Unit Value Bias (Indices) Reconsidered

Price- and Unit-Value-Indices in Germany

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Jens Mehrhoff*, Deutsche Bundesbank

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*This paper represents the author's personal opinion and does not necessarily reflect the view of the Deutsche Bundesbank or its staff.
1. Introduction and Motivation

2. Unit value index (UVI) and Drobisch's Index (PU^D)

3. Price and unit value indices in German foreign trade statistics (Tests of hypotheses)

4. Properties and axioms (uv, UVI, PU^D)

5. Decomposition of the Unit Value Bias (PU^p/PL L- and S-effect)

6. Interpretation of the S-effect in terms of covariances (using a generalized theorem of Bortkiewicz)

7. Conclusions
1. Introduction and Motivation

Export and Import Price Index Manual (XMPI Man. IMF, 2008)

Unit Value Indices (UVIs) are used in

Prices of trade (export/import), land, air freight and certain services
(consultancy, lawyers etc)

Literature (UVIs cannot replace price indices)

Diewert 1995 (NBER paper), 2004 etc.
von der Lippe 2006 GER
http://mpra.ub.uni-muenchen.de/5525/1/MPRA_paper_5525.pdf
Silver (2007), Do Unit Value Export, Import, and Terms of Trade
Indices Represent or Misrepresent Price Indices, IMF Working Paper
WP/07/121
1. Introduction and Motivation

2000 Jan – 2007 Dec

--- Price indices --- Unit value indices
1. Unit value for the k\textsuperscript{th} commodity number (CN)

\[ \tilde{p}_{k0} = \frac{\sum p_{kj0} q_{kj0}}{\sum q_{kj0}} = \sum_j p_{kj0} \frac{q_{kj0}}{Q_{k0}} = \sum_j p_{kj0} m_{kj0} \]

\[ k = 1, \ldots, K \quad \text{Unit values are not defined over all CNs} \]

Examples for CNs

<table>
<thead>
<tr>
<th>HS (Harmonized System)</th>
<th>Germany (Warenverzeichnis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>19 05 90 Other Bakers' Wares, Communion Wafers, Empty Capsules, Sealing Wafers</td>
<td>19 05 90 45 Cakes and similar small baker's wares (8 digits)</td>
</tr>
<tr>
<td>23 09 10 Dog or Cat Food, Put up for Retail Sale</td>
<td>23 09 10 11 to 23 09 10 90 twelve (!!) CNs for dog or cat food</td>
</tr>
</tbody>
</table>
2. German Unit Value Index (UVI) of exports/imports
the usual Paasche index (unit values instead of prices)

\[
PU_{0t}^P = \frac{\sum_k \tilde{p}_{kt} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{kt}} = \frac{\sum_k \sum_{m_k} p_{kjt} q_{kjt}}{\sum_k Q_{kt} \left( \sum_j \frac{m_k p_{kj0} q_{kj0}}{Q_{k0}} \right)}
\]

Aggregation in two stages; 
k = 1, ..., K ,

\[
j = 1, ..., n_K \text{ commodities in the } k^{th} \text{ CN; } \sum n_k = n \text{ (all commodities)}
\]

3. The Unit value index (UVI) should be kept distinct from
Drobisch's index (1871)

\[
P_{0t}^{DR} = \frac{\sum_k \sum_j p_{jkt} q_{jkt}}{\sum_k \sum_j q_{jkt}} = \frac{\sum_k \sum_j p_{jkt} q_{jkt}}{\sum_k Q_{kt}}
\]

\[
= \frac{\sum_k \sum_j p_{jk0} q_{jk0}}{\sum_k \sum_j q_{jk0}} = \frac{\sum_k \sum_j p_{jk0} q_{jk0}}{\sum_k Q_{k0}}
\]
Drobisch's index

\[ p_{0t}^{DR} = \frac{\tilde{p}_t}{p_0} = \frac{V_{0t}}{\tilde{Q}_{0t}}, \quad \tilde{Q}_{0t} = \frac{Q_t}{Q_0} \]

However, Drobisch is better known for

\[ \frac{1}{2} \left( P_{0t}^L + P_{0t}^P \right) \]

<table>
<thead>
<tr>
<th>Information about quantities available</th>
<th>Information about quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td>the same commodity in different outlets</td>
<td>&quot;normal&quot; usage of the term &quot;low level&quot;</td>
</tr>
<tr>
<td>different goods grouped by a classification</td>
<td>situation of a UVI (Σq needed for unit value)</td>
</tr>
</tbody>
</table>

It does not make sense to consider **absolute unit values** ("Euro per kilogram")
Austrian Import prices rose from \( \approx 20 \, \text{€ per kilogram} \) in 1995 to 25 € … in 2005

"Because we use weights as units an increasing import price index could be explained by either rising prices or reduced weights due to quality improvement"
2. UVI and price indices (PI): System of possible indices

$2^4 = 16$ indices:
- type of index (price vs quantity)
- Prices ($p$) vs unit values ($uv$)
- Laspeyres vs Paasche
- Export vs import

$$V = \frac{\sum p_t q_t}{\sum p_0 q_0} = P^P Q^L = P^L Q^L$$

<table>
<thead>
<tr>
<th></th>
<th>Price-indices</th>
<th>Quantity-indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p$</td>
<td>$uv$</td>
</tr>
<tr>
<td>$P$</td>
<td>$P^L$</td>
<td>$PU^L$</td>
</tr>
<tr>
<td>$Q$</td>
<td>$Q^L$</td>
<td>$QU^L$</td>
</tr>
<tr>
<td>Laspeyres</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P^P$</td>
<td>$PU^P$</td>
<td></td>
</tr>
<tr>
<td>$Q^P$</td>
<td>$QU^P$</td>
<td></td>
</tr>
<tr>
<td>Paasche</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ottawa Group 2009
Unit Value Bias Reconsidered
## 3. Indices in Germany

<table>
<thead>
<tr>
<th>Data</th>
<th>Price index</th>
<th>Unit value index</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td>Survey based (monthly), sample; more demanding (weights!)</td>
<td>Customs based (by-product), census, Intrastat: survey</td>
</tr>
<tr>
<td><strong>Formula</strong></td>
<td>Laspeyres</td>
<td>Paasche</td>
</tr>
<tr>
<td><strong>Quality adjustment</strong></td>
<td>Yes</td>
<td>No (feasible?)</td>
</tr>
<tr>
<td><strong>Prices, aggregates</strong></td>
<td>Prices of specific goods at time of contracting</td>
<td>Average value of CNs; time of crossing border</td>
</tr>
<tr>
<td><strong>New / disappearing goods</strong></td>
<td>Included only when a new base period is defined; vanishing goods replaced by similar ones constant selection of goods *</td>
<td>Immediately included; price quotation of disappearing goods is simply discontinued variable universe of goods</td>
</tr>
<tr>
<td><strong>Merits</strong></td>
<td>Reflect pure price movement (ideally the same products over time)</td>
<td>&quot;Representativity&quot; inclusion of all products; data readily available</td>
</tr>
<tr>
<td><strong>Published in</strong></td>
<td>Fachserie 17, Reihe 11</td>
<td>Fachserie 7, Reihe 1</td>
</tr>
</tbody>
</table>

CN = commodity numbers  
* All price determining characteristics kept constant
### 3. Indices in Germany

#### (2) Overview of Hypothesis

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Argument</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2. Volatility U &gt; P</strong></td>
<td>U no pure price comparison (U is reflecting changes in product mix [structural changes])</td>
</tr>
<tr>
<td><strong>3. Seasonality U &gt; P</strong></td>
<td>U no adjustment for seasonally non-availability</td>
</tr>
<tr>
<td><strong>4. U suffers from heterogeneity</strong></td>
<td>Variable vs. constant selection of goods, CN less homogeneous than specific goods</td>
</tr>
<tr>
<td><strong>5. Lead of P</strong></td>
<td>Prices refer to the earlier moment of contracting (contract-delivery lag; exchange rates)</td>
</tr>
<tr>
<td><strong>6. Smoothing (due to quality adjustment)</strong></td>
<td>Quality adjustment in P results in smoother series</td>
</tr>
</tbody>
</table>
4. Properties and axioms: 4.1. unit values: one CN, two commodities

\[ p_{10} = p_{1t} = p \]
\[ p_{20} = p_{2t} = \lambda p \]
\[ \mu = m_{2t}/0.5 \]
\[ m_{10} = m_{20} = 0.5 \]

\[ \Delta = \tilde{p}_{kt} - \tilde{p}_{k0} = \frac{p}{2} (1 - \lambda)(1 - \mu) \]

<table>
<thead>
<tr>
<th>( \lambda &lt; 1 )</th>
<th>( \lambda &gt; 1 ) and ( \mu &lt; 1 ) → ( \Delta &lt; 0 )</th>
<th>( \lambda &gt; 1 ) and ( \mu &gt; 1 ) → ( \Delta &gt; 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I &lt; 1 )</td>
<td>less of the more expensive good 2 unit value <em>declining</em></td>
<td>more of the more expensive good 2 unit value <em>rising</em></td>
</tr>
<tr>
<td>( \lambda &lt; 1 )</td>
<td>( \lambda &lt; 1 ) and ( \mu &lt; 1 ) → ( \Delta &gt; 0 )</td>
<td>( \lambda &lt; 1 ) and ( \mu &gt; 1 ) → ( \Delta &lt; 0 )</td>
</tr>
<tr>
<td>( \lambda &gt; 1 )</td>
<td>less of the cheaper good 2 unit value <em>rising</em></td>
<td>more of the cheaper good 2 unit value <em>declining</em></td>
</tr>
</tbody>
</table>

"… 'unit value' indices … may therefore be affected by changes in the mix of items as well as by changes in their prices. Unit value indices cannot therefore be expected to provide good measures of average price change over time" (SNA 93, § 16.13)
1) UVI mean of uv-ratios

\[ PU^P_{0t} = \sum_k \frac{\tilde{p}_{kt}}{\tilde{p}_{k0}} \frac{\tilde{p}_{k0} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{kt}} \]

2) Ratio of unit values ≠ mean of price relatives

\[ \frac{\tilde{p}_{kt}}{\tilde{p}_{k0}} = \sum_j \frac{p_{kjt}}{p_{kj0}} \left( \frac{p_{kj0} q_{kjt}}{\tilde{p}_{k0} Q_{kt}} \right) \]

the weights do not add up to unity, but to

\[ \frac{Q_{k0}}{Q_{kt}} \cdot Q_{0t}^{L(k)} = \frac{Q_{0t}^{L(k)}}{\tilde{Q}_{0t}^k} \]

3) Proportionality (identity)

Contribution of k to S-effect
### Axioms Drobnisch's (price) index and the German UVI (= PU^P)

<table>
<thead>
<tr>
<th>Axiom</th>
<th>Drobnisch*</th>
<th>German PU^P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportionality</td>
<td>$U(p_0, \lambda p_0, q_0, q_t) = \lambda$ (identity = 1)</td>
<td>no</td>
</tr>
<tr>
<td>Commensurability</td>
<td>$U(\Lambda p_0, \Lambda p_t, \Lambda^{-1} q_0, \Lambda^{-1} q_t) = U(p_0, p_t, q_0, q_t)$</td>
<td>no</td>
</tr>
<tr>
<td>Linear homogen.</td>
<td>$U(p_0, \lambda p_t, q_0, q_t) = \lambda \cdot U(p_0, p_t, q_0, q_t)$</td>
<td>yes</td>
</tr>
<tr>
<td>Additivity** (in current period prices)</td>
<td>$U(p_0, p^<em>_t, q_0, q_t) = U(p_0, p_t, q_0, q_t) + U(p_0, p^+_t, q_0, q_t)$ for $p^</em>_t = p_t + p^+_t$</td>
<td>yes</td>
</tr>
<tr>
<td>Additivity** (in base period prices)</td>
<td>$[U(p^<em>_0, p_t, q_0, q_t)]^{-1} = [U(p_0, p_t, q_0, q_t)]^{-1}$ $+ [U(p^+_0, p^+_t, q_0, q_t)]^{-1}$ for $p^</em>_0 = p_0 + p^+_0$</td>
<td>yes</td>
</tr>
<tr>
<td>Product test</td>
<td>Implicit quantity index of PU^D or PU^P</td>
<td>$\Sigma q_t/\Sigma q_0$</td>
</tr>
<tr>
<td>Time reversibility</td>
<td>$U(p_0, p_t, q_0, q_t) = U^\leftarrow =$ $[U(p_0, p_t, q_0, q_t)]^{-1} = [U^\rightarrow]^{-1}$</td>
<td>yes</td>
</tr>
<tr>
<td>Transitivity</td>
<td>$U(p_0, p_2, q_0, q_2) = U(p_0, p_1, q_0, q_1) \cdot U(p_1, p_2, q_1, q_2)$</td>
<td>yes</td>
</tr>
</tbody>
</table>

* Balk1995, Silver 2007, IMF Manual; applies also to subindex $\tilde{P}_{kt}/\tilde{P}_{k0}$

** Inclusive of (strict) monotonicity
5. Decomposition of the discrepancy \( D \)

Value index \( V_{0t} = PU_{0t}^L QU_{0t}^P = PU_{0t}^P QU_{0t}^L \)

**Bortkiewicz Formula**

\[
C = \sum_i \left( \frac{p_{it}}{p_{i0}} - P_{0t}^L \right) \left( \frac{q_{it}}{q_{i0}} - Q_{0t}^L \right) \frac{p_{i0}q_{i0}}{\sum p_{i0}q_{i0}}
\]

\[
= V_{0t} - Q_{0t}^L P_{0t}^L = Q_{0t}^L \left( P_{0t}^P - P_{0t}^L \right)
\]

**Discrepancy (uv-bias)**

\[
D = \frac{PU_{0t}^P}{P_{0t}^L} = \left( \frac{C}{Q_{0t}^L P_{0t}^L} + 1 \right) \left( \frac{Q_{0t}^L}{QU_{0t}^L} \right) = \frac{P_{0t}^P}{P_{0t}^L} \cdot \frac{PU_{0t}^P}{P_{0t}^L} = L \cdot S
\]

\[
L = \frac{Q_{0t}^P}{Q_{0t}^L} = \frac{Q_{0t}^P}{S \cdot QU_{0t}^L} = \frac{PU_{0t}^P}{S \cdot P_{0t}^L}
\]

\[
S = \frac{Q_{0t}^L}{QU_{0t}^L} = \frac{Q_{0t}^P}{L \cdot QU_{0t}^L} = \frac{PU_{0t}^P}{L \cdot P_{0t}^L}
\]
5. The two effects $L$ and $S$ - 1 -

<table>
<thead>
<tr>
<th></th>
<th>$S &lt; 1$</th>
<th>$S = 1$</th>
<th>$S &gt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L &lt; 1$</td>
<td>$PU^P &lt; P^P &lt; P^L$</td>
<td>$PU^P &lt; P^L$</td>
<td>indefinite</td>
</tr>
<tr>
<td>$L = 1$</td>
<td>$PU^P &lt; P^L = P^P$</td>
<td>$PU^P = P^P = P^L$</td>
<td>$PU^P &gt; P^L = P^P$</td>
</tr>
<tr>
<td>$L &gt; 1$</td>
<td>indefinite</td>
<td>$PU^P &gt; P^L$</td>
<td>$PU^P &gt; P^P &gt; P^L$</td>
</tr>
</tbody>
</table>
5. The two effects L and S - 2 -

Deflator X and M respectively taken for $P^p$

S and L independent?

---

Laspeyres effect (% pt) -- Structural component (% pt) ⋯ Discrepancy (%)
Normal reaction:
**L and S negative** more likely in the case of imports

**exports**

**imports**
Interpretation L-Effect: contributions to the covariance (Szulc)

\[ R = \frac{P_P - P_L}{P_L} = \sum_i \left[ \left( \frac{p_i^1/p_i^0 - P_L}{P_L} \right) \left( \frac{q_i^1/q_i^0 - Q_L}{Q_L} \right) \left( \frac{p_i^0 q_i^0}{\sum p_i^0 q_i^0} \right) \right] \]

\[ R \text{ a "centred" covariance} \quad \frac{s_{XY}}{\overline{X}\cdot\overline{Y}} \quad L = R + 1 \]


**No L-effect** (\( L = 1 \)) if

1. all \( p^1/p^0 \) equal (\( P_L \)) or = 1
2. all \( q^1/q^0 = Q_L \) or = 1
3. covariance = 0

**No S-effect** (\( S = Q^L/QU^L = 1 \)) if

1. no CNs, only individual goods (or: each \( n_k = 1 \), perfectly homogeneous CNs)
2. all \( q^1/q^0 \) equal (or = 1)
3. all \( m_{kjt} = m_{kj0} \) \( \forall j, k \)
4. all prices in 0 equal \( p_{kj0} = \overline{p}_{k0} \)
   
   *prices in \( t \) are irrelevant*
6. Contribution of a CN (k) to S as ratio of two linear indices

1. \[ S = \frac{Q_0^L}{QU_0^L} = \sum_k \frac{Q_0^{L(k)}}{Q_0^L} \cdot \frac{\tilde{p}_{k0}}{\tilde{p}_{k0}} \frac{Q_{kt}}{Q_{kt}} \]

2. Generalized theorem of Bortkiewicz for two linear indices \( X_t \) and \( X_0 \)

\[ X_t = \frac{\sum x_t y_t}{\sum x_0 y_t} \]

\[ X_0 = \frac{\sum x_t y_0}{\sum x_0 y_0} \]

\[ X_t = 1 + \frac{S_{xy}}{X \cdot Y} \]

\[ w_0 = \frac{x_0 y_0}{\sum x_0 y_0} \]

\[ \sum \frac{x_t}{x_0} w_0 = \overline{X} = X_0 \]

\[ S_{xy} = \sum \left( \frac{x_t}{x_0} - \overline{X} \right) \left( \frac{y_t}{y_0} - \overline{Y} \right) w_0 = \frac{\sum x_t y_t}{\sum x_0 y_0} - \overline{X} \cdot \overline{Y} \]

The "usual" theorem (page 15) is a special case →
6. Generalized Theorem of Bortkiewicz

**Theorem for the L-effect**

\[
\frac{X_t}{X_0} = 1 + \frac{s_{xy}}{X \cdot Y}
\]

<table>
<thead>
<tr>
<th>(x_0 = p_0)</th>
<th>(y_0 = q_0)</th>
<th>(X_t = P^p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_t = p_t)</td>
<td>(y_t = q_t)</td>
<td>(X_0 = P^L)</td>
</tr>
</tbody>
</table>

\[
C = \sum_i \left( \frac{p_{it}}{p_{i0}} - \frac{p^L_{it}}{p^L_{i0}} \right) \left( \frac{q_{it}}{q_{i0}} - \frac{Q^L_{0t}}{Q^L_{0t}} \right) \sum p_{i0} q_{i0}
\]

1. for \(S\)

\[
S = \frac{Q^L_{0t}}{QU^L_{0t}}
\]

<table>
<thead>
<tr>
<th>(x_0 = q_0)</th>
<th>(y_0 = 1)</th>
<th>(X_t = Q^L_{0t}(k))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_t = q_t)</td>
<td>(y_t = p_0)</td>
<td>(X_0 = \tilde{Q}^k_{0t})</td>
</tr>
</tbody>
</table>

\[
\sum \left( \frac{q_{kjt}}{q_{kj0} - Q^L_{0t}} \right) \left( \frac{p_{kj0}}{p_{kj0} - \tilde{p}_{k0}} \right) \frac{q_{kj0}}{\sum q_{kj0}}
\]

2. for \(1/S\)

<table>
<thead>
<tr>
<th>(x_0 = q_0)</th>
<th>(y_0 = p_0)</th>
<th>(X_t = \tilde{Q}^k_{0t})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_t = q_t)</td>
<td>(y_t = 1)</td>
<td>(X_0 = Q^L_{0t}(k))</td>
</tr>
</tbody>
</table>

\[
\sum \left( \frac{q_{kjt}}{q_{kj0} - Q^L_{0t}} \right) \left( \frac{1}{p_{kj0} - \tilde{p}_{k0}} \right) \frac{p_{kj0} q_{kj0}}{\sum p_{kj0} q_{kj0}}
\]
6. Two commodities example with both, S and L effect (example of page 12)

\[ p_{10} = p_{1t} = p \]
\[ p_{20} = p_{2t} = \lambda p \]
\[ \mu = m_{2t} / 0.5 \]
\[ m_{10} = m_{20} = 0.5 \]

\[ \pi = p_{1t} / p_{10} \]
\[ p_{2t} / p_{20} = \eta \pi \]

<table>
<thead>
<tr>
<th>L-effect</th>
<th>( \lambda &gt; 1 )</th>
<th>( \Delta &lt; 0 \rightarrow S &lt; 1 )</th>
<th>( \Delta &gt; 0 \rightarrow S &gt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda &lt; 1 )</td>
<td>( \Delta &gt; 0 \rightarrow S &gt; 1 )</td>
<td>( \Delta &lt; 0 \rightarrow S &lt; 1 )</td>
<td>( \mu &lt; 1 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S-effect</th>
<th>( S = \frac{Q_{0t}^L}{\tilde{Q}_{0t}} = 1 + \frac{(1-\lambda)(1-\mu)}{1+\lambda} = 1 + \frac{\Delta}{\tilde{p}_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{0t}^L = \frac{\pi(1 + \eta \lambda)}{1 + \lambda} )</td>
<td>( = 1 )</td>
</tr>
<tr>
<td>( P_{0t}^P = \frac{\pi(2 - \mu + \eta \lambda \mu)}{2 - \mu + \lambda \mu} )</td>
<td>( = 1 )</td>
</tr>
</tbody>
</table>

\[ L = \frac{P_{0t}^P}{P_{0t}^L} = \frac{2 - \mu + \eta \lambda \mu}{1 + \eta \lambda} \cdot \frac{1 + \lambda}{2 - \mu + \lambda \mu} \]
7. Conclusions

\[
\Delta^* = \tilde{p}_t - \tilde{p}_0 = \frac{p}{2} \left[ \pi (2 - \mu (1 - \eta \lambda)) - (1 + \lambda) \right]
\]

if \( \pi = \eta = 1 \)

\[
\Delta^* = \Delta
\]

\[
C = s_{xy}^{(L)} = \frac{2 \tilde{Q}_{0t} \lambda (1 - \eta)(1 - \mu)}{(1 + \lambda)^2}
\]

\[
C = 0
\]

\[
\Delta^* = \tilde{p}_t - \tilde{p}_0 = \pi \frac{s_{xy}^1}{\tilde{Q}_{0t}} + \frac{s_{xy}^L (1 + \lambda)^2}{2 \tilde{Q}_{0t}} + \pi (1 - \lambda \eta) - (1 - \lambda)
\]

7. What remains to be done

- Analysis of the time series of UVIs and PIs on various levels of disaggregation, cointegration and Granger-Causality

- Microeconomic interpretation of S-effect (in terms of utility maximizing behaviour)
No structural change between CNs (that is $Q_{k0} = Q_{kt}$) yields

$$V_{0t} = PU^P_{0t} = PU^L_{0t} \quad \text{and} \quad QU^L_{0t} = QU^P_{0t} = 1$$

This is, however, not sufficient for the S-effect to vanish

$$S = Q^L_{0t} \neq 1$$

No mean value property of PUP

$$PU^P = \sum_k \sum_j p_{kjt} \left( \frac{p_{kjt} q_{kjt}}{\sum \sum p_{kjt} q_{kjt}} \right)$$

$$P^P = \sum_k \sum_j p_{kjt} \left( \frac{p_{kjt} q_{kjt}}{\sum \sum p_{kjt} q_{kjt}} \right)$$

The same applies to Laspeyres

$$PU^L = \sum_k \sum_j p_{kjt} \left( \frac{q_{kjt}}{Q_{k0}} \right)$$

$$P^L = \sum_k \sum_j p_{kjt} \left( \frac{q_{kjt}}{Q_{k0}} \right)$$

$q^*_{kjt} = q_{kjt} \frac{Q_{kt}}{Q_{k0}}$

a fictitious quantity in $t$
The relation $S = \frac{P_U^P}{P^P}$ instead of $S = \frac{Q^L}{Q^L_U}$ is not interesting.

\[
PU^P = \sum_k \frac{\tilde{p}_{kt} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{kt}} = \sum_k P^P_{(k)} \frac{Q_{kt} \sum_j p_{kj0} m_{kjt}}{\sum_k Q_{kt} \sum_j p_{kj0} m_{kjt}}
\]

\[
P^P = \sum_k \sum_j p_{kjt} q_{kjt} = \sum_k P^P_{(k)} \frac{Q_{kt} \sum_j p_{kj0} m_{kjt}}{\sum_k Q_{kt} \sum_j p_{kj0} m_{kjt}}
\]

Sum of weights!

**UVI in XMPI Manual**

§ 2.14
Drobisch's formula

\[
P_U = \left( \frac{\sum_m p_m^1 q_m^1}{\sum_m q_m^1} \right) / \left( \frac{\sum_n p_n^0 q_n^0}{\sum_n q_n^0} \right)
\]