \( t \) characteristics

\[
\prod_{i \in N^t} \left( \hat{p}^t_{i|z_i^t} \right)^{\frac{1}{N^0}} = \exp \left( \sum_{i \in N^t} \ln \hat{p}^t_{i|z_i^t} \right)
\]
Equivalences: Characteristics and imputation approaches give the same results

- Linear hedonic and arithmetic aggregator (for characteristics)
- Log-linear (semi-log) and arithmetic aggregator
- Log-log (double-log) and geometric aggregator

\[
\prod_{k=0}^{K} \left( \hat{\beta}_k^t \right) Z_k^0 = \exp \left( \sum_{k=0}^{K} Z_k^0 \ln \hat{\beta}_k^t \right) = \exp \left( \frac{1}{N^0} \sum_{k=0}^{K} \sum_{i \in N^0} z_{i,k}^0 \ln \hat{\beta}_k^t \right) = \exp \left( \frac{1}{N^0} \sum_{i \in N^0} \sum_{k=0}^{K} z_{i,k}^0 \ln \hat{\beta}_k^t \right) = \prod_{i \in N^0} \left( \hat{\rho}_{i,t}^0 \right)^{1/N^0}
\]

- Axiomatic property
- Hill and Melser (2008); Hill (2013); de haan and Diewert (2013); Rambaldi
Weights – A question:

Why not weight each transaction’s price change by its relative period 0 (period t) values?

\[
\prod_{i \in N^0} \left( \frac{\hat{p}_{i|z_i}^0}{\sum_{i \in N^0} \hat{p}_{i|z_i}^0} \right) \left( \frac{\hat{p}_{i|z_i}^0}{\sum_{i \in N^0} \hat{p}_{i|z_i}^0} \right) = \prod_{i \in N^0} \left( \frac{\hat{p}_{i|z_i}^0}{\hat{p}_{i|z_i}^0} \right) \left( \frac{\hat{p}_{i|z_i}^0}{\sum_{i \in N^0} \hat{p}_{i|z_i}^0} \right)
\]
A second question

Why not weight each transaction using “quasi-superlative” index number formula?

\[
\prod_{i \in N^0} \left( \frac{\hat{p}_{i|z_i}^t}{\hat{p}_i^0} \right)^{\hat{w}_i^t} = \prod_{i \in N^0} \left( \frac{\hat{p}_{i|z_i}^t}{\hat{p}_i^0} \right)^{\hat{w}_i^t}
\]

where \( \hat{w}_i^t = \frac{1}{2} \left( \frac{\hat{p}_i^0}{\sum_{i \in N^0} \hat{p}_i^0} + \frac{\hat{p}_i^t}{\sum_{i \in N^0} \hat{p}_i^t} \right) \)
And a third..

- Why is it only quasi-superlative?
- Use of period 0 and period $t$ transactions requires:

$$
\prod_{i \in S(0-t)} \left( \frac{\hat{p}_i^t Z_i^0}{\hat{p}_i^0 Z_i^0} \right) \prod_{i \in S(t-0)} \left( \frac{\hat{p}_i^t Z_i^t}{\hat{p}_i^0 Z_i^t} \right) \prod_{i \in S(0 \cap t)} \left( \frac{p_i^t Z_i^{0 \cap t}}{p_i^0 Z_i^{0 \cap t}} \right)
$$

And differs from

\[ \sqrt{\prod_{i \in S(0-t)} \left[ \frac{\hat{p}_{i|z_i}^t}{\hat{p}_{i|z_i}^0} \right] w_i^0} \times \prod_{s(t-0)} \left[ \frac{\hat{p}_{i|z_i}^t}{\hat{p}_{i|z_i}^0} \right] w_i^t \]

- Hill and Melser (2008)
- Akin to a Fisher: Laspeyres and Paasche cross
- Substitution effect; use of predicted vs. raw weights.
What the paper does..

- **Equivalences:** finds equivalences for reasonable forms of the imputation and characteristics approaches. Cuts down on choice by consolidating approaches and the many types of each. Validates them - axiomatic.

- **Weights:** shows how weights can be introduced at lower level - for price changes of *individual properties* within a strata.

- **Substitution effects:** shows how substitution effects can be included via a “quasi” superlative formulation – redefines a superlative index.

- **Re-visits the theory** on superlative hedonic RPPIs.
Also, ..

- In the practical context of thin markets – **sparse data** - and vagrancies of regular hedonic estimation

- Only estimates a reference period hedonic regression – with regular re-linking.

\[
\prod_{i \in N^t} \left[ \frac{p_{i|Z_i}^t}{\hat{p}_{i|Z_i}^0} \right] \left( \frac{\hat{p}_{i|Z_i}^0}{\sum_{i \in N^t} \hat{p}_{i|Z_i}^0} + \frac{p_{i|Z_i}^t}{\sum_{i \in N^t} p_{i|Z_i}^t} \right)^{1/2} = \exp \left( \sum_{i \in N^t} (\hat{w}_i^0 + w_i^t) / 2 \left[ \ln p_{i|Z_i}^t - \ln \hat{p}_{i|Z_i}^0 \right] \right)
\]

- Sample selectivity bias but limited substitution bias

But needs double imputation workarounds

For weights

\[ w_{i}^{*t} = p_{i}^{t} \left( \frac{\hat{p}_{i|z_{i}^{0}}^{0}}{p_{i}^{0}} \right) = w_{i}^{t} e_{i}^{0} \]

\[ \sum_{i \in N^{t}} p_{i}^{t} \left( \frac{\hat{p}_{i|z_{i}^{0}}^{0}}{p_{i}^{0}} \right) \]

For prices

\[ \hat{p}_{i|z_{i}^{t}}^{**t} = p_{i|z_{i}^{0}}^{t} \left( \frac{\hat{p}_{i|z_{i}^{t}}^{0}}{p_{i}^{0}} \right) \approx \hat{p}_{i|z_{i}^{t}}^{t} \]

\[ w_{i}^{*\tau} = 0.5 \left( \hat{w}_{i}^{0} + w_{i}^{*t} \right) \]
Use an indirect volume measure

Value index/volume index = implicit price index

\[
\Pi_{i \in N^0} \left( \frac{\hat{p}_i^0 \mid z_i^t}{\hat{p}_i^0 \mid z_i^0} \right)^{\hat{w}^F_i} = \frac{\Pi_{i \in N^0} \left( \hat{p}_i^0 \right)^{\hat{w}^F_i}}{\Pi_{i \in N^0} \left( \hat{p}_i^0 \mid z_i^0 \right)^{\hat{w}^F_i}}
\]
The end