SPECULATIVE BUBBLE ON THE MOROCCAN REAL ESTATE MARKET: IDENTIFICATION AND CYCLES

Pr. Firano Zakaria
University of Mohammed V Rabat-Agdal
firanou@yahoo.fr

Abstract
This paper present several approaches to identify and dated the speculative bubble at real estate’s market. Using the price index real estate (IPA1), statistical and structural approaches were combined in order to detect the existence of a bubble on the Moroccan real estate market. The results obtained affirm that the Moroccan real estate market knew a speculative bubble during the period 2006-2008 explained mainly by the boom of the credit during the same period. The use of the Markov switching model affirmed also that the speculative bubble in Morocco is cyclic and corroborates consequently the critic formulated by Evans (1991) concerning the approaches traditional of detection of the financial bubbles. Thus, the analysis of the series of the bubble, extracted using the kalman filter, affirms the existence of two regimes namely: an explosive regime and a normal regime. The first regime describes the periods of explosion of the bubble and lasts about 9 quarters, while the second, of which during 14 quarters, describes the periods to return to the average cycle.

Keys words: real estate bubble, market efficiency, financial stability, pricing, equilibrium
JEL-Classification: G12, E44
1. Introduction
The real estate markets are frequently affected by speculative bubbles following the example of financial markets. The real estates are the subject of purchase and sale by taking account of anticipations of the agents. For this reason, the price determination on the market of the real estate is a fundamental question which as well worries the actors of the market as the public authorities because of the impact of this market on economic development and financial stability. The fall of the prices on the real estate markets has fatal consequences for the economy and in particular on the value of the balance-sheets of the various economic agents. Indeed, the fall of the real assets prices generate a fall of the financial assets prices on the whole of the markets, a deceleration of the growth and a loss of confidence resulting in a significant decrease returned of the sectors into connection with the real sector.

The international financial crisis showed that the models of economic growth deteriorated with the real estate sector tend to suffer from a great brittleness because of the strong correlation which can exist between the price level on the real estate market and the growth prospects economic.

In this direction and because of the importance of the real estate sector in Morocco, this study is interested in the analysis of the trend of real estate price and the checking of the assumption, according to which, there can be a speculative bubble on the real estate market in Morocco. For this reason, paper proposes to use several approaches to detect the speculative bubbles by using the real price indexes and of the rents.

In finance theory the fundamental value constitutes only one specific solution of the equation of Euler, the second party of the solution is resulting from the suppression of the assumption of transversality and makes it possible to converge towards a more general solution. This additional solution should lead to the emergence or the acceptance of a speculative bubble. However, so that this solution is in conformity with the assumption of rational anticipation and so that it is also accepted and allowed by the whole of the economic agents, it is necessary that this solution is rational and independent of the endogenous behaviors of the market. On the theoretical plan and so that the difference between the price and the fundamental value is rational (rational bubble), it is necessary that the equation that is to say a single and sufficient solution of the Euler equation.

We accept that:

\[ E_t P_{t+1} = E_t \left[ \delta E_{t+1} D_{t+2} + \delta^2 E_{t+2} D_{t+3} + \cdots + B_{t+1} \right] \quad (1.1) \]

If we use the iterative expectation law, we can write the following form:

\[ E_t P_{t+1} = \delta E_t D_{t+2} + \delta^2 E_t D_{t+3} + \cdots + E_t B_{t+1} \quad (1.2) \]

While replacing \( E_t P_{t+1} \) in the equation of Euler then we obtains that:

\[ \delta (E_t P_{t+1} + E D_{t+1}) = \delta E D_{t+1} + \delta^2 E_t D_{t+2} + \delta^3 E_t D_{t+3} + \cdots + \delta E_t B_{t+1} \quad (1.3) \]

Then:

\[ P_t = \delta (E_t P_{t+1} + E D_{t+1}) = P_t^* + \delta E_t B_{t+1} \quad (1.7) \]

According to this formulation, it arises that the solution with speculative bubble (rational) is a solution of the equation of Euler and represents a more general solution in absence of condition of transversality. The deviation of the prices compared to their fundamental value is thus rational and always adheres to the assumptions of rational anticipation and objectivity of the economic model. However, so that the solution with the component is single it is necessary that the equation above is in equivalence with the formula of Euler. For this purpose, it is necessary that the bubble follows a martingale process, according to which the
prediction of the future value of this martingale is its present value. On this register we imposes the following definition of the rational bubble:

\[ E_t B_{t+1} = \frac{B_t}{\delta} = (1 + x)B_t \]  

(1.8)

According to this formulation, the solution of the Euler equation is single and includes in addition to the basic components (dividends or rents), other components related to the future trajectories of the prices. From this point of view, the investors are not satisfied any more to only formulate anticipations on the future outputs, but also on the future prices, they thus contribute to increase the bubble in a rational way while being based on Self-fulfilling prophecy\(^1\) of future price. By adopting anticipations on the trajectory of the prices, the participants on the market continue to accept a fair game which supports informational efficiency. In other words, the prices include anticipations of the outputs and the future prices, therefore the probability of generating important gains is almost impossible.

It should be noted also that the fact of considering that \((1 + x)\) than 1 that is higher makes it possible to consider than the bubble is always ascending and consequently we draws aside the possibility of having negative speculative bubbles indicating the possibility of having negative prices \(\lim_{n \to \infty} E(B_{t+n}) = +\infty\) and thus it is noted that \(B_t > 0\).

3. Empirical studies

Several empirical approaches were developed of which most important will be thereafter detailed. The tests of the variance are the oldest tests as regards detection of the speculative bubbles. This approach was initiated by Shiller (1979, 1981) and is based on the fact that the financial assets price must be always in agreement with the fundamental value explained by the current value of the sum of the outputs. A volatility of the prices more important than that of the fundamental value can induce the existence of factors other than those fundamental intervening in the market pricing.

The approach of West (1987) is different from that of Shiller (1981), in the direction where it is based on the checking of two assumptions to knowing; H0: according to which the courses of action are fixed by a model adhering to the assumption of efficiency of the markets, while H1: indicates that the prices, in addition to the basic component, they are composed of a speculative bubble. The results obtained on market indexes SP 500 (1871-1980) and the index Dow-Jones (1928-1978) made it possible to reject the null assumption of absence of bubble. Demirguc-Kunt (1990) uses the same approach on very small samples and manage to conclude that it is difficult to affirm the conclusion of West (existence of bubbles) on similar samples.

Diba and Grossman (1988a, 1988b) examining the assumption of Co-integration of the two series. Thus, if the series of the dividends are not stationary and are integrated of order 1, the price on the market must be also integrated same order, under the assumption of speculative inexistence of bubbles. In practice the two authors used the test of Bhargava and the method of Engel-Granger to affirm or cancel the existence of speculative bubbles.

Evans (1991) stresses in its item that the bubbles are supposed to appear and disappear throughout the formation process of the prices within the markets of the capital. Into this direction, it rejects the relevance of the tests of roots unit and also of cointegration in detection of the periodic speculative bubbles of nature. The simulations used by the author affirm that when the bubbles appear on the markets, the stock exchange exchange rates do not seem to be more explosive than the dividends within the meaning of the unit tests of roots, which lets predict a weakness of these tests and leaves the problems of largely open detection of the bubbles.

\(^1\) Anticipation Self-fulfilling is a form of rational anticipation which constitutes a form of answer as for the indetermination of the future of the economic world and which describes the beliefs of the individuals.
Philips, Shi and Yiu, (2011) propose a generalization of the test ADF (sup ADF) which makes it possible to detect the bubbles on the markets of the capital. The implementation on index SP500 shows clearly that between the periods 1871 and 2010, several bubbles were identified. Matthew S. Yiu and Al (2012) forward an implementation of the method suggested by Philips, Shi and Yiu, (2011) on the real estate market of Hong Kong. Their results confirm the presence of several positive bubbles on this market of which most important is that of 1997.

While taking as a starting point the work by Bong and Al (2010), we choose to approximate the real assets price through the price index of the real assets (IPAI) available. The incomes generated by the detention of the real assets are the rents which the tenant in the event of use of the real estate pours and consequently they are regarded as being the fundamental factors. In the absence of such statistics, allowing to describe the yield, we privilege the use of a proxy rents with knowing the index of the rent which belongs to the components of consumer price index (IPC). For this purpose, the whole of the tests and the models which will be used to check the presence of bubbles on the Moroccan real estate market will be carried out by using these two indicators.

4. Statistical approach’s
The statistical tests were the first to be used to be able to cancel or affirm the existence of speculative bubble on the markets of the capital. These tests base on a fundamental idea, according to which, the two generating processes of the prices and the dividends must be Co-integrated, in the event of absence of speculative bubble. Diba and Grossman (1988) were the first to implement this type of approach by suggesting the use of the tests of unit roots and the Co integration tests, in particular test ADF, the test of Granger and Engel and the test of Bhargava. Other recent approaches proposed to adopt new tests of unit roots allowing of stage the limits of the traditional tests. Indeed, the latter adopt the assumption of existence of deterministic bubbles answering the definition of Blanchard and Al (1979, 1982), without having to claim that the bubbles can be characterized by several regimes. Thus, other statistics, taking account of the criticism of Evans (1991), were used, it acts in particular of those of Bussetti-Taylor (2004), of Philips and Al (2011, 2012).

4.1. First generation test’s
Diba and Grossman (1988) proposed to identify the speculative bubbles by using three statistical tests with knowing test ADF, the test of co-integration and the test of Bhargava. In theory, they used the definition of the rational bubbles to describe the unobservable party of the assets prices, which is anything else only the difference between the real price and the fundamental value. In this direction, the linear combination, described according to the definition of the rational bubbles, between the prices and the fundamental ones (dividends or rent) makes it possible to identify so yes or not one is in the presence of bubble. In other words, if the prices and the dividends (or rents) are co-integrated same order that implies that the variation of the prices is largely faded with the variations of the intrinsic elements to the economy. The procedure of test is based on a use of several statistics making it possible to validate the null assumption of nonexistence of bubble by indicating that the assets prices and the outputs are integrated same order. Indeed, if the prices and the outputs are no stationary in level and are it in first difference, in this respect the assumption of nonexistence of bubble is checked. The first test used is that based on the statistics of the test of augmented Dickey Fuller (ADF) which makes it possible to check the existence or not unit root in the series in question.

| Table 1: Test ADF on the series of the prices and the outputs |
|-------------------|-----------------|----------------|
| Series            | ADF probability | Lag used in ADF |
| Log of outputs (LD) | 0.0006          | 9              |
It arises according to the analysis of the statistics ADF which the prices and the outputs are stationary of difference, while, the series in level post heterogeneous results. Indeed, the series of the prices is no stationary in level, on the other hand, the series of the outputs (rent) are stationary. On the basis of these result, it proves that for a first test that the prices seem to be not cointegrate with the series of the outputs. For this purpose, the variations of the real assets prices can not arrive of a volatility of the fundamental ones.

Diba and Al (1988) also propose to use the test of Bhargava and al. (1986) to test the nature of the relation between the assets prices and the outputs which result from this. Indeed, if the prices are completely explained by the fundamental ones, from this point of view, the residues of the linear relation between the prices and the rents should be stationary. For this purpose, the test of Bhargava makes it possible to check this assumption directly.

### Table 2: Bhargava test

<table>
<thead>
<tr>
<th>Series</th>
<th>Bhargava stat</th>
<th>Observation number</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP-LD</td>
<td>-1.56</td>
<td>51</td>
</tr>
</tbody>
</table>

According to the statistical table of Lee and Schmidt (1994) and knowing that the number of observation is between 50 and 100 observation then -1.56 is largely higher than the critical points than 1%,-18.3 and -19.3. In this respect one can conclude that the linear combination between the real assets prices and the price of the rent is no stationary, it acts of a stationary series of difference.

To confirm this first intuition, we calls upon a third test, suggested by the authors, of co-integration between the two variables. The tests of Johansen and Granger-Engel were used to check the existence of this relation.

### Table 3 : Johansen test

<table>
<thead>
<tr>
<th>Assumption</th>
<th>Eigenvalues</th>
<th>Statistical test</th>
<th>Critical Value to 5%</th>
<th>Critical probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>0.20</td>
<td>13.65</td>
<td>15.49</td>
<td>0.09</td>
</tr>
<tr>
<td>At most 1</td>
<td>0.05</td>
<td>2.78</td>
<td>3.84</td>
<td>0.09</td>
</tr>
</tbody>
</table>

The test of the trace indicates that the series are not co-integrates with the threshold of 5%

### Table 4 : Granger test

<table>
<thead>
<tr>
<th>Z-statistic</th>
<th>Critical probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log real prices</td>
<td>-7.147741</td>
</tr>
<tr>
<td>Log rent</td>
<td>-6.648238</td>
</tr>
</tbody>
</table>

The approaches suggested by Diba and Grossman (1988) rest on tests of unit roots on the series of the assets prices and the series of the dividends (rents) to check the correspondence between the two variables and to affirm or cancel the existence of speculative bubble. This methodology rests indeed on the adoption of a step of cointegration. However, the criticism of Evans (1991) called in question this approach based on the tests of unit roots usual (in particular Co integration tests). Indeed, it proved that these tests do not make it possible to collect the case where the bubbles are formed and deflate on several occasions. Consecutive collapses of the bubbles complicate the possibility of detecting in an effective way the existence of bubbles on the various markets of capital.

### 4.2. Second generation test’s

The tests of unit roots and the tests of co-integration thus do not make it possible to provide such a convincing analysis of the existence of speculative bubble on the markets of the
capital. Indeed, these tests can be skewed if it is necessary to note that the series is characterized by a regime change according to which, it can vary between a stationary state and an explosive state. The speculative bubbles appear and disappear according to an unknown frequency, which gets a quasi-cyclic character to them whose usual tests of unit root’s and co-integration are adapted.

Thus and in answer to the criticism of Evans (1991) of other tests were developed in order to arrive to a better checking of the presence of speculative bubbles. Among these approaches, those proposed by Bussetti et al. (2004) and Philips, Wu, Yu (2009, 2011, 2012) “PWY” seem to be more relevant.

Bussetti and Taylor (BT, 2004) propose statistics to test the assumption according to which a series is stationary compared to an alternative assumption which suggests that the series passes from a stationary regime to a regime I (1). The test is based on the calculation of the following statistics:

$$sup_{\tau \in [0,1-\tau_0]} BT_{\tau} \quad (1.10)$$

where:

$$BT_{\tau} = \frac{1}{s_0^2 (T-\tau T)^2} \sum_{t=\tau T+1}^{T} (y_T - y_{t-1})^2 \quad (1.11)$$

And $s_0^2$ is an estimate of the variance over all the temporal period of the series there and $\tau$ is under interval chooses to work out the test.

The use BT test (2004) provided results more or less similar to those obtained using the statistical tests previously used. Thus, one can reject with the threshold of 5% the assumption of stationary of the series of the price index real estate to the detriment of the alternative assumption, according to which, the real assets prices forward of a stationary state in an explosive state.

<table>
<thead>
<tr>
<th>Log of price index of the real assets</th>
<th>1.4858</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical value</td>
<td></td>
</tr>
<tr>
<td>90%</td>
<td>0.5057</td>
</tr>
<tr>
<td>95%</td>
<td>1.0153</td>
</tr>
<tr>
<td>99%</td>
<td>5.9401</td>
</tr>
</tbody>
</table>

The breaking values were obtained using simulation (5000 iterations) on a sample of 51 observations and with an interval $\tau_0=0.1$.

PWY (2009, 2011, 2012) use a sup ADF (SADF) according to which usual test ADF is retorted on small fragments of the series in a sequential way, on several occasions, by prolonging each time the samples used (windows). Then, and on the basis of the value superior exit of various simulations, a statistical test of inference is worked out to make it possible to check the existence or not explosive behavior thus testifying to the presence of bubble. They manage to confirm the power of their test compared to the other traditional tests like unit roots and Co integration as regards checking of the presence of speculative bubbles. Moreover, PWY (2012) proposes an improvement of their test to manage to detect several speculative bubbles by generalizing procedure SADF. Indeed, their new test makes it possible to provide a more suitable framework of analysis of the explosive behavior of the long series and to provide a more relevant appreciation of the various speculative bubbles which can emerge over one more or less long period.

The new test suggested by PWY (2012) is called GSADF “generalization of sup ADF). Based on the same principle that the SADF, the GSADF are conceived to be overall and more

\[^1\] Homm and Breitung (2011) also affirm, using simulations Monte Carlo, the supremacy of the test of PWY (2011).
flexible device, as regards fixing of the initial points and in the determination of the windows to be tested. Thus these two tests make it possible to collect any explosive behavior in the series of the prices and to ensure a better amendment of the test the number of observation used.

Statistical test SADF is based on a sequence of test ADF usually used to detect the existence of unit roots. If it is supposed that the sample of regression (ADF) starts as from one moment \( \tau_1 \) and finishes at one final moment \( \tau_2 \), with \( \tau_2 = \tau_1 + \tau w \) and \( \tau w \) is the fraction of the sample used at the time of the regression. In this case the number of observation used in the regression is of \( Tw = T * \tau w \). The test is thus based on a repetition of test ADF on several temporal breaches \( \tau w \) who starts starting from the starting point \( \tau_1 \) up to the point to arrive \( \tau_2 \). Thus \( \tau w \) is selected enters \( \tau_0 \) and \( 1 \) and \( \tau_0 \) is given in such a way that the estimate of test ADF is effective. These new statistics ADF repeated are commonly called SADF and indicated:

\[
SADF = \sup_{\tau w \in [\tau_0 , 1]} ADF_{\tau w} \quad (1.12)
\]

What one can write in the following asymptotic form:

\[
SADF = \sup_{\tau w \in [\tau_0 , 1]} \left\{ \frac{\tau w \left[ \int_0^{\tau w} WdW - \frac{1}{2} \tau w \right] - \int_0^{\tau w} WdW (\tau w)}{\tau w^{1/2} \left[ \int_0^{\tau w} W^2 dr - \left[ \int W(\tau) d\tau \right]^2 \right]^{1/2}} \right\} \quad (1.13)
\]

Under the assumption that \( W \) is a stationary process.

Test GSADF is based on a reproduction of test ADF on a sequence of small samples even broader than that deployed in test SADF. Indeed, the fact of changing the starting point of the test “ \( \tau 1 \) ” made so that it is there several windows on which one can carry out test ADF. From this point of view the GSADF is defined in the following form:

\[
GSADF(r0) = \sup_{r2 \in [0, 1]} \{ ADF \} \quad (1.14)
\]

Another fundamental contribution of the authors is to arrive to set up of the tests of detection of the bubbles dates’ formation, by using recursive tests with SADF statistics. More precisely, the strategy consists in comparing statistics resulting from one canvassed recursive of statistics SADF and GSADF (BSADF, BGSADF) with the breaking values of SADF, which makes it possible to detect the dates of formation of the bubbles or the start dates of the explosion of the variable.
Tests SADF and GSADF can detect the existence of speculative bubbles on the level of the series of the prices through replication on several under sample of the series of test ADF in answer to the criticism formulated by Evans (1991). Authors PWY (2011, 2012) set up statistical tables which describe the breaking values of the two tests.

By implementing the two tests to the series of the prices and the rents, the results obtained are forwarded in the table below:

<table>
<thead>
<tr>
<th>Table 6</th>
<th>SADF Test</th>
<th>GSADF Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of real assets prices</td>
<td>0.7058</td>
<td>2.6392</td>
</tr>
<tr>
<td>Log of index of the rents</td>
<td>-1.2689</td>
<td>0.1204</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Critical value</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.7879782</td>
<td>1.1915780</td>
<td>1.5360598</td>
</tr>
<tr>
<td></td>
<td>3.4615806</td>
<td>1.5360598</td>
<td>2.1555409</td>
</tr>
<tr>
<td></td>
<td>3.5906605</td>
<td>1.5360598</td>
<td>2.1555409</td>
</tr>
</tbody>
</table>

The whole of the tests were carried out on a sample of 51 observations and with an interval \( r_0=0.4 \) the breaking values were obtained on the basis of 5000 Monte Carlo simulation.

The analysis of the results obtained affirm that the series of the real assets prices is explosive. Indeed, test GSADF is higher than the breaking values with the threshold of 1%, 5% and 10%, which implies that the series east can be characterized by the existence of a speculative bubble.

In addition, with regard to the series of rent index, it arises according to the two tests that the H0 assumption cannot be rejected and consequently the series indicates the presence of an explosive behavior making it possible to explain the explosion of the series of the real prices.

Using recursive test BGSADF developed by PWY (2012) we can provide an estimate of the date of formation of this bubble in Morocco. This is presented in the following graph:

![Figure 3: identification of the boost period of the real estate prices index](image)

According to the development of recursive test BGSADF we note that the value of the test largely exceeds the breaking value of test GSADF (1.53). Thus we can affirm that there was a formation of bubble during the period going from December 2006 to March 2009. These results can be confirmed using the tests of change regime which make it possible to detect the points of change of the tendencies of the time series. In this direction, one uses the test of Zivot-Andrews and the statistics of the Chow type implemented to test ADF (sup
The results to which one led make it possible to affirm the results of tests SADF and GSADF.

<table>
<thead>
<tr>
<th>Table 7</th>
<th>Zivot Andrews</th>
<th>Test of sup DCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of real assets prices</td>
<td>-4.64 (0.06)</td>
<td>2.3486 (0.00)</td>
</tr>
<tr>
<td>Critical value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90%</td>
<td>-5.57</td>
<td>1.5762</td>
</tr>
<tr>
<td>95%</td>
<td>-5.08</td>
<td>1.9327</td>
</tr>
<tr>
<td>99%</td>
<td>-4.82</td>
<td>2.2685</td>
</tr>
</tbody>
</table>

By comparing the two tests with their breaking values, the assumption of absence of speculative bubbles is difficult to reject. In this direction, we can affirm that there was a deviation of the prices compared to their fundamental value. These last two tests also confirm that the explosion of the series of the assets prices intervened as from 2006, to corroborate the results of the preceding tests.

The use of the statistical tests of the first and the second generation affirms that the series of the real estate’s price index is explosive and seems to be independent, not co-integrate, of the developments of the rents. This conclusion can to check the assumption according to which the trend of prices indicates the presence of a speculative bubble.

5. Structural approach

The examination of the statistical properties of the series of the real assets prices and the series of the rents confirmed the presence of an explosive nature in the first series thus representing the formation of a speculative bubble lasting the period 2006-2008. However, the statistical tests are limited only to the econometric properties of the analyzed series, without taking account of an economic design and a definition more structural of the speculative bubbles. For this purpose, other economic approaches (structural) were suggested in order to check in a relevant way the assumption according to which the trend of prices indicates the presence of a speculative bubble.

The central idea of West (1987) model is to test the assumption according to which the price is equal to the fundamental value, against the assumption that the price in addition to the fundamental value includes another component, which is the speculative bubble.

Let us note that:

\[ S_t = P_t^f + B_t \text{ où } S_t = P_t^f \]

In the case of the West model, we note that the bubble \( B_t \) is defined according to the model of Blanchard and Watson (1982) and that the discount factor is unknown and should be estimated. It also supposes that the dividends follow an auto-regression process (AR (P=1)).

\[ D_t = \theta D_{t-1} + \varepsilon_t \text{ avec: } |\theta| < 0 \]

It is also known that according to the assumption of rational anticipation that, \( \sum_{i=1}^{\infty} b^i D_{t+i} = \gamma D_t \) then we can write that \( \gamma = b\theta/(1 - b\theta) \). If the assets price is equal to the fundamental value in this case:

\[ S_t = P_t^f = \gamma D_t \quad (1.17) \]

According to this design, the West model thus proposes to estimate three equations. Equations 1.16 and 1.17 can be estimated with ordinary least square (OLS), while, the first equation has a character forward looking, which requires the adoption of an estimate with instrumental variables or by using the Generalized method of moments (GMM).
In order to check the existence of bubble in the series of the assets prices it is necessary to compare the two estimates, direct and indirect, of $\gamma$. In the event of divergence of the two estimators, that confirms that there exist other factors that those fundamental making it possible to explain the trend of prices. The series used are relating to the price index of the assets and the index of the rents. This last is an approximation of the real returns on assets. The step of West consists in initially estimating the process of the dividends using model ARIMA. For this purpose, the Stepwise approach was used to obtain the optimal delays. The model obtained is the following:

$$\log(D_t) = 0.96(0.00)\log(D_{t-1}) + 0.14(0.00)$$  \hspace{1cm} (1.18)

For the two other equations the estimate was carried out using the instrumental variables and the results obtained are presented in the following table:

<table>
<thead>
<tr>
<th>Table 8 : Estimation of 1.16 and 1.17 equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>$b$</td>
</tr>
<tr>
<td>$\gamma$</td>
</tr>
</tbody>
</table>

According to the estimates the discount factor is equal to 0.5, which is the rate required by the holders of the real assets neighbor the 50%. In other words, the purchasers of the real assets wait a yield equal to 50% for each real assets, which is a more or less important rate. Concerning the factor $\gamma$ it was estimated using the instrumental variables and the value obtained is equal to 1.0007.

According to the specification of West (1987), we can determine $\gamma$ according to the following relation: $\gamma = \hat{b}\theta/(1 - \hat{b}\theta)$. The use of a test of Wald to compare the two estimates of $\gamma$ allows to obtain the following results:

<table>
<thead>
<tr>
<th>Table 9: Wald test $\hat{\gamma} = \frac{\hat{b}\theta}{1 - \hat{b}\theta} = \gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical test</td>
</tr>
<tr>
<td>Chi-square</td>
</tr>
</tbody>
</table>

On the basis of result of the test of Wald, the coefficient $\gamma$ seem to be different for the two types of estimate. So we can affirm that the fundamental do not determine only the formation of the assets prices. For this purpose, we can affirm that there exists a speculative bubble in the formation process of the real assets prices. Although the West model (1987) makes it possible to validate in a way structural the existence or not of a speculative bubble, it remains unable to quantify the development of it from where the recourse to innovating approaches adopting the kalman filter.

On the basis of work of Campbell and Shiller (1988), who could develop the equation of Euler under the assumption of constancy of the yields and using a Taylor development. A new linear representation of the prices can be considered:

$$P_t = c + \alpha P_{t+1} + (1 - \alpha)D_{t+1} - x_{t+1}$$  \hspace{1cm} (1.19)

The analysis of this relevant relation emphasizes that the price of a financial credit is according to the outputs which it can get following its detention and also on a future price level anticipated. Thus, the model of Campbell et al. (1988) makes it possible to obtain a general solution for the equilibrium prices of assets.

By retaining that the fundamental value of the financial assets is equal to:
\[ p_t^f = \frac{(c - x)}{(1 - \alpha)} + (1 - \alpha) \sum_{i=0}^{n} \alpha^i E(D_{t+i}) \] (1.20)

with; 
\[ c = -\ln(\alpha) - (1 - \alpha)(\ln\left(\frac{1+D/P}{p}\right)) \] and \[ \alpha = \frac{1}{1+D/P} \]

On the basis of this definition of the Campbell et al. (1988), fundamental value and proposes to define the variation of the assets prices as being the variation of the two following components:

\[ \Delta P_t = \Delta P_t^f + \Delta B_t \] (1.21)

\( \Delta P_t^f \) is the variation of the fundamental value and \( \Delta B_t \) is the variation of the speculative bubble.

In this case, the fluctuations in prices are described according to the following formulation:

\[ \Delta P_t = (1 - \alpha) \sum_{i=0}^{n} \alpha^i E(D_{t+i} - D_{t+i-1}) + \Delta B_t \] (1.22)

According to work of Wu (1995 and 1997) the first party of the equation above can be in relation to the developments of the hopes of output of the dividends and is approximated according to a process ARIMA (H, 1, 0) with drift. The author thus proposes to forward the variation of the dividends according to the following form:

\[ \Delta W_t = s_t + \sum_{i=0}^{n} \delta_i \Delta d_{t-i} + \epsilon_t \] (1.23)

with, \( \epsilon \to N(0, \sigma^2) \) and \( s_t \) is a drift.

In order to circumvent this constraint, Wu (1995) proposes approach the securities anticipated in the equation of Campbell (1988) by their last achievements and their securities.

It is thus noted that:

\[ Y = AY_{t-1} + \eta \] (1.24)

where;

\[ Y = \begin{pmatrix} \Delta d_t \\ \Delta d_{t-1} \\ \vdots \\ \Delta d_{t-n} \end{pmatrix}, \eta = \begin{pmatrix} \varepsilon \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \text{et } A = [\delta_1 \cdots \delta_n] \]

If it is considered that \( g = (1,0,0,...,0) \), we can thus write that:

\[ \Delta d_t = gY_t \] (1.25)

In this direction, we can rewrite the value of the expected dividends in the following form \(^3\) :

\(^3\) With \( Y_{t+1} = A^t Y_t \) a form of capitalization.
\[ E_t(\Delta D_{t+i}) = gE_tY_{t+i} = gA^iY_t \]  \hspace{1cm} (1.26)

If it is noted that \( E_t(D_{t+i}) = E(D_t + \sum_{j=1}^{i} D_{t+j}) \) then:

\[ E_t(D_{t+i}) = D_t + g \sum_{j=1}^{i} A^jY_t \]  \hspace{1cm} (1.27)

While replacing in the equation describing the variation of the prices, we thus obtains that:

\[ \Delta P_t = (1 - \alpha) \sum_{i=0}^{n} \alpha^i [D_t + g \sum_{j=1}^{i} A^jY_t - D_{t-1} - \sum_{j=1}^{i} gA^jY_{t-1}] + \Delta B_t \]  \hspace{1cm} (1.28)

In this direction we can finally write that:

\[ \Delta P_t = \Delta D_t + \Delta Y_t + \Delta B_t \]  \hspace{1cm} (1.29)

This relation thus makes it possible to collect the variations of the prices and explained by the means of the variations of the dividends and the variations of the bubbles. Wu (1995 and 1997) proposes to model the variations of the prices by having recourse to the kalman filter. The choice of this approach is based on impossibility to quantify the component \( B_t \) since it remains unobservable while belonging to the curve of the prices.

To estimate the securities of \( X \) and his variance we thus has recourse to the securities of \( \varepsilon \) and of \( \nu1 \). For this purpose, we can write that:

\[ x_t = x_{t|t-1} + \frac{c(2) * \sigma^2_{t|t-1} * \varepsilon}{\nu1} \]  \hspace{1cm} (1.34)

\[ \sigma^2_t = \sigma^2_{t|t-1} + \frac{c(2)^2 * \sigma^2_{t|t-1}}{\nu1} \]  \hspace{1cm} (1.35)

They is two estimates are estimators consist and without skews, which minimize the conditional variance.

In a last stage, we uses the method of maximum likelihood in order to estimate the parameters \( C(\cdot) \). The function of probability used is:

\[ L = \frac{1}{2} \sum_t \log(\nu1) - \frac{1}{2} \sum_t \frac{\varepsilon}{\nu1} \]  \hspace{1cm} (1.36)

The use of kalman filter should make it possible to extract the component bubble, unobservable, of the series of the real assets prices, by taking account of the developments of the fundamental value. From this point of view, we considers the equation proposed by Wu (1995):

\[ \Delta P_t = \Delta D_t + \Delta Y_t + \Delta B_t \]  \hspace{1cm} (1.37)

With:
- \( \Delta P_t \) : is the variation of the price index of the real assets;
- \( \Delta D_t \) : is the variation of the index of the rents (approximation of the real returns on assets);
- \( \Delta B_t \) : is the variation of the speculative bubble.

In order to apply this approach, it is necessary to identify two equations of measure and two equations of transition or state:
The equations of measure, integrating the observable variables P and D, are:

\[
\Delta P_t = \Delta D_t + \Delta Y_t + \Delta B_t \quad (1.38)
\]

\[
\Delta d_t = u_t + \sum_{i=0}^{n} \delta_i \Delta d_{t-i} + \varepsilon_t \quad (1.39)
\]

The equations of transition are:

\[
Y = AY_{t-1} + \eta \quad (1.40)
\]

\[
\Delta B_t = \gamma \Delta B_{t-1} + \mu^4 \quad (1.41)
\]

The estimates using the maximum of probability are carried out over one period going from 2000 to 2012 in quarterly frequency. The variation of the price index of the assets can be negative which also predicts the possibility of negative variations in the component bubble Cagan (1956). The series observed difference first are stationary and do not reveal any regime change (see preceding section).

The estimate of the equation of Wu (1995) by kalman filter provided the results according to:

<table>
<thead>
<tr>
<th>Table 10 : kalman filter estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coefficients</strong></td>
</tr>
<tr>
<td>C(1)</td>
</tr>
<tr>
<td>C(2)</td>
</tr>
<tr>
<td>C(3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>State variables</strong></th>
<th><strong>Value</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta D_t$</td>
<td>0.164742 (0.00)</td>
</tr>
<tr>
<td>$\Delta Y_t$</td>
<td>0.066667 (0.00)</td>
</tr>
<tr>
<td>$\Delta B_t$</td>
<td>-0.736229 (0.00)</td>
</tr>
</tbody>
</table>

| **LogL** | -1676067 |

Estimate using the maximum of probability 50000 iterations

The estimates gave more or less satisfactory results. Indeed, the two parameters C(1) and C(2), relating to the formation processes of the rents as C(3) post consistent estimates. Knowing that C(3) represents the coefficient connecting the variation of the bubble compared to its last:

\[
\Delta B_t = 0.699 \times \Delta B_{t-1} \quad (1.42)
\]

we can consider that:

\[
B_t = 0.699 \times \Delta B_{t-1} + B_{t-1} \quad (1.43)
\]

where:

\[
B_t = 0.699 \times B_{t-1} + B_{t-1} - B_{t-2} \quad (1.44)
\]

\[
B_t = 1.699 \times B_{t-1} - B_{t-2} \quad (1.45)
\]

If it is noted that the speculative bubble is characterized by a continuity in the belief in the rise and that the economic agents integrate this perception car director, then:

\[
B_t > B_{t-1} > B_{t-3} \ldots > B_{t-n}
\]

\[4\] We suppose in this formulation that the cov($\eta, \mu$) is null.
Graphically, it appears that the fundamental party, which must normally mainly explain the pricing of the rates on the real estate market seems to have a less important impact. By opposition, the component bubble largely explains the extreme movements of the real assets prices.

It proves that the development of the price index of the real assets is impacted by the development of the rents in Morocco. Indeed, the component bubble explains best the developments and the variations of the real assets prices. ANOVA analysis makes it possible to confirm this official report:

<table>
<thead>
<tr>
<th>Table 11: Source of the variations of the real price index</th>
<th>Bubble</th>
<th>Rent index (fundamental component)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANOVA Test</td>
<td>9.32 (0.00)</td>
<td>0.193 (0.66)</td>
</tr>
</tbody>
</table>

The test of the variance affirms that the volatility of the real index is resulting mainly from the component bubble and that the development of the rent in Morocco contributes marginally to the variations of the real prices.

Analysis of the bubble during the periods 2000 and 2012 fact of arising several significant rises during two periods 2006-2008 and 2010-2012. Thus the bubble during the period 2009-2010 knew the east, can be explained by the negotiable instruments of the international financial crisis and its transmission with the Moroccan economy.

Indeed, according to the formulation of Blanchard and Watson (1982) we can note that the bubble on the Moroccan real estate market is following form:

\[
\begin{align*}
B_{t+1} &= B_t \left(1 + \frac{0.5}{\pi}\right) + \mu_t \quad \text{with probability } \pi \\
B_{t+1} &= \mu_t \quad , \quad (1 - \pi)
\end{align*}
\]  
(1.46)
With \( \mu \) is the average tendency of long run and discount rate (capitalization) is of 50%.

6. Speculative bubbles cycles

Several work suggested using the Markov switching model to test the phases of boost and bust of the bubbles. The characteristic of these model lies in their capacity to describe in an empirical way the phase’s hawser and depression of the assets prices. Indeed, the criticism formulated by Evans (1991) on the cyclic nature of the bubbles constituted a catalyst for this type of work which tries to identify the regimes of the speculative bubbles.

The model is characterized by their capacity to detect heterogeneous states of the world. In the continuation of this paragraph, one proposes a short description of these models, however, for more details, it is necessary to see work of Hamilton [1994], Kim and Nelson [1999] and Wang [2003].

The simple following process is considered:

\[
y_t = \mu_{st} + \epsilon_t \quad (1.48)
\]

Where \( s_t \) the states by which the stochastic process passes during its existence, \( \epsilon_t \) is a Gaussian white noise and \( \mu \) makes it possible to detect the transition between the various states from the endogenous variable.

In this case, it should be noted as depending on the states of the world of “there” we obtains securities different as well with regard to the expectation \( \mu \) as from the moments of the residues of the equation.

If it is supposed that the world knows only two states, 1 and 2, for this purpose, the system can arise following system:

\[
y_{1t} = \mu_{s1} + \epsilon_{1t} : \text{first state}
\]
\[
y_{2t} = \mu_{s2} + \epsilon_{2t} : \text{second state}
\]

Where: \( \epsilon_{1t} \sim (0, \sigma_1^2) \) et \( \epsilon_{2t} \sim (0, \sigma_2^2) \)

This formulation induces clearly that the two processes \( y_1 \) and \( y_2 \) are different. When we are in the presence of state 1 then the hope of \( y_1 \) and \( \mu_{s1} \), while the emergence of state 2 fact of arising that the expectation of is there \( \mu_{s2} \).

The variances of the residues, forwarded in each process, describe predictive uncertainties of the model in each state of the system. In other words, the variances of the residues of the models, describe volatilities of each state of the system, for example, for a state 1 we could have a volatility more increased compared to that noted on the level of the second state.

In this section and using the model of Markov, it is proposed to measure the dating of the bubbles by specifying their birth date and also their date of deflation. It should thus make it possible to detect the phases ascending and downward prices.

By adopting the formulation of Blanchard et al. (1982), we considers that the process of bubble is controlled by two types of phase: a first ascending phase and a second depression.*

\[
\begin{cases}
B_{t+1} = B_t \left( \frac{1+r}{\pi} \right) + \mu_t \\
B_{t+1} = \mu_t , \quad (1 - \pi)
\end{cases}
\]

With, \( r \): the rate of capitalization and \( \pi \) probability of birth of the bubble.
In this case and by using the model of Markovian regime change we can estimate both state of the bubbles according to the following formulation:

\[
\begin{align*}
B_{1t} &= \mu_{s1} + \varepsilon_{1t}, \\
B_{2t} &= \mu_{s2} + \varepsilon_{2t},
\end{align*}
\]

The first state is relative to a speculative bubble hawser, during which the prices tend to increase, while the second state describes the phase depression or the bubble with tendency to burst itself under the negotiable instruments of the falls of the prices. The model with change with Markovian regime’s used makes it possible to approximate two types of state. The estimates using the maximum likelihood made it possible to lead to the following results:

<table>
<thead>
<tr>
<th>States</th>
<th>Mean</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boost (E2)</td>
<td>0.047453 (0.09)</td>
<td>2.118854 (0.00)</td>
</tr>
<tr>
<td>Bust (E1)</td>
<td>-3.903149 (0.00)</td>
<td>1.290944 (0.01)</td>
</tr>
</tbody>
</table>

LogL: -89.6689, the estimates were carried out by supposing the normal distribution.

According to the results obtained we are in the presence of two types of bubble on the Moroccan real estate market, first positive whose average of the rise is of 0.04 and we second phase of fall of the prices whose hope is equal to -3.9. Moreover and according to the analysis of the variance (volatility) of the two states, it arises that the phase of fall of the prices is less volatile than this rise. This can be explained by the fact why when the market with tendency to be more bull volatility increases in answer to the increased request of the purchasers of good against the phases depressions which are often interpreted same manner by the unit of the agents and where weakened volatility.

Most interesting in terms of analysis using the model with Markovian regime change, is the production of the matrix of probability of transition from a regime to another “D”. The results obtained affirm that; when we are in the presence of the first state (E1) we has a probability of 93% of persisting there and 6% of chance to forward towards a regime bull. In this direction and normal situation there are 6% of speculative probability of seeing being born a bubble. However, when we are in the second state of rise of the prices we has only 89% of chance that the bubble persists, while 11% of chance are in favor of a return to the state first.

\[
Matrix of transition probabilities = \begin{bmatrix} 0.93 & 0.11 \\ 0.07 & 0.89 \end{bmatrix}
\]

Figure 5: Probabilities of transition from a normal state in an explosive state of the real estate bubble between 2000 and 2012
The analysis of the development of the probability of supervening of an explosion of a bubble indicates that the frequency of formation of bubble is very limited. Indeed, it is noted that they exist only two periods which were affected by a rather important explosion of the prices, there is the two jumps of the curve in worms to cross the threshold of 100%. Most important to note is that this situation of euphoria persists only for one more or less short period being neighborly the 6 and 9 quarters. Thus, the analysis of the duration of the two regimes resulting from the model of Markov emphasizes the results presented in the following table:

<table>
<thead>
<tr>
<th>Table 13: duration of bubbles cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>states</td>
</tr>
<tr>
<td>Boost (E2)</td>
</tr>
<tr>
<td>Bust (E1)</td>
</tr>
</tbody>
</table>

The explosion of the bubble during the periods 2006 and 2008 coincides with the beginning of the boom of the credit with Morocco. Indeed, at this period the growth rate of the credit in Morocco was of two digits testifying to a broad expansion in the distribution of the loan to the economy and in particular the real estate market.

The analysis of the correlation between the bank credits and the real estate bubble emphasizes a significant rate of correlation between the two series, as we can note it on table 14.

| Table 14: correlation between the development of the bank credits and the real estate bubble |
|----------------|-----------------|-----------------|-----------------|
| Lag            | 0               | 1               | 2               | 3               |
| Correlation    | 38.13%          | 46.72%          | 52.71%          | 44.32%          |

As, the graphic comparison between the series of the bubble and the development of the bank credits and the real estate credits confirm as it is there a certain relation of dependence between the decision of granting of credit and the explosion of the bubble on the Moroccan real estate market. We notes since figure 6 which the boom of the appropriations which Morocco during the period 2006-2008 knew coincides largely with the explosion of the bubble over the same period. Indeed, the analysis of the correlation indicates that the rise of the real prices intervened two quarters after the beginning of the boom of the credit. This confirms that the increase in appropriations is among the principal determinants of the bubble which was formed over the period 2006-2008.

Figure 6: Development of the bank credits, the real estate credits and the real estate bubble between 2001 and 2010
7. Conclusion
The objective of this paper is to present several steps suitable for check the existence of speculative bubble on the Moroccan real estate market. In absence of a series of the real assets prices, the price index of the real assets was used to deduce the existence from it or not from a speculative bubble. Two approaches were recommended namely: a statistical approach and a structural approach. The first methodology use of the statistical tests suggested in the theoretical and empirical literature and which rests on the identification of the explosive character of the series of the prices and the fundamental value. In addition, the structural approach is based on a theoretical definition of the bubble and a test of estimate of the various forms of the Euler identity.
The results obtained confirmed that the real assets prices deviate significantly from the fundamental value, whose explanatory capacity is less important. For this purpose, a speculative bubble characterized the development of the real assets prices during the period 2006-2008. In addition, the analysis of the dating of the bubble on the real estate market lets predict the existence of two regimes which control the formation of the speculative bubble. The first state describes one period of rise of the prices, an explosion of the bubble, and a second state relative to a return at the normal or average state. According to these results obtained using the Markovian regimes we can affirm that the criticism of Evans (1991) on the cyclic nature of the financial bubbles is confirmed. Indeed, on the level of the Moroccan real estate market the explosion of the bubble lasts about 9 quarters and the normal cycle is of 14 quarters lifespan. At the end of each cycle begins another and so on.
The formation of the speculative bubble on the Moroccan real estate market is largely faded with the developments; that the credit market in Morocco recorded. Indeed, the boom of the credit market during the period 2006-2008 contributed to the explosion of the real estate bubble during the same period with a two quarters shift. This known as, the easing of the conditions of granting of credit during this phase to feed positively the beliefs of the economic agents intervening on the real estate market, until managing to form a certain consensus on the continuity of the rise of the real prices thus giving rise to a bubble on the market. This anticipation car-director characterized the market throughout one 2 year to return to a normal situation, after the transmission of the crisis to the Moroccan economy. However, the analysis of the trends of real prices and the speculative bubble, lets still predict the existence of a disconnection of the prices of fundamental and a continuation of the positive tendency of the bubble. For this purpose and to avoid a radical deflation of the bubble, it is convenient to lead preventive policies being able to mitigate the negotiable instruments of a crash landing of the prices on the real estate market.
References

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Title</th>
<th>Source</th>
</tr>
</thead>
</table>