Scanner Data and the Treatment of Quality Change in Rolling Year GEKS Price Indexes

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Abstract: The recently developed rolling year GEKS procedure makes maximum use of all matches in the data in order to construct price indexes that are (approximately) free from chain drift. A potential weakness is that unmatched items are ignored. In this paper we use imputation Törnqvist price indexes as inputs into the rolling year GEKS procedure. These indexes account for quality changes by imputing the 'missing prices' associated with new and disappearing items. Three imputation methods are discussed. The first method makes explicit imputations using a hedonic regression model which is estimated for each time period. The other two methods make implicit imputations; they are based on time dummy hedonic and time-product dummy regression models and are estimated on pooled data. We present empirical evidence for New Zealand from scanner data on eight consumer electronics products and find that accounting for quality change can make a substantial difference.

Key words: hedonic regression, imputation, multilateral index number methods, quality adjustment, scanner data, transitivity.

JEL Classification: C43, E31.

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1. Introduction

Barcode scanning data, or scanner data for short, contain information on the prices and quantities sold of all individual items. One obvious advantage of using scanner data for compiling the Consumer Price Index (CPI) is that price indexes can cover product categories completely rather than being based on a small sample of items as is usual practice. Another advantage of using scanner data is that the construction of superlative indexes, such as Fisher or Törnqvist indexes, is now feasible.[1] Superlative indexes treat both time periods in a symmetric fashion and have attractive properties, like taking into account the consumers' substitution behavior. Most statistical agencies still rely today on fixed-weight, Laspeyres-type indexes to compile the CPI.

Scanner data typically show substantial item attrition; many new items appear and many 'old' items disappear. This makes it difficult if not impossible to construct price indexes using the standard approach where the prices of a more or less fixed set of items are tracked over time. Chain linking period-on-period price movements seems an obvious solution, but that can lead to a drifting time series under certain circumstances. Ivancic, Diewert and Fox (2011) resolved the problem of chain drift by adapting the well-known GEKS (Gini, 1931; Eltetö and Köves; 1964; Szulc, 1964) method for comparing prices across countries to comparing prices across time. Their rolling year (RY) GEKS approach makes optimal use of the matches in the data and yields price indexes that are approximately free from chain drift.

A potential weakness of matched-item approaches, including RYGEKS, is that the price effects of new and disappearing items are neglected. High-tech products, such as consumer electronics, usually experience rapid quality changes; new items are often of higher quality than existing ones. It is well established in the literature that adjusting for quality change is an essential part of price measurement (see e.g. ILO et al., 2004). The purpose of this paper is to show how quality-adjusted RYGEKS price indexes can be estimated. These indexes provide us with a benchmark measure that can be used to assess the performance of easier-to-construct price indexes.

[1] Statistics Netherlands has been using scanner data for supermarkets in the CPI since 2004. In 2010, a new computation method was introduced and the coverage was expanded to include more supermarket chains. However, the new Dutch method does not make use of weighting information at the individual item level. Van der Grient and de Haan (2011) describe the method and explain the choice for using an unweighted index number formula at the elementary aggregation level.

The RYGEKS procedure combines bilateral superlative indexes, which compare two time periods, with different base periods. In the original setup, the bilateral indexes only take account of the matched items, i.e. the items that are available in both periods compared. Quality mix changes can occur within the set of matched items. For example, overall quality will improve over time when consumers increasingly purchase higher-quality items. Changes in the quality mix of a matched set do not need special attention here, however; they will be handled appropriately using matched-item superlative price indexes.

The issue at stake is how to account for quality changes associated with new and disappearing items. We do this by estimating bilateral imputation price indexes, which serve as inputs into the RYGEKS procedure. Imputation price indexes adjust for quality changes by imputing the unobservable or 'missing' prices to construct price relatives for the new and disappearing items.

The paper is structured as follows. Section 2 outlines the RYGEKS procedure. Sections 3 to 5 discuss three regression-based bilateral imputation Törnqvist indexes. The method outlined in section 3 makes explicit imputations using a hedonic regression model which is estimated on cross-section data for each period. The two other methods are based on making implicit imputations. In section 4 we discuss a result derived by de Haan (2004) regarding how a weighted least squares time dummy hedonic model that is estimated on the pooled data of two periods implicitly defines an imputation Törnqvist index. A non-hedonic variant of the weighted time dummy model, referred to as the time-product dummy model, is described in section 5. We show that this model leads to a matched-item price index and therefore does not offer a solution to the quality-change problem.

In section 6 we summarize the discussion by listing the steps to be followed for estimating Imputation Törnqvist (IT) RYGEKS indexes using the time dummy hedonic approach and point to a few additional issues. In section 7 we describe our data set and explain that it unfortunately does not enable us to estimate separate regression models for each period. The monthly scanner data cover purchases in New Zealand over a three-year period on eight consumer electronics goods: camcorders, desktop computers, digital cameras, DVD players/recorders, laptop computers, microwaves, televisions and portable media players. Section 8 presents empirical evidence and shows that hedonic imputations have, on average, a significant downward effect on the RYGEKS indexes.

We compare our ITRYGEKS indexes with indexes estimated as rolling year versions of the weighted multi-period time dummy method. The latter are also quality-adjusted and approximately drift-free, and appear to perform quite well.

Section 9 concludes the paper and suggests some topics for further work in this area.

2. The rolling window GEKS method

Suppose that we know the prices p_i^t and expenditure shares s_i^t for all items *i* belonging to a product category *U* in all time periods $t = 0,...,T$. For the moment, we assume that there are no new or disappearing items so that *U* is fixed over time. The Törnqvist price index going from the starting period 0 to period *t* (>0) is defined as

$$P_T^{0t} = \prod_{i \in U} \left(\frac{p_i^t}{p_i^0} \right)^{\frac{s_i^0 + s_i^t}{2}}; \qquad t = 1,...,T. \tag{1}$$

This index compares the prices in each period *t* (>0) directly with those in the starting or base period 0. The Törnqvist index is superlative and has useful properties from both the economic approach and the axiomatic approach to index number theory (ILO *et al.*, 2004). However, the index series defined by (1) is not transitive. That is, the results of the price comparisons between two periods depend on the choice of base period.[2] In (1), the starting period 0 was chosen as the base or price reference period, but this choice is rather arbitrary if we want to compare any pair of time periods.

To illustrate the non-transitivity property, let us take period 1 as the base instead of period 0 and make a comparison with period *T*. The Törnqvist index P_T^{1t} going from period 1 to period *T* is

$$P_T^{1T} = \prod_{i \in U} \left(\frac{p_i^T}{p_i^1} \right)^{\frac{s_i^1 + s_i^T}{2}}. \tag{2}$$

Using period 0 as the base (as in (1)), the price change between periods 1 and *T* will be calculated as the ratio of the index numbers in periods *T* and 1:

[2] In spatial price comparisons, transitivity is also known as circularity. This is an important requirement because the choice of base country should not affect measured price level differences across countries.

$$\frac{P_T^{0T}}{P_T^{01}} = \frac{\prod_{i \in U} \left(\frac{p_i^T}{p_i^0} \right)^{\frac{s_i^0 + s_i^T}{2}}}{\prod_{i \in U} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{s_i^0 + s_i^1}{2}}} = P_T^{1T} \left[\prod_{i \in U} (p_i^0)^{s_i^1 - s_i^T} \prod_{i \in U} (p_i^1)^{s_i^T - s_i^0} \prod_{i \in U} (p_i^T)^{s_i^0 - s_i^1} \right]^{\frac{1}{2}}. \tag{3}$$

In general, the bracketed factor in (3) will differ from 1 and we have $P_T^{0T} / P_T^{01} \neq P_T^{1T}$, indicating non-transitivity.

In a time series context, transitivity implies that the period-on-period chained, or shifting base, index equals the corresponding direct (fixed base) index since the choice of base period does not matter. Put differently, transitive price indexes will be free from chain drift. Chain drift is defined as a situation where the chain price index, unlike its direct counterpart, differs from unity when the prices of all items return to their initial (period 0) values.[3] Empirical research on scanner data has shown that, in spite of their symmetric structure, superlative indexes can exhibit substantial chain drift under high-frequency chaining (see Feenstra and Shapiro, 2003; Ivancic, Diewert and Fox, 2011; de Haan and van der Grient, 2011).

There are circumstances when high-frequency chaining is recommended though, for example, when there are a large number of new and disappearing items. Chaining enables us to maximize the set of matched products (those products that are available in the periods compared). To resolve the problem of chain drift, Ivancic, Diewert and Fox (2011) adapted the GEKS method, which is well known from price comparisons across countries, to price comparisons across time. Below, we outline their methodology for constructing transitive price indexes.[4]

The proposed GEKS index is equal to the geometric mean of the ratios of all possible 'bilateral' price indexes, based on the same index number formula, where each period is taken as the base. Taking 0 as the index reference period (the period in which

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[3] This definition seems a little restrictive. In a less formal way, chain drift can alternatively be described as a situation in which the chain index drifts further and further away from the underlying 'true' trend. If there are no new or disappearing items, the 'true' trend can be measured by the direct index according to some preferred formula. Random deviations from the trend do not reflect drift and should not bother us too much.

[4] In the context of price indexes for seasonal goods, Balk (1981) describes a method that is equivalent to the GEKS method. Kokoski, Moulton and Zieschang (1999) also pointed to the possibility of adapting the GEKS approach to intertemporal price comparisons.

the index equals 1) and denoting the link periods by l $(0 \le l \le T)$, the GEKS price index going from 0 to t is

$$P_{GEKS}^{0t} = \prod_{l=0}^{T} \left[P^{0l} / P^{tl} \right]^{1/(T+1)} = \prod_{l=0}^{T} \left[P^{0l} \times P^{lt} \right]^{1/(T+1)}; \qquad t = 0,...,T. \tag{4}$$

Equation (4) presupposes that the bilateral indexes satisfy the time reversal test, (that $P^{0t} = 1/P^{t0}$). The GEKS index will then also satisfy this test. It can easily be shown that the GEKS index is transitive and can therefore be written as a period-on-period chained index.

Using the second expression of (4), the GEKS index going from period 0 to the last (most recent) period T can be expressed as

$$P_{GEKS}^{0T} = \prod_{t=0}^{T} \left[P^{0t} \times P^{tT} \right]^{1/(T+1)}. \tag{5}$$

So far, the number of time periods (including the index reference period 0) was fixed at $T+1$. In practice, we want to extend the series as time passes. If we add data pertaining to the next period $(T+1)$, then the GEKS index for this period is

$$P_{GEKS}^{0,T+1} = \prod_{t=0}^{T+1} \left[P^{0t} \times P^{t,T+1} \right]^{1/(T+2)}. \tag{6}$$

Extending the time series in this way has two drawbacks. The GEKS index for the most recent period $T+1$ does not only depend on the data of periods 0 and $T+1$ but also on the data of all intermediate periods. Hence, when the time series is extended, there will be an increasing loss of characteristicity.[5] Furthermore, the GEKS method suffers from revision: the price index numbers for periods $1,...,T$ computed using the extended data set will differ from the previously computed index numbers.

To reduce the loss of characteristicity and circumvent the revision of previously computed price index numbers, Ivancic, Diewert and Fox (2011) propose a rolling year approach. This approach makes repeated use of the price and quantity data for the last 13 months (or 5 quarters) to construct GEKS indexes. A window of 13 months has been chosen as it is the shortest period that can deal with seasonal products. The most recent month-on-month index movement is then chain linked to the existing time series. Using

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[5] Caves, Christensen and Diewert (1982) define characteristicity as the "degree to which weights are specific to the comparison at hand".

$P_{GEKS}^{0,12}$ as the starting point for compiling a monthly time series, the rolling year GEKS (RYGEKS) index for the next month becomes

$$P_{RYGEKS}^{0,13} = P_{GEKS}^{0,12} \prod_{t=1}^{13} \left[P^{12,t} \times P^{t,13} \right]^{1/13} = \prod_{t=0}^{12} \left[P^{0t} \times P^{t,12} \right]^{1/13} \prod_{t=1}^{13} \left[P^{12,t} \times P^{t,13} \right]^{1/13} . \tag{7}$$

One month later, the RYGEKS index is

$$P_{RYGEKS}^{0,14} = P_{RYGEKS}^{0,13} \prod_{t=2}^{14} \left[P^{13,t} \times P^{t,14} \right]^{1/13} . \tag{8}$$

This chain linking procedure is repeated each next month.

Ivancic, Diewert and Fox (2011) used bilateral matched-item Fisher indexes in the above formulas. Following de Haan and van der Grient (2011), we will use bilateral Törnqvist indexes since their geometric structure facilitates a decomposition analysis, as will be shown later on. Both the Fisher index and the Törnqvist index satisfy the time reversal test and usually generate very similar results.

Unlike GEKS indexes, RYGEKS indexes are not by definition free from chain drift. Nevertheless, it is most likely that any chain drift will be very small. Since each 13-month GEKS series is free from chain drift, we would expect chain linking the GEKS index changes not to lead to a drifting series. Empirical evidence from scanner data on goods sold at supermarkets lends support to our expectation that RYGEKS indexes are approximately drift free. See Ivancic, Diewert and Fox (2011); de Haan and van der Grient (2011); Johansen and Nygaard (2011); and Krsinich (2011).

Although matched-item GEKS indexes are free from chain drift, this does not necessarily mean they are completely drift free; there may be other causes for a drifting or biased time series. Greenlees and McClelland (2010) show that matched-item GEKS price indexes for apparel suffer from significant downward bias. The prices of apparel items typically exibit a downward trend so that any matched-item index will measure a price decline. The problem here is a lack of explicit quality adjustment.[6] Of course this quality-change problem carries over to RYGEKS indexes.

The problem can in principle be dealt with by using bilateral imputation price indexes as inputs into the RYGEKS procedure rather than their matched counterparts,

[6] As mentioned by van der Grient and de Haan (2010), this problem may be partly due to the use of a too detailed item identifier, in which case items that are comparable from the consumer's perspective would be treated as different items.

provided that the imputations make sense. Imputation price indexes use all the matches in the data and, in addition, impute the 'missing prices' that are associated with new and disappearing items. In section 3, we discuss the (hedonic) imputation Törnqvist index and decompose this index into three factors: the contributions of matched items, new items and disappearing items.

3. Hedonic imputation Törnqvist price indexes

The issue considered in this section (and in sections 4 and 5) is how the unmatched new and disappearing items should be treated in a bilateral Törnqvist price index, where we compare two time periods. For the sake of simplicity, we compare period 0 with period *t* $(t = 1,...,T)$. In section 6 we will show how to handle all the bilateral price comparisons that show up in the RYGEKS framework.

We will denote the set of items that are available in both period 0 and period *t* by U^{0t}. For these matched items, we have base period prices p_i^0 and period *t* prices p_i^t, so that we can compute price relatives p_i^t / p_i^0. The set of disappearing items, which were observed in period 0 but are no longer available in period *t*, is denoted by $U^0_{D(t)}$. Here, the base period price is known but the period *t* price is unobservable. To compute price relatives for the disappearing items, values $\hat{p}_i^t$ have to be predicted (imputed) for the 'missing' period *t* observations. The set of new items, which are observed in period *t* but were not available in period 0, is denoted by $U^t_{N(0)}$. In this case the period *t* prices are known but the base period prices are 'missing' and must be imputed by $\hat{p}_i^0$ to be able to compute the price relatives. Note that $U^{0t} \cup U^0_{D(t)} = U^0$, the total set of items in period 0, and $U^{0t} \cup U^t_{N(0)} = U^t$, the total set of items in period *t*. Using the observed and imputed prices, the imputation Törnqvist price index – which equals the square root of the product of the imputation geometric Laspeyres and Paasche indexes – is given by

$$P_{SIT}^{0t} = \left[\prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^0} \prod_{i \in U^0_{D(t)}} \left(\frac{\hat{p}_i^t}{p_i^0} \right)^{s_i^0} \right]^{\frac{1}{2}} \left[\prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^t} \prod_{i \in U^t_{N(0)}} \left(\frac{p_i^t}{\hat{p}_i^0} \right)^{s_i^t} \right]^{\frac{1}{2}}$$

$$= \prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{\frac{s_i^0 + s_i^t}{2}} \prod_{i \in U^0_{D(t)}} \left(\frac{\hat{p}_i^t}{p_i^0} \right)^{\frac{s_i^0}{2}} \prod_{i \in U^t_{N(0)}} \left(\frac{p_i^t}{\hat{p}_i^0} \right)^{\frac{s_i^t}{2}} . \tag{9}$$

P_{SIT}^{0t} is a so-called single imputation Törnqvist index. Statistical agencies use the term imputation for estimating missing observations, and so single imputation would be the usual approach. In the index number literature, double imputation has also been used. In a double imputation price index, the observed prices of the unmatched new and disappearing items are replaced by predicted values. Hill and Melser (2008) discuss all kinds of different imputation indexes based on hedonic regression. They argue that the double imputation method may be less prone to omitted variables bias since the biases in the numerator and denominator of the estimated price relatives for the unmatched items are likely to cancel out, at least partially.[7] Syed (2010) focuses on consistency rather than bias and makes a similar case. However, the single imputation variant is our point of reference because, as will be shown in section 4 below, this links up with the use of a weighted time-dummy variable approach to hedonic regression.

In Appendix 1 it is shown that P_{SIT}^{0t} can be decomposed as[8]

$$P_{SIT}^{0t} = \prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{\frac{s_{i(0t)}^0 + s_{i(0t)}^t}{2}} \left[\frac{\prod_{i \in U_{D(t)}^0} \left(\frac{\hat{p}_i^t}{p_i^0} \right)^{s_{iD(t)}^0}}{\prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_{i(0t)}^0}} \right]^{\frac{s_{D(t)}^0}{2}} \left[\frac{\prod_{i \in U_{N(0)}^t} \left(\frac{p_i^t}{\hat{p}_i^0} \right)^{s_{iN(0)}^t}}{\prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_{i(0t)}^t}} \right]^{\frac{s_{N(0)}^t}{2}}, \qquad (10)$$

where $s_{i(0t)}^0$ and $s_{i(0t)}^t$ denote the expenditure share of item *i* with respect to the set U^{0t} of matched items in period 0 and period *t*, respectively; $s_{iD(t)}^0$ is the period 0 expenditure share of *i* with respect to the set $U_{D(t)}^0$ of disappearing items, and $s_{iN(0)}^t$ is the period *t* expenditure share of *i* with respect to the set $U_{N(0)}^t$ of new items; $s_{D(t)}^0 = \sum_{i \in U_{D(t)}^0} s_i^0$ is the aggregate period 0 expenditure share of disappearing items, and $s_{N(0)}^t = \sum_{i \in U_{N(0)}^t} s_i^t$ is the aggregate period *t* expenditure share of new items.

The first factor in (10) is the matched-item Törnqvist index. The second factor equals the ratio, raised to the power of $s_{D(t)}^0 / 2$, of the imputation geometric Laspeyres index for the disappearing items and the geometric Laspeyres index for the matched items. The third factor is the ratio, raised to the power of $s_{N(0)}^t / 2$, of the imputation

[7] Equation (9) is similar to equation (4) in Silver and Heravi (2007), except that they use predicted prices for all items, including the matched ones. So they define a full imputation Törnqvist index. A drawback of their approach is that the index becomes fully dependent on the choice of the hedonic model and the estimation technique.

[8] A similar decomposition holds for the double imputation Törnqvist index.

geometric Paasche index for the new items and the geometric Paasche index for the matched items.

The product of the second and third factor can be viewed as an adjustment factor by which the matched-item Törnqvist price index should be multiplied in order to obtain a quality-adjusted price index. If someone would prefer the matched-item index as a measure of aggregate price change, then from an imputations perspective they are either assuming that the second and third factors cancel each other out (which would be a pure coincidence) or that the 'missing prices' are imputed such that both factors are equal to 1. The latter occurs if $\hat{p}_i^t$ for the disappearing items is calculated through multiplying the period 0 price by the matched-item geometric Laspeyres index $\prod_{i \in U^{0t}} (p_i^t / p_i^0)^{s_{i(0t)}^0}$ and if $\hat{p}_i^0$ for the new items is calculated through dividing the period t price by the matched-item geometric Paasche index $\prod_{i \in U^{0t}} (p_i^t / p_i^0)^{s_{i(0t)}^t}$. There is no a priori reason to think this would be appropriate.

The imputations should measure the Hicksian reservation prices, which are the prices that would have been observed if the items had been available on the market. Of course, these fictitious prices can only be estimated by using some kind of modelling. Hedonic regression is an obvious choice in this respect.[9] The hedonic hypothesis states that a good is a bundle of, say, K price determining characteristics. We will denote the fixed 'quantity' of the k-th characteristic for item i by z_{ik} $(k = 1,...,K)$. Triplett (2006) and others have argued that the functional form should be determined empirically, but we will only consider the logarithmic-linear model specification:

$$\ln p_i^t = \alpha^t + \sum_{k=1}^{K} \beta_k^t z_{ik} + \varepsilon_i^t ; \qquad t = 0,...,T , \qquad (11)$$

where β_k^t is the parameter for characteristic k in period t and ε_i^t is an error term with an expected value of zero. The log-linear model specification has been frequently applied and usually performs quite well. It has three advantages: it accounts for the fact that the (absolute) errors are likely to be bigger for higher priced items, it is convenient for use in a geometric index such as the Törnqvist, and it can be compared with the models we will be using in sections 4 and 5.

[9] This is true for product varieties which are comparable in the sense that they can be described by the same set of characteristics so that their prices can be modeled by the same hedonic function. We do not address the problem of entirely new goods, which have different characteristics than existing goods due to, for example, new production techniques.

We assume for now that model (11) is estimated separately for each time period by least squares regression. Using the estimated parameters $\hat{\alpha}^t$ and $\hat{\beta}_k^t$, the predicted prices are denoted by $\hat{p}_i^t = \exp(\hat{\alpha}^t + \sum_{k=1}^{K} \hat{\beta}_k^t z_{ik})$. The predicted values for $i \in U_{D(t)}^0$ and $i \in U_{N(0)}^t$ serve as imputations in the single imputation Törnqvist price index P_{SIT}^{0t} given by (9). It is easily verified that P_{SIT}^{0t} satisfies the time reversal test.

An issue is whether we should use either Ordinary Least Squares (OLS) or some form of Weighted Least Squares (WLS) regression. From an econometric point of view, WLS could help increase efficiency (reduce the standard errors of the coefficients) when heteroskedasticity is present. With homoskedastic errors, OLS would seem to be appropriate. Silver (2003) pointed out, however, that we have multiple observations for item *i*, equal to the number of sales q_i^t, rather than a single observation. Running an OLS regression on a data set where each item counts q_i^t times is equivalent to running a WLS regression where the q_i^t serve as weights. This type of WLS would reflect the economic importance of the items in terms of quantities sold.

Instead of quantities or quantity shares[10] we could alternatively use expenditure shares as weights in the regressions. In section 4 we will explain that a particular type of expenditure-share weighting is 'optimal' when estimating a (two-period) pooled time-dummy variable hedonic model. But in the current situation, where we look at separate regression models for each time period, the weighting issue has not been completely settled.

4. The weighted time dummy hedonic method

The hedonic imputation method discussed in section 3 has the virtue of being flexible as the characteristics parameters are allowed to change over time. In spite of this, it may be useful to constrain the parameters to be the same in the periods compared to increase efficiency. Again, we will be looking at bilateral price comparisons (to be used in an RYGEKS framework) where the starting period 0 is directly compared with each period *t*, and where *t* runs from 1 to *T*. Replacing the β_k^t in the log-linear hedonic model (11) by time-independent parameters β_k yields

[10] When using the item's quantity share, i.e., the quantity sold divided by the aggregate quantity sold, as its weight, the weights of the different items add up to 1 while leaving the estimates unaffected. Note that aggregating quantities across different items has no particular economic interpretation.

$$\ln p_i^t = \alpha^t + \sum_{k=1}^{K} \beta_k z_{ik} + \varepsilon_i^t \,; \qquad t = 0,...,T \,. \tag{12}$$

Model (13) should be estimated on the pooled data of the two periods compared. Using a dummy variable D_i^t that has the value 1 if the observation relates to period t $(t \neq 0)$ and the value 0 if the observation relates to period 0, the estimating equation for the bilateral time dummy variable method becomes[11]

$$\ln p_i^t = \alpha + \delta^t D_i^t + \sum_{k=1}^{K} \beta_k z_{ik} + \varepsilon_i^t \,; \qquad t = 0,...,T \,, \tag{13}$$

where ε_i^t is an error term with an expected value of zero, as before. Note that the time dummy parameter δ^t shifts the hedonic surface upwards or downwards. The estimated time dummy and characteristics parameters are $\hat{\delta}^t$ and $\hat{\beta}_k$. Since model (13) controls for changes in the characteristics, $\exp \hat{\delta}^t$ is a measure of quality-adjusted price change between periods 0 and *t*. The predicted prices in the base period 0 and the comparison periods *t* are $\hat{p}_i^0 = \exp(\hat{\alpha} + \sum_{k=1}^{K} \hat{\beta}_k z_{ik})$ and $\hat{p}_i^t = \exp(\hat{\alpha} + \hat{\delta}^t + \sum_{k=1}^{K} \hat{\beta}_k z_{ik})$, so we have $\exp \hat{\delta}^t = \hat{p}_i^t / \hat{p}_i^0$ for all i $(t = 1,...,T)$.

The question arises as to what regression weights would properly reflect the economic importance of the items when estimating equation (13) by WLS. In Appendix 2 it is shown that if the weights for the matched items are the same in periods 0 and *t*, i.e., if $w_i^0 = w_i^t = w_i^{0t}$ for $i \in U^{0t}$, then the time dummy index can be expressed as

$$P_{TD}^{0t} = \exp \hat{\delta}^t = \left[\prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{w_i^{0t}} \prod_{i \in U_{D(t)}^0} \left(\frac{\hat{p}_i^t}{p_i^0} \right)^{w_i^0} \prod_{i \in U_{N(0)}^t} \left(\frac{p_i^t}{\hat{p}_i^0} \right)^{w_i^t} \right]^{\frac{1}{w^{0t} + w_{D(t)}^0 + w_{N(0)}^t}} , \tag{14}$$

where, as before, U^{0t} is the set of matched items (with respect to periods 0 and *t*), $U_{D(t)}^0$ is the set of disappearing items, and $U_{N(0)}^t$ is the set of new items; $w^{0t} = \sum_{i \in U^{0t}} w_i^{0t}$, $w_{D(t)}^0 = \sum_{i \in U_{D(t)}^0} w_i^0$ and $w_{N(0)}^t = \sum_{i \in U_{N(0)}^1} w_i^t$

Following up on the work of Diewert (2003), de Haan (2004) suggested taking the average expenditure shares as weights for the matched items, i.e., $w_i^{0t} = (s_i^0 + s_i^t)/2$ for $i \in U^{0t}$, and taking half of the expenditures shares for the unmatched items (in the periods they are available), i.e, $w_i^0 = s_i^0 / 2$ for $i \in U_{D(t)}^0$ and $w_i^t = s_i^t / 2$ for $i \in U_{N(0)}^t$. Since now $w^{0t} + w_{D(t)}^0 + w_{N(0)}^t = 1$, substitution of the proposed weights into (14) gives

[11] Diewert, Heravi and Silver (2009) and de Haan (2010) compare the (weighted) hedonic imputation and time dummy approaches.

$$P_{TD}^{0t} = \exp \hat{\delta}^t = \prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{\frac{s_i^0 + s_i^t}{2}} \prod_{i \in U_{D(t)}^0} \left(\frac{\hat{p}_i^t}{p_i^0} \right)^{\frac{s_i^0}{2}} \prod_{i \in U_{N(0)}^t} \left(\frac{p_i^t}{\hat{p}_i^0} \right)^{\frac{s_i^t}{2}} . \tag{15}$$

The weighted time dummy hedonic index (15) is a special case of the single imputation Törnqvist price index given by (9), where the 'missing prices' for the unmatched items are imputed according to the estimated time dummy model. Note that the regression weights are identical to the weights used to aggregate the price relatives in the Törnqvist formula. So the notion of economic importance is the same in the weighted regression and in the index number formula, which is reassuring. Note further that the time dummy index satisfies the time reversal test.

If there are no new or disappearing items, then (15) reduces to the matched-item Törnqvist index. Thus, the result is independent of the set of characteristics included in the model. This is a desirable property: in this case we want the resulting price index not to be affected by the model specification but to be based on the standard matched-model methodology. If model (13) was estimated by OLS rather than WLS regression on the pooled data of periods 0 and t, then in the matched-items case the time dummy index would equal the (unweighted) Jevons index. The use of an unweighted index number formula is obviously undesirable, so the weighting issue is particularly important for the time dummy method.

5. The weighted time-product dummy method

As mentioned previously, the hedonic hypothesis states that a product can be seen as a bundle of characteristics that determines the quality, hence the price, of the product. The number of relevant characteristics differs across product groups. In practice the set of characteristics is typically rather limited, often because sufficient information is lacking. But what if detailed information on characteristics is missing? This is not an unrealistic situation. Statistical agencies are increasingly getting access to highly disaggregated data on prices and quantities purchased, but the data sets often include only loose item descriptions. Obtaining sufficiently detailed information on item characteristics can be difficult or costly.

Let us look at the extreme case where no price determining characteristics at all are known and see what happens if the only 'characteristic' of an item that is included in the time dummy model is a dummy variable that identifies the item. Suppose that we have N different items, both matched and unmatched ones. The estimating equation for the bilateral time dummy model then becomes

$$\ln p_i^t = \alpha + \delta^t D_i^t + \sum_{i=1}^{N-1} \gamma_i D_i + \varepsilon_i^t , \tag{16}$$

where the item or product dummy variable D_i has the value 1 if the observation relates to item i and 0 otherwise; γ_i denotes the corresponding parameter. To prevent perfect multicollinearity, the dummy for an arbitrary item N is excluded from model (16).[12]

Model (16) is a so-called fixed-effects model, which has been applied by several researchers to estimate price indexes, e.g., by Aizcorbe, Corrado and Doms (2003) and Krsinich (2011). In the international price comparisons literature, where countries are compared instead of time periods, the method is known as the Country-Product Dummy (CPD) method.[13] In the present intertemporal context we will refer to it as the Time-Product Dummy (TPD) method. The period 0 and period t predicted prices for item i are given by $\hat{p}_i^0 = \exp(\hat{\alpha} + \hat{\gamma}_i)$ and $\hat{p}_i^t = \exp(\hat{\alpha} + \hat{\delta}^t + \hat{\gamma}_i)$. The estimated fixed effect for item i equals (the exponential of) $\hat{\gamma}_i$, and the estimated two-period time dummy index is $\exp \hat{\delta}^t = \hat{p}_i^t / \hat{p}_i^0$, as before.

In the general exposition of section 4 we did not specify the set of characteristics included in the time dummy model, so the main results also apply in the present context. We list the most important properties:

- The TPD method automatically imputes the 'missing prices' for the unmatched items.[14]

[12] Alternatively, we could leave out the intercept term and add a dummy variable for this item (plus a time dummy for the base period). This would not affect the results.

[13] There is a large literature on international price comparisons and the associated measurement problems. An elementary introduction can be found in Eurostat and OECD (2006). For more advanced overviews, see Diewert (1999) and Balk (2001) (2008).

[14] This property of 'filling gaps' in an incomplete data set was the reason for Summers (1973) to propose the (multilateral) CPD method as an alternative to the (G)EKS method. It has been argued that another advantage of the CPD method is the possibility for calculating standard errors. But the interpretation of these standard errors is not straightforward if, as with scanner data, we observe the entire finite population of items. For example, if all items are observed and matched, the bilateral weighted TPD index equals the Törnqvist price index, which has no sampling error but does have a standard error attached to it. This standard error is in fact a measure of model error rather than sampling error (unless one would be willing to assume that the finite population is a sample from a 'super population').

- The TPD index satisfies the time reversal test.
- If all items are matched during the two time periods compared, and if the model is estimated by OLS regression, then the bilateral TPD index equals the Jevons price index.
- If a WLS regression is run on the pooled data of (the two) periods 0 and t with appropriate expenditure share weights, then the resulting TPD index is a single imputation Törnqvist index.

One interpretation of the TPD model is as follows. If, in agreement with the time dummy variable method, all characteristics parameters are assumed constant over time, then the combined effect on the price is ‘fixed’ for each item. So the TPD method can be viewed as a variant of the time dummy method where item-specific effects are measured through dummy variables. It could even be argued that the TPD method is ‘better’ than the time dummy hedonic method. The TPD method takes into account the combined effect of all characteristics, whereas the hedonic method suffers from omitted variables bias when some relevant price-determining characteristics are unobservable. Also, because the weighted TPD method makes implicit imputations for the unmatched items, the TPD index may seem preferable to the matched-item Törnqvist index.

There are a number of issues involved, however. First, if sufficient information on characteristics is available, the TPD method is less efficient than the time dummy hedonic method (although with enough observations, as we have in our data set, there will not be a problem with degrees of freedom). Second, the item-specific effects will be inaccurately estimated in the bilateral case because we have only one price observation for an unmatched item. Third, and most importantly, because these effects are measured through dummy variables, the observed prices of the unmatched items in the periods they are available are equal to the predicted prices. Put differently, the bilateral TPD method implicitly defines a double imputation index.

The third point has an interesting implication. Substituting $p_i^0 = \hat{p}_i^0$ for $i \in U_{D(t)}^0$ and $p_i^t = \hat{p}_i^t$ for $i \in U_{N(0)}^t$ into decomposition (10), and recalling that $\hat{p}_i^t / \hat{p}_i^0 = \exp \hat{\delta}^t$, the weighted bilateral TPD index turns out to be a weighted mean of the matched-item geometric Laspeyres and Paasche price indexes:

$$P_{TPD}^{0t} = \exp \hat{\delta}^t = \left[\prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_{i(0t)}^0} \right]^{\frac{s_M^0}{s_M^0 + s_M^t}} \left[\prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_{i(0t)}^t} \right]^{\frac{s_M^t}{s_M^0 + s_M^t}}, \tag{17}$$

where s_M^0 and s_M^t denote the aggregate expenditures shares of the matched items in the two periods. If $s_M^0 > s_M^t$ $(s_M^0 < s_M^t)$, the weight attached to the matched-item geometric Laspeyres index will be greater (smaller) than the weight attached to the matched-item geometric Paasche index. If $s_M^t = s_M^0$, then (17) reduces to the matched-item Törnqvist index. In the unweighted case, the bilateral TPD index would be equal to the matched-item Jevons index. This result was derived earlier by Silver and Heravi (2005), so our result is a generalization of theirs.

It can be seen that, conditional on the weights for the matched items, expression (17) is insensitive to the choice of weights for the unmatched items. That is, due to the least squares orthogonality property with respect to the regression residuals, the new and disappearing items become redundant in the bilateral TPD method; essentially, they are dropped out from the estimation, and a matched-item index results. This method therefore does not resolve the quality-change problem. Furthermore, there seems to be no good reason to prefer the resulting index (17) over the (symmetric and superlative) matched-item Törnqvist index.[15]

6. Estimating hedonic imputation Törnqvist-RYGEKS indexes

In sections 3 and 4 we discussed two variants of hedonic imputation in Törnqvist price indexes: explicit imputation, based on a log-linear model which is estimated separately for each time period, and implicit imputation, based on a weighted version of the time dummy method. In both cases, the bilateral indexes compare each time period *t* directly with the base period 0. For estimating hedonic imputation Törnqvist RYGEKS indexes, we need all kinds of bilateral price comparisons. However, the general idea stays the same, and the two methods can be easily extended to other comparisons.

Recall expression (5) for the GEKS price index, which we repeat here:

$$P_{GEKS}^{0T} = \prod_{t=0}^{T} \left[P^{0t} \times P^{tT} \right]^{1/(T+1)}, \tag{18}$$

[15] Diewert (2004) discusses the weighted bilateral country-product dummy (CPD) approach in the context of price comparisons between two countries. His expression for the implicit index number formula is essentially equivalent to our equation (17) for the intertemporal case. He notes that the index number formula "can deal with situations where say item n* has transactions in one country but not the other" – an unmatched item in our language – and that "the prices of item n* will be zeroed out".

where T denotes the most recent period; when using monthly data, T will be equal to 12. This expression holds for the hedonic imputation Törnqvist price indexes as they satisfy the time reversal test. In addition to bilateral indexes P^{0t} going from 0 to t, we require bilateral indexes P^{tT} going from t to T. The construction of P^{tT} is similar to that of P^{0t}; we only need to change the two periods compared. Extending this to a rolling year framework is also straightforward. We move the 13-month window one month forward, estimate GEKS price indexes again, compute the latest monthly index change and chain link this change to the existing series. This procedure is repeated each month.

As will be explained later, our data set does not allow us to estimate hedonic models for each time period separately. This means we are unable to apply the explicit imputation variant but are only able to implement the weighted time dummy variant. For convenience, we list the steps to be followed for estimating Imputation Törnqvist Rolling Year GEKS (ITRYGEKS) price indexes using bilateral time dummy hedonic indexes.

We distinguish eight steps:

1. Select an appropriate set of price-determining characteristics for the product category in question that will be used in the log-linear time dummy hedonic model.[16]
2. Estimate bilateral time dummy models by weighted least squares regression using data pertaining to the first 13 months $(0,...,12)$, where the weights are expenditure shares as defined in section 4.
3. Compute the corresponding bilateral time dummy price index numbers.
4. Calculate the GEKS index numbers for months $1,...,12$ according to equation (17) using these bilateral time index numbers; the index for period 0 is equal to 1.
5. Repeat steps 2, 3, and 4 for the period covering months $1,...,13$.
6. Compute the most recent GEKS index change by dividing the index number for month 13 by the index number for month 12.
7. Chain link the index change through multiplication to the existing series.
8. Repeat steps 5, 6, and 7 for subsequent 13-month windows.

[16] For practical advice on the estimation of hedonic regression models, see ILO *et al*. (2004), Triplett (2006), and Destatis (2009).

There are two issues that may need further clarification. First, the time dummy method assumes that the characteristics parameters are constant over time. In a rolling-year framework, this assumption is relaxed since the parameters are constrained to be the same for no more than 13 months. There is an inconsistency in assuming fixity of the parameters during, say, the first 13-month period (months 0,…,12) and then during the second 13-month period (months 1,…,13) because the parameters relating to months 1,…,12 are allowed to take on different values in the two 13-month windows, which is at variance with the underlying assumption. However, the flexibility of the rolling year approach is a very useful property, and it seems to us that this type of inconsistency is not a major problem.[17] Note that the rolling year approach is also flexible in the sense that it facilitates changing the set of characteristics included in the hedonic model when deemed necessary or when entirely new characteristics are introduced.

Second, one may wonder why we are not using a more straightforward approach to estimating transitive, quality-adjusted price indexes. In particular, pooling the data of many periods and running a time dummy regression would generate transitive indexes because the results of a pooled regression are insensitive to the choice of base period. To mitigate the problem that the indexes will increasingly rely on model predictions as the number of matched items decreases over time, we could restrict the regression to 13 months and apply a rolling year procedure; this would also circumvent the problem of revisions.

The point is that our choice for the regression weights that implicitly produces a single imputation Törnqvist price index in the two-period case cannot be extended to the multi-period case because we would have multiple weights for the observations of the matched items in the starting period 0. In the empirical section 8 we will nevertheless estimate rolling year multilateral time dummy hedonic indexes, using monthly varying expenditure shares as regression weights, to investigate how this easier-to-implement method performs.

Two other important questions addressed in section 8 are the following. What is the effect of imputing the 'missing prices' in Törnqvist-RYGEKS indexes as compared to their matched-item counterparts? Are different product categories equally affected by the imputations?

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[17] Even if the 'true' parameter values within each 13-month window were constant, the estimated values for the bilateral comparisons will generally differ because they are estimated on different data sets.

7. New Zealand consumer electronics scanner data

Statistics New Zealand has been using scanner data for consumer electronics products from market research company GfK for a number of years, to inform expenditure weighting. This data is very close to full-coverage of the New Zealand market, and contains sales values and quantities aggregated to quarterly levels for combinations of brand, model and up to 6 characteristics.

Recently a much more detailed dataset was purchased for the three years from mid 2008 to mid 2011 for eight products: camcorders, desktop computers, digital cameras, DVD players and recorders, laptop computers, microwaves[18], televisions, and portable media players. Monthly sales values and quantities are disaggregated by brand, model and around 40 characteristics.

Table 1 shows the eight products ordered by their expenditure weights. These are the average of the monthly expenditure weights, across the three years from mid-2008 to mid-2011. Televisions have by far the most significant weight of 44%, followed by laptop computers which have an average expenditure weight of 26%. Desktop computers have only 20% of the weight of laptop computers, at 6%.

Table 1. Average expenditure weights (%) for each product

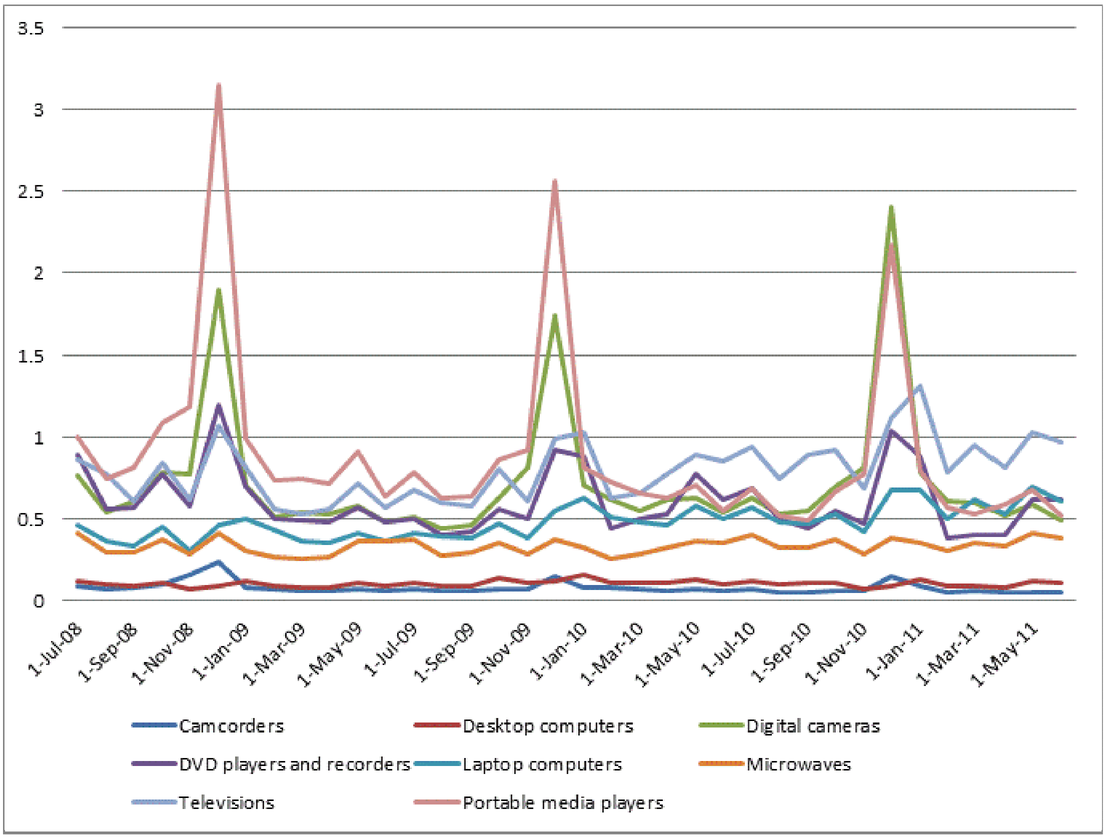
Televisions	43.7
Laptop computers	26.0
Digital cameras	8.1
Portable media players	6.8
Desktop computers	6.0
DVD players and recorders	4.1
Microwaves	3.0
Camcorders	2.2

Figure 1 shows quantities sold of each product across time. For confidentiality reasons, the total quantities are scaled so that portable media players = 1 in July 2008,

[18] Microwaves are not really a ‘consumer electronics’ product but, as a product with less rapid technological change, they can provide a useful comparison in terms of how different price index methods perform.

which preserves the relative quantities between products and across time. As expected, the quantities are strongly seasonal, with significant December/Christmas peaks for portable media players and camcorders. For televisions and computers, the highest number of sales tend to be the following month, in January.

Figure 1. Relative quantities sold of each product



Also for confidentiality reasons, any brand where a single retailer has a share of more than 95%[19] of total sales for the month is renamed to ‘tradebrand’ in GfK’s output system; similarly at the model level when a single retailer accounts for more than 80% of the sales of that model.[20]

We define an ‘item’ as a unique combination of brand, model and the full set of characteristics[21] available in the data. This can be seen as equivalent to the ‘barcode’

[19] For all the products looked at in this paper except microwaves, which has a threshold of 99%.

[20] Ideally we would hope to find a way to protect confidentiality without this aggregation to ‘tradebrand’ if we were to adopt scanner data in production.

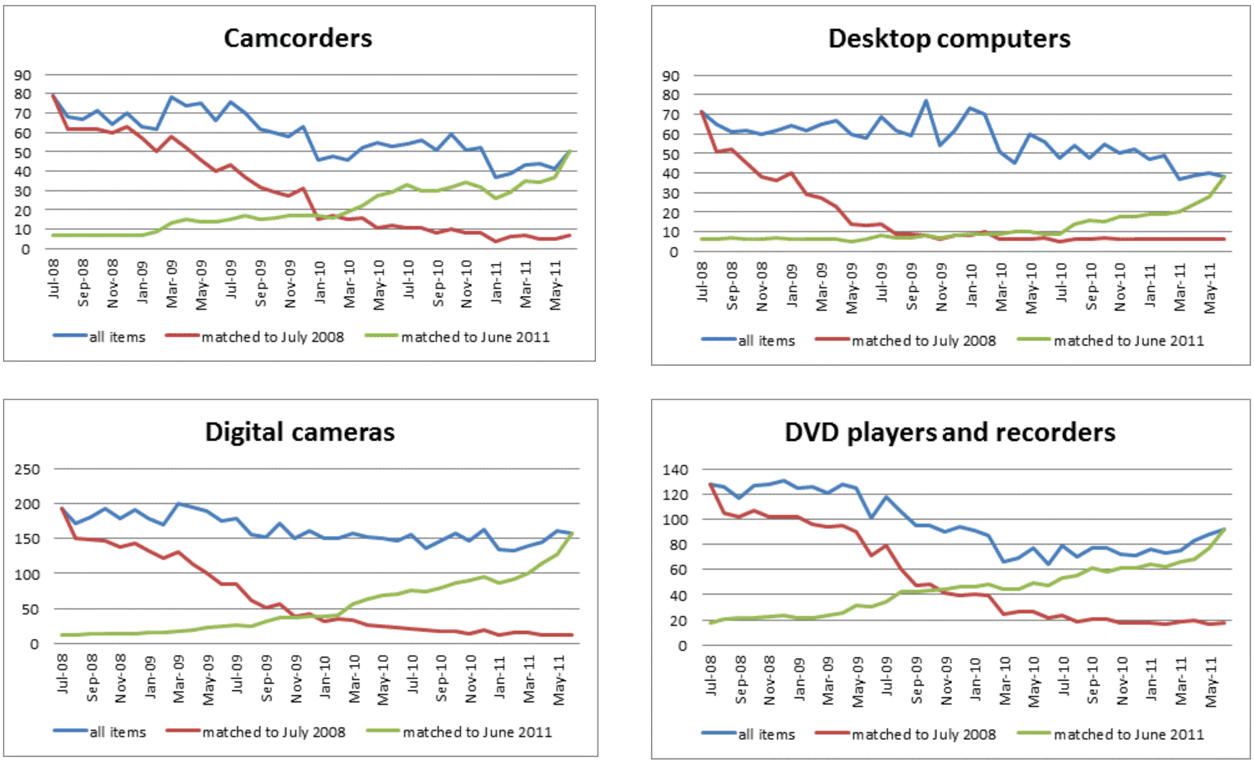
[21] Appendix 3 shows the set of characteristics used for each of the eight products.

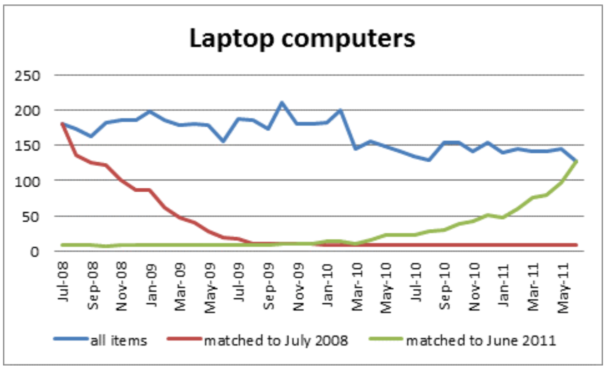
because it corresponds to the full level of detail on characteristics of the products. Note that the data is aggregated across outlets. The service associated with particular outlets can be viewed as part of the total quality of a product, so any change in the composition of the sample in terms of outlets should ideally be controlled for. We are not able to do this.

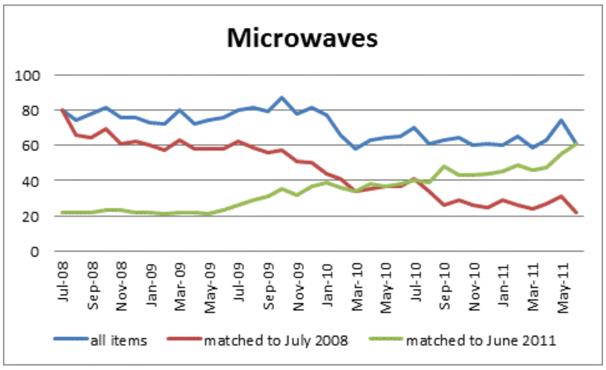
A key feature of scanner data is that it reflects the high level of ‘churn’ in the specific items available and being sold from month to month. That is, there are many new specifications of the product becoming available in the market and, conversely, old specifications dropping out of the market as they become obsolete.

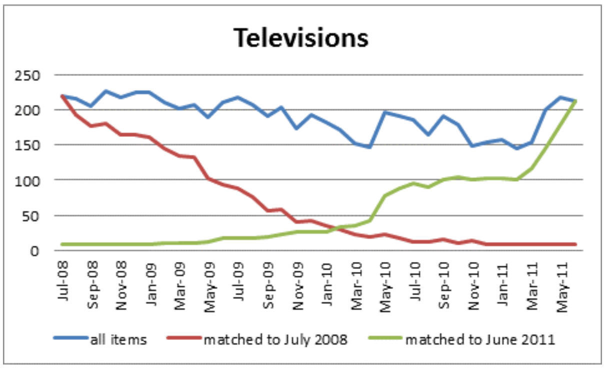
Figure 2 shows, for each of our eight products, the number of distinct items available over the three year period, alongside the percentage of items matched to those available at the start and end of the three-year period. For each product, ‘all items’ shows the number of distinct items being sold in each month. For example, in July 2008, there are over 200 different specifications of televisions being sold, while there are only around 70 different specifications of desktop computers being sold.

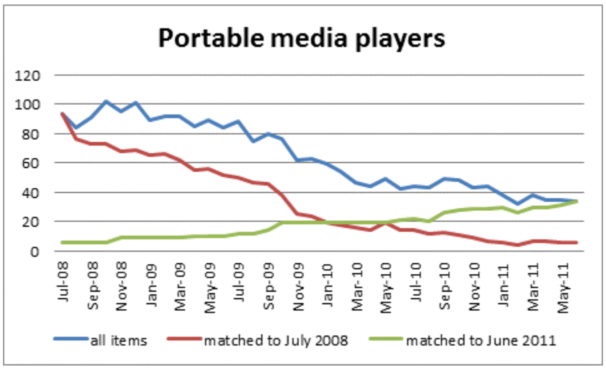
Figure 2. Total and matched items mid-2008 to mid-2011









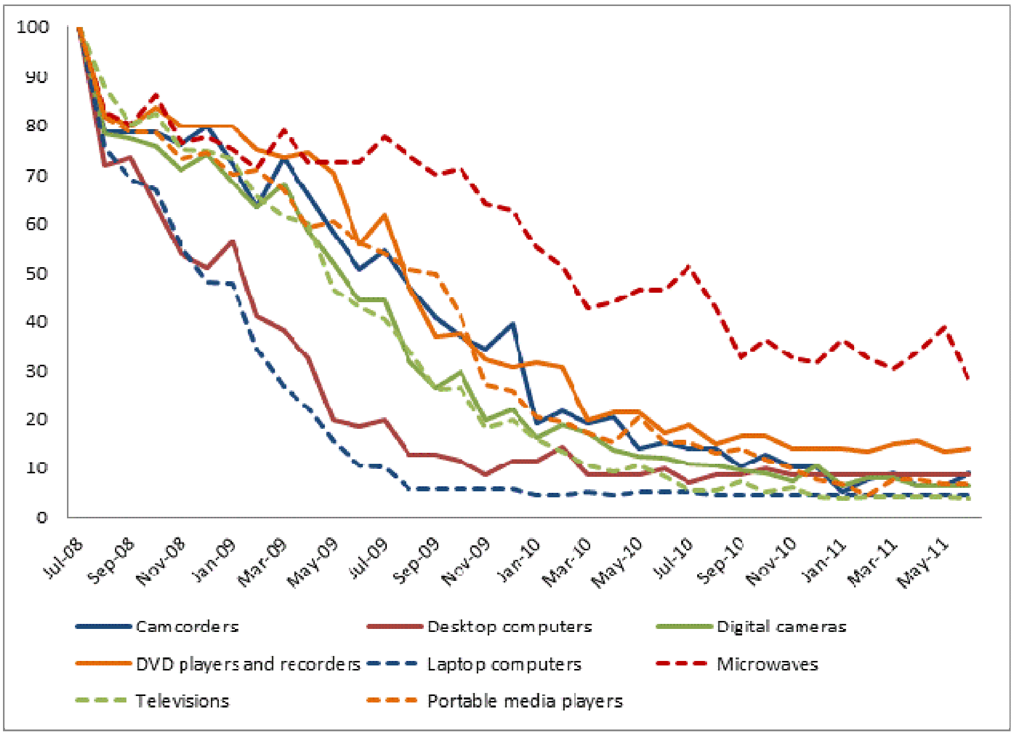


For most of the products, the number of items being sold is gradually decreasing over the three year period. It is not clear whether this is a real-world effect or whether it is a consequence of the tradebrand aggregation. Perhaps concentration of particular brands or models being sold by a particular retailer is increasing over time. While this requires further investigation, for the main purpose of this paper – comparing different methodologies on the same set of scanner data – it seems unlikely to be an issue.

'Matched to July 2008' shows, for each product, the number of distinct items sold in each month that were also being sold at the start of the three year period, and similarly 'Matched to June 2011' shows the number of distinct items sold in each month that are also being sold in the final month of the three year period. For high technology products, such as desktop and laptop computers, the rates of new and disappearing items are very high while for low-technology products such as microwaves, the churn is far less.

Figure 3 allows us to more easily compare attrition rates across different products by showing the percentage of July 2008 items still being sold for all products on one graph. This emphasises that computers – both desktops and laptops – have significantly higher attrition rates than the other products.

Figure 3. Percentage of July 2008 items available



8. Empirical evidence

Sections 3-5 provide the theoretical basis for Imputation Törnqvist RYGEKS indexes based on three different imputation approaches – the explicit hedonic, the weighted time dummy hedonic and the weighted time-product dummy approach.

Explicit hedonic imputation cannot be applied in the case of scanner data with predominantly categorical characteristics,[22] which can have new categories appearing or disappearing. This is because no prediction can be made for the new category of a categorical variable using only data from the period in which it does not exist.[23] However, the time dummy hedonic method bases the prediction on the main effects estimated for all the characteristics in the pooled data from both periods relevant to the bilateral index being estimated.

[22] Such as this GfK consumer electronics scanner data where, in fact, we treat even the few numeric characteristics as categorical – see Appendix 4 for an explanation of the approach taken to the regression modeling, and summaries of the adjusted R-squares.

[23] Unlike new or disappearing items defined in terms of either new values of numeric variables, or new combinations of existing categories of categorical variables (or a combination of both).

Section 5 explains why implicit imputation via the weighted time-product dummy method is very similar to not imputing at all for new and disappearing items and is therefore very close to the RYGEKS index. Any difference between the two reflects the changing expenditure share of matched items, which is reflected in the weights used for the time-product dummy regression modelling but not the RYGEKS.

We focus, therefore, on the weighted time dummy hedonic method, which we will refer to as the ITRYGEKS(TD). We produced ITRYGEKS(TD) indexes for each of the eight consumer electronics products, using a 13-month rolling window. These are compared below, in figures 5 to 12, to indexes estimated by a range of other methods:

- RYGEKS - a rolling year GEKS index based on bilateral matched-item Törnqvist indexes, with a 13-month rolling window;
- RYTD - a pooled time dummy hedonic index with monthly expenditure share weights and a 13-month rolling window;
- ITRYGEKS(TPD) - the ITRYGEKS with implicit imputation based on the weighted time-product dummy method;
- RYTPD - a pooled time-product dummy index with monthly expenditure share weights and a 13-month rolling window;
- The monthly chained Törnqvist.

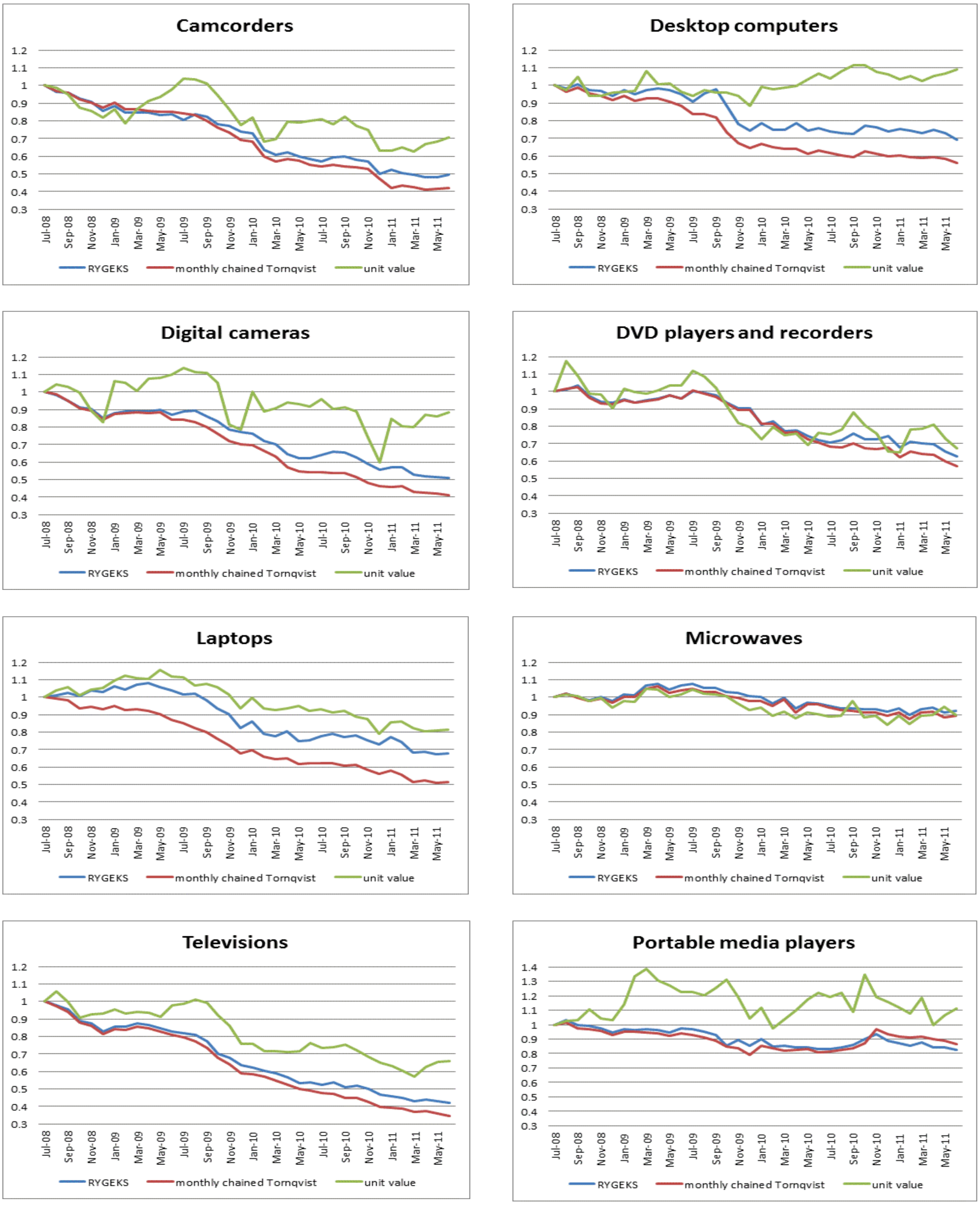
We also include a unit value index, calculated as the total expenditure divided by the total quantity sold, for each product. This gives us an index of the prices unadjusted for quality change which, in comparison to the quality-adjusted methods, enables us to see how quality is changing over time. It also highlights seasonal patterns in the average price.[24]

Chain drift in the monthly chained Törnqvist

Recent research on supermarket scanner data – Ivancic, Diewert and Fox (2011), de Haan and van der Grient (2011) – shows that high-frequency chained superlative indexes can have significant, and usually downward, chain drift. In figure 4 we test this for the case of consumer electronics data by comparing the monthly chained Törnqvist to the RYGEKS which is, by definition, free of chain drift.

[24] For example, note the strong seasonal dips in figure 4 in the average price for digital cameras corresponding to cheaper cameras being sold over the Christmas period. See Krsinich (2012) for more analysis of unadjusted prices and quantities in this data.

Figure 4. RYGEKS, chained Törnqvist and unit value indexes



Over the three year period the monthly-chained Törnqvist decreases faster than the RYGEKS for all products except portable media players. We would expect the RYGEKS and the monthly chained Törnqvist to suffer similarly from any bias due to neglecting the price movements of new and disappearing items – i.e. bias due to incorporating only matched items into the estimation – so it appears that the difference between the monthly chained Törnqvist and the RYGEKS is evidence of downwards chain drift in the monthly chained Törnqvist for most of these products.

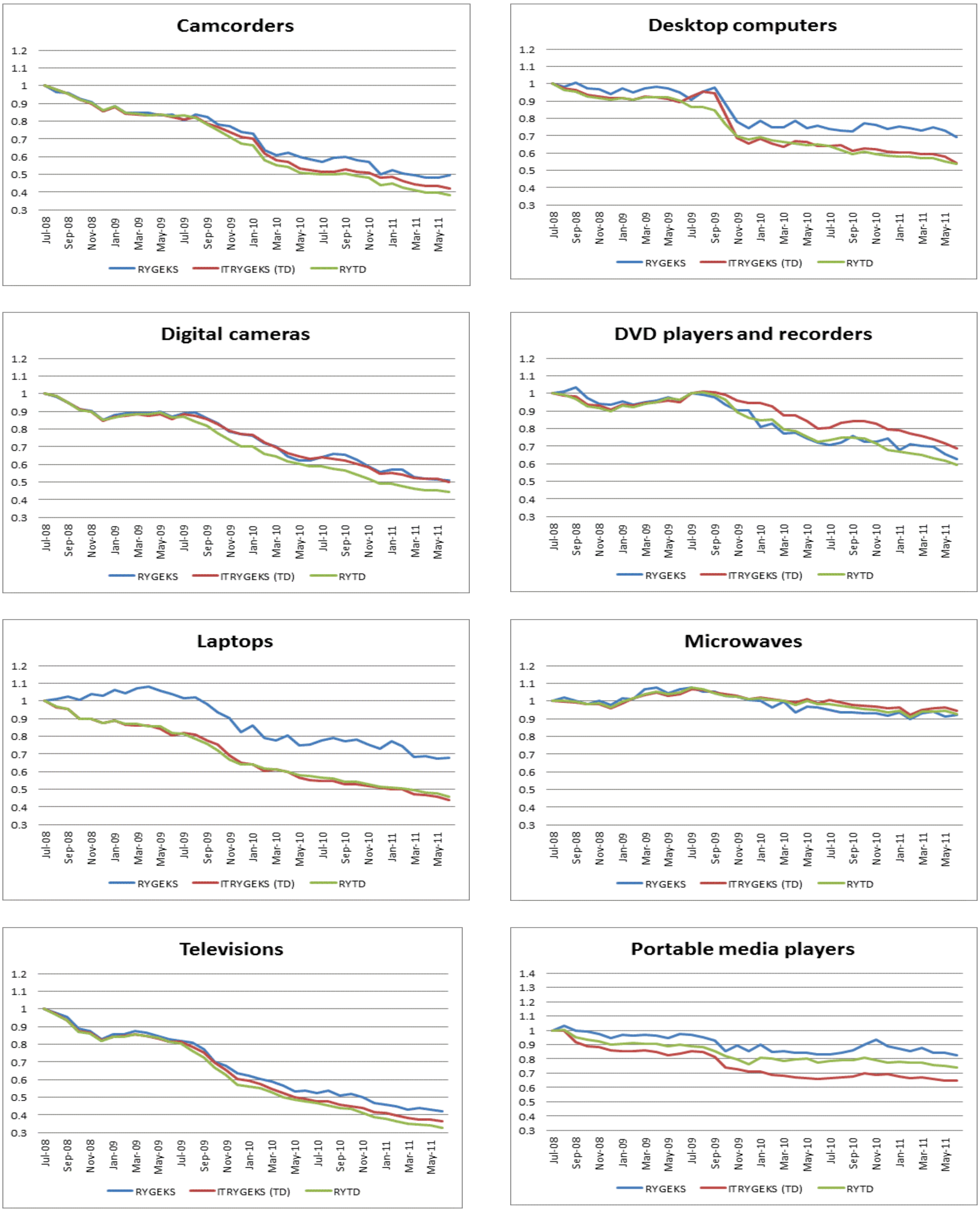
Given the seasonal pattern in total quantities shown by figure 1 and the seasonality of average prices at the product level, we would expect to see some chain drift. It is reasonable to expect that, at particular seasons (such as Christmas) or during sales, consumers are more likely to buy particular products, or particular specifications of given products. However, consumers do not tend to stockpile, say, televisions during sales in the way they stockpile bottles of soft drink or rolls of toilet paper. Therefore it seems reasonable that the chain drift we find for consumer electronics is less significant than the chain drift for supermarket products reported by others.

Quality change bias in the RYGEKS

The ITRYGEKS(TD) adjusts for quality change associated with new and disappearing items by imputing their price movements based on time dummy hedonic models. Figure 5 compares this benchmark index with the RYGEKS, to determine whether there is quality change bias in the RYGEKS for high-technology goods such as consumer electronics. We also include results for the easier-to-implement RYTD.

As shown in figure 5, there is a significant upward quality-change bias in the RYGEKS for computers (both desktops and laptops) and portable media players. To a lesser extent, there is upward quality-change bias for camcorders and televisions. For both microwaves and, surprisingly, digital cameras there is no evidence of quality-change bias in the RYGEKS. The results for DVD players and recorders are interesting – for this product the RYGEKS appears to be biased downwards, which would indicate a net quality *decrease* due to new and disappearing items. This requires further investigation.

Figure 5. RYGEKS, ITRYGEKS(TD) and RYTD



The easier-to-implement RYTD method gives very similar results to the ITRYGEKS(TD) for computers (both desktops and laptops). For the other products the results are mixed. RYTD gives similar results to ITRYGEKS(TD) for camcorders and televisions, and sits between the RYGEKS and ITRYGEKS(TD) for portable media players. For DVD players and recorders the RYTD actually appears to suffer from the same level of quality-change bias as the RYGEKS, though it is less volatile. For digital cameras, the RYTD sits below both the RYGEKS and the ITRYGEKS(TD), which perhaps suggests that the quality effect of characteristics is changing at a faster rate than the 13-month pooling window of the RYTD can reflect. Across all the products, though, the RYTD is closer to the ITRYGEKS(TD) than the RYGEKS is.

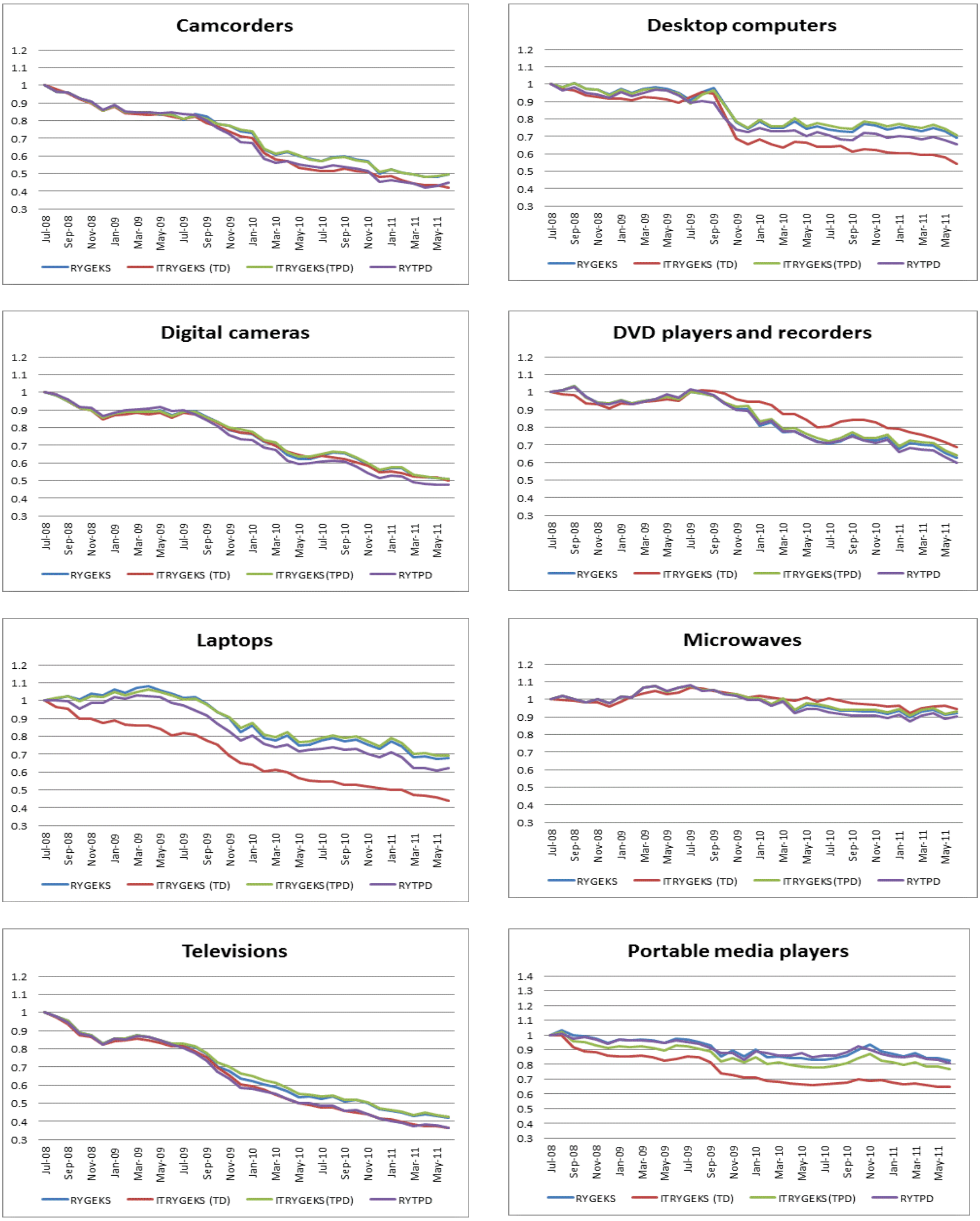
When there are no characteristics available

In the absence of any, or sufficient, information on quality characteristics in the data[25], it is not possible to apply either the ITRYGEKS(TD) or the RYTD. The RYGEKS, ITRYGEKS(TPD) and the RYTPD can all be applied in this situation by using the item identifier itself as the ‘characteristic’ being controlled for in the regression models. In section 5 we explained that the ITRYGEKS(TPD) effectively does no imputation for new and disappearing items and will therefore give virtually the same result as the RYGEKS - any differences will be due to changes in the expenditure share of the matched items over time. Figure 6 compares both the RYGEKS and the ITRYGEKS(TPD) to the benchmark ITRYGEKS(TD).

Silver and Heravi (2005) mention that the equivalence of the TPD method in the two-period case to a matched-item index does not carry over to the case where there are more than two periods, but “it can be seen that in the many-period case, the .... [TPD] measures of price change will have a tendency to follow the chained matched-model results.” In a preliminary version of their 2011 paper, Ivancic, Diewert and Fox (2009) compared matched-item Fisher-GEKS and expenditure-share weighted multilateral TPD indexes and found that these were very similar. So we would expect a rolling year version of expenditure-share weighted TPD – i.e. the RYTPD, which is also included in figure 6 – to also give similar results to the RYGEKS.

[25] This will generally be the case with supermarket scanner data, which may separately include one or two important characteristics, such as weight, but for which most descriptions of the item (i.e. the barcode) will be stored in free-text fields. While there may be potential for parsing characteristics from these fields, it is likely to be resource-intensive and product-specific.

Figure 6. RYGEKS, ITRYGEKS(TD), ITRYGEKS(TPD) and RYTPD

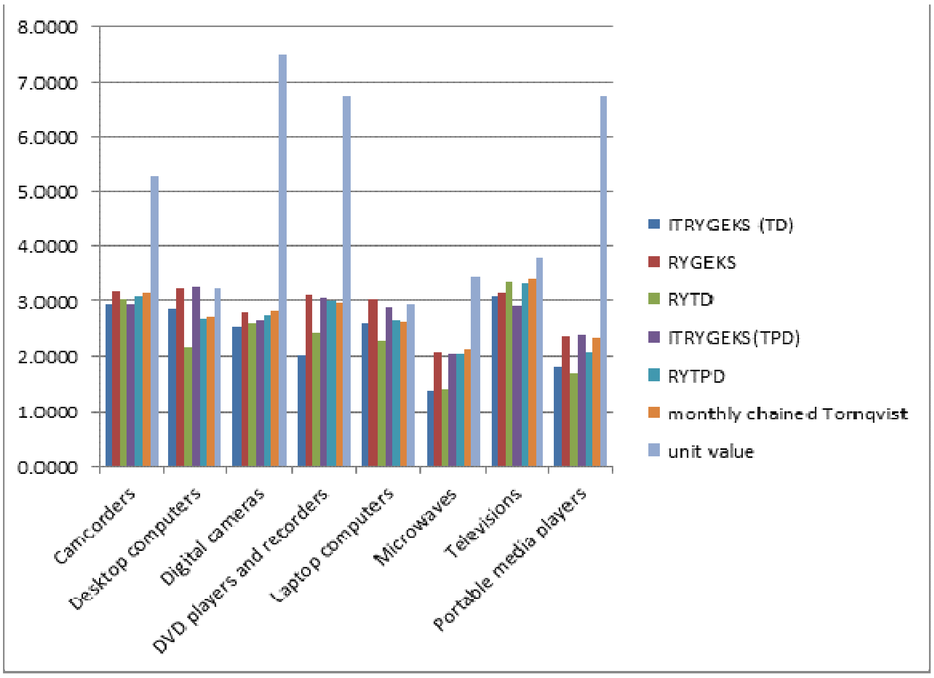


As expected, the RYGEKS and the ITRYGEKS(TPD) are very similar, with the exception of portable media players.[26] But, to our surprise, the RYTPD generally sits in between the RYGEKS and the ITRYGEKS(TD). This suggests that the RYTPD is less biased by the quality change due to new and disappearing items than the RYGEKS is. This calls for further research.

Volatility

For each product and method we calculated a volatility measure, which we define as the average of the absolute monthly percentage changes. This is shown below in figure 7. As we would expect, the index from the unit value is significantly more volatile than the indexes produced using the other methods, for many of the products. In particular this is the case for digital cameras and portable media players which, as shown in figures 1 and 4, have strong seasonality in their quantities sold and average prices.

Figure 7. Volatility of each method



[26] Portable media players is often an exception to the patterns we see between these methods. This requires further research but is likely to be a consequence of some very dominant items and/or sudden shifts in expenditure weights. Perhaps, for portable media players, we are approaching some of the extreme situations simulated by Ribe (2012) where the RYGEKS (and presumably, but perhaps to a lesser extent, the ITRYGEKS) start to exhibit perverse behavior.

The ITRYGEKS(TD) and RYTD tend to be the least volatile of the methods except for televisions – where the ITRYGEKS(TPD) is the least volatile and the RYGEKS' volatility lies between ITRYGEKS(TD) and RYTD; camcorders – where the ITRYGEKS(TPD)'s volatility lies between that of the ITRYGEKS(TD) and the RYTD; and desktop computers – where the volatility of the RYTPD and the monthly chained Törnqvist lie between that of the RYTD and the ITRYGEKS(TD).

Aggregation across products

The different methods for calculating price indexes from the scanner data can give quite different results at the product level. To see how these aggregate up the CPI hierarchy we created a quasi 'consumer electronics' aggregate of the eight products, using the monthly expenditures from the GfK data to weight together[27] the individual products' price indexes for each of the methods compared. This is shown below in figure 8.

Another important question is how a traditional CPI compares to our results from scanner data at the aggregate consumer electronics level. We therefore include in the comparison (using the same weighting as for the scanner data-derived indexes) the price changes for the corresponding products from the New Zealand Consumers Prices Index.[28]

Figure 8 shows that, at this aggregate level, the aggregation of existing New Zealand CPI product-level indexes gives a result that is very similar to the RYGEKS.[29] While these matched-item methods[30] do not adjust for the quality change associated with new and disappearing items, they do adjust for changes in quality mix of the matched items. This component of the quality change is shown in the difference between the unit value index and the RYGEKS, which indicates that, as we would expect, the quality mix of matched items is improving over time.

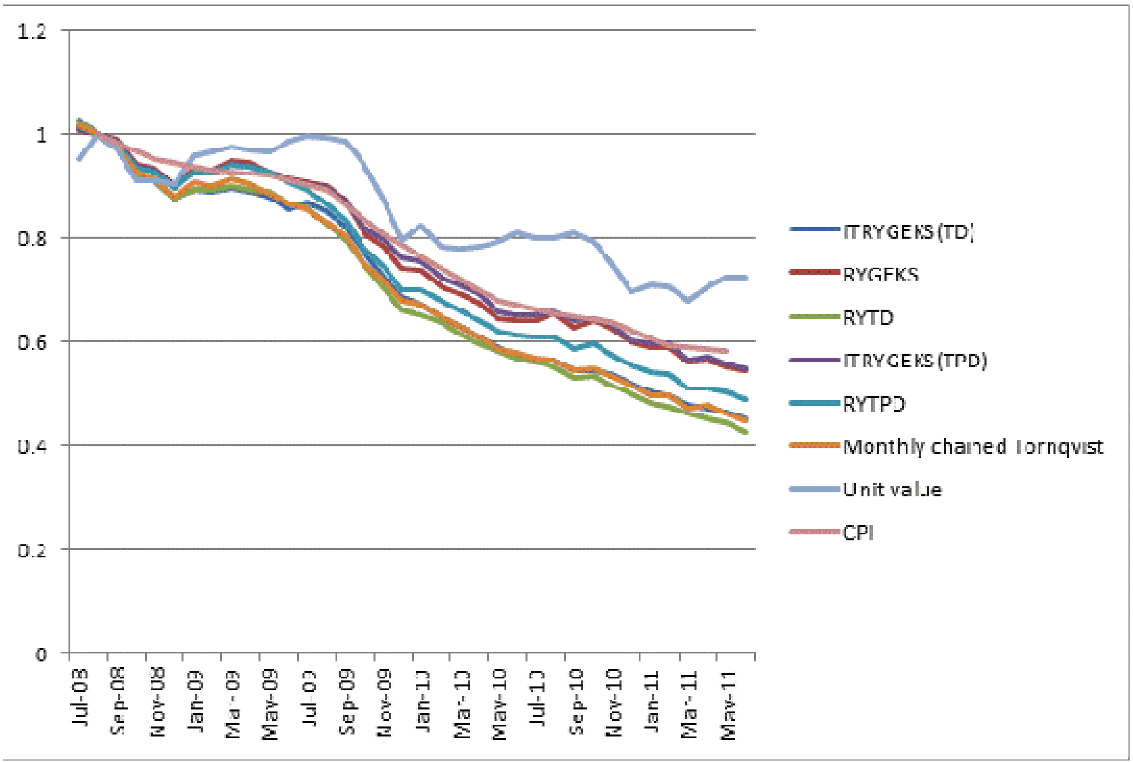
[27] Using upper level Törnqvist aggregation. Note that fixing the expenditure shares as at the start of the three-year period – i.e. a Laspeyres-type approach – made virtually no difference. So there is no evidence of substitution bias across these products.

[28] Note that the New Zealand CPI is quarterly. The indexes shown in figure 9 are rebased to August 2008, i.e. the middle of the third quarter.

[29] And also the ITRYGEKS(TPD) which, as we have shown earlier, is equivalent to the RYGEKS except for changes in the expenditure weight of the matched items.

[30] The existing New Zealand CPI is approximately a matched-item approach in terms of what operational practice is trying to achieve with replacement of items and manual quality adjustment.

Figure 8. Products aggregated to 'consumer electronics' for each method



The difference between the RYGEKS and the benchmark ITRYGEKS(TD) shows that, at this aggregate level, the introduction and disappearance of items are resulting in a further net quality improvement over time (and therefore a quality-adjusted price decrease).

Surprisingly, the monthly chained Törnqvist index is very close to our benchmark ITRYGEKS(TD) index. It appears that the mostly downward chain drift[31] is cancelling out against the upwards bias due to quality change of new and disappearing items. We have seen evidence of chain drift at the product level, and we know that the monthly chained Törnqvist will suffer from any quality change bias due to new and disappearing items, so this result at the aggregate level should be seen as coincidental. However it would be interesting to see whether these two biases cancel out so neatly for other product groups.

[31] Except for portable media players, for which the chain drift of the monthly chained Törnqvist is upwards.

Difference from the ITRYGEKS(TD) benchmark

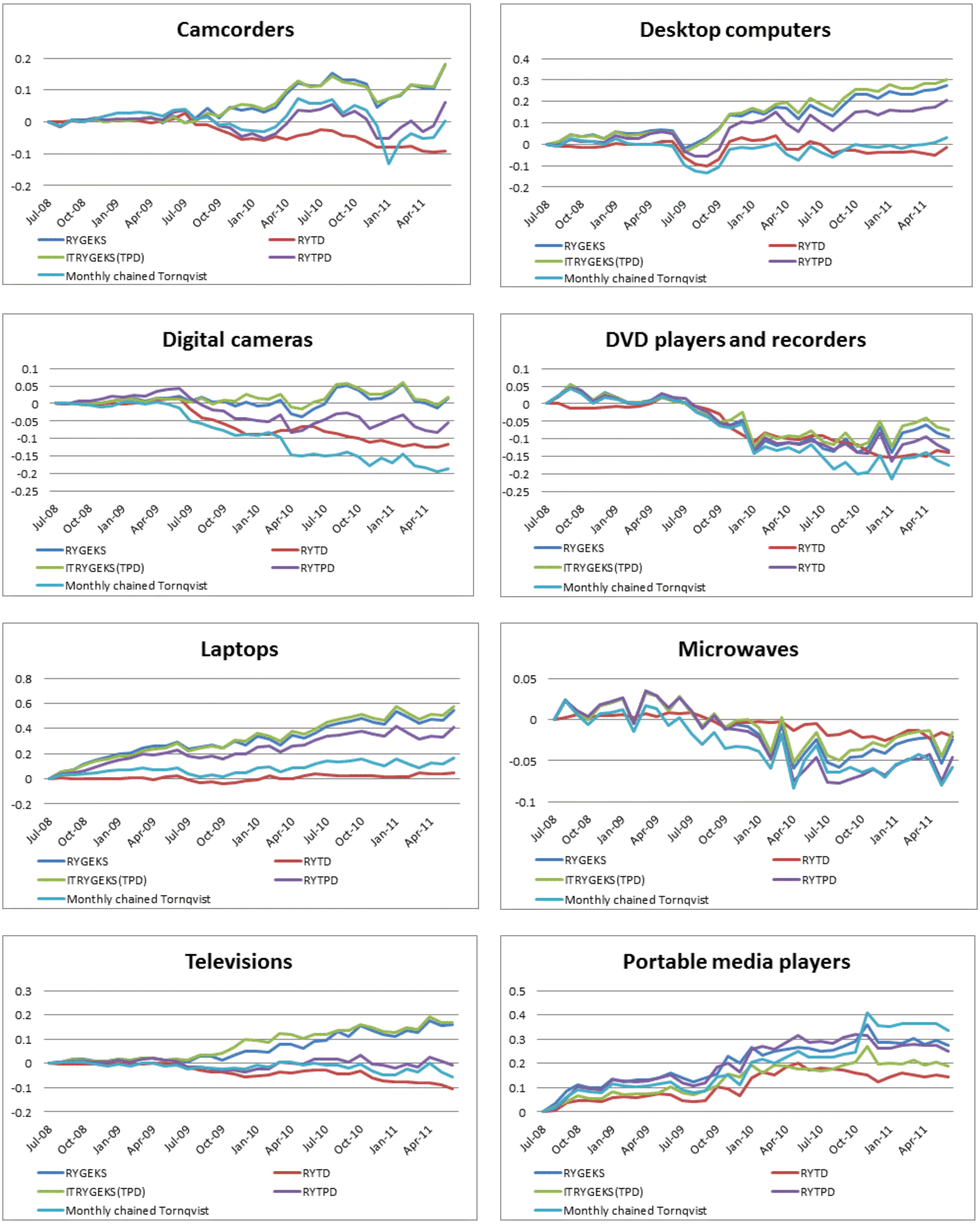
In the theoretical section we established why the ITRYGEKS(TD) can be considered a benchmark index against which other methods can be compared: easier-to-implement methods such as the RYTD; methods not requiring characteristics such as the RYGEKS or the RYTPD; or more familiar methods such as the monthly chained Törnqvist. In figure 9 we show the full set of index methods in terms of how they differ from the ITRYGEKS(TD).[32]

The results shown in figure 9 differ across products, though there are some general tendencies: the RYGEKS and ITRYGEKS(TPD) tend to sit furthest away from the ITRYGEKS(TD) in an upwards direction while the RYTD tends to sit near the bottom of the group though not necessarily the closest to the ITRYGEKS(TD). At the product level, the following observations can be made.

- At the end of the three year period, the RYTD is closest to the benchmark ITRYGEKS(TD) for computers (both desktops and laptops) and portable media players. For microwaves, it is closest over the 3 year period, though by the end the RYGEKS and ITRYGEKS(TPD) are equally close.
- The RYGEKS and the ITRYGEKS(TPD) are closest to the ITRYGEKS(TD) for digital cameras while the ITRYGEKS(TPD) is closest for DVD players and recorders.
- For camcorders, the chained Törnqvist is closest to the ITRYGEKS(TD) at the end of the period, though note that its difference is very volatile over the three years.
- For televisions (the most highly weighted of these products – refer to table 1) the RYTPD is the closest to the ITRYGEKS(TD) both during, and at the end, of the three year period.

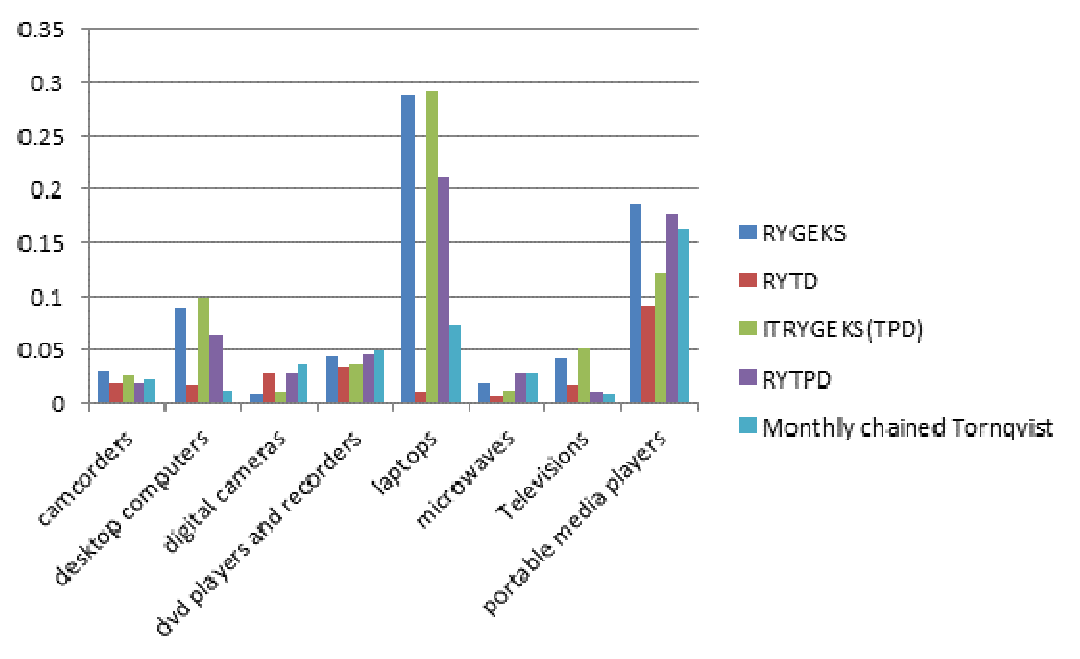
[32] We take the difference between each different index and the ITRYGEKS(TD), divided by the ITRYGEKS(TD) to standardize the comparison.

Figure 9. Comparison of all other methods to ITRYGEKS(TD)



We summarise the information in figure 9 to an 'average difference from ITRYGEKS(TD)' by taking the geometric mean of the absolute of the (standardised) differences between each index and the ITRYGEKS(TD). This is shown below in figure 10.

Figure 10. Average difference from ITRYGEKS(TD) for each method



Across the eight products, RYTD can arguably[33] be said to perform the best. As we might expect, the RYGEKS performs least well for the highest technology products, desktop and laptop computers, and for portable media players, which have volatile expenditure shares of matched items.[34]

The RYTPD performs better than the RYGEKS for all products except digital cameras, DVD players and microwaves. It performs particularly well for televisions. The monthly chained Törnqvist also performs well for televisions. Given that the difference between the RYGEKS and the ITRYGEKS(TD) for televisions indicates that there is quality-change bias due to new and disappearing items (which should exist similarly for the matched-item chained monthly Törnqvist), this suggests that chain drift and new goods bias are cancelling out to a certain extent at this product level in the

[33] If we consider the products to be of equal weight in this assessment.

[34] As shown in figure 7 by the large difference between RYGEKS and ITRYGEKS(TPD).

monthly chained Törnqvist, and given the very high expenditure weight of televisions this is likely to be driving the cancelling out at the aggregate level shown earlier in figure 8.

9. Conclusions

The Imputation Törnqvist (IT) RYGEKS method explicitly or implicitly imputes price movements for new and disappearing items based on regression models. The paper outlines three variants of the ITRYGEKS: explicit hedonic imputation, and implicit imputation via either a weighted time dummy hedonic method or a weighted time-product dummy method. We explain why, for the mainly categorical characteristics in the consumer electronics scanner data, the use of the time dummy hedonic method – the ITRYGEKS(TD) – is the appropriate method to estimate fully quality-adjusted price indexes.

We confirm that the monthly chained Törnqvist is not a viable method for consumer electronics, as it has downward chain-drift for most of the products examined, with the exception of microwaves and portable media players.

The RYGEKS shows evidence of quality-change bias when compared to the benchmark ITRYGEKS(TD), particularly for computers.

The easier-to-implement RYTD gives similar results to the ITRYGEKS(TD), in particular for computers. Portable media players are an exception to this, presumably because the 13 month windows of the RYTD smooth out the effect of volatility in the expenditure shares of matched items for this product.[35]

In some cases, such as supermarket data, there will be few or no characteristics available and so neither the ITRYGEKS(TD) nor the RYTD, which are both based on time dummy hedonic models, will be feasible. Our results suggest that in this situation the RYTPD does some adjustment for quality change and is therefore preferable to the RYGEKS. Further empirical and theoretical work is required to fully understand this.

Aggregation of the eight products using their relative expenditure weights shows that the current New Zealand Consumers Price Index gives results that are very similar

[35] Though, arguably, this might be seen as a desirable characteristic of the RYTD in the case of volatile expenditure shares.

to the matched-item RYGEKS. The RYTD and the monthly chained Törnqvist both track the benchmark ITRYGEKS(TD) closely but in the case of the monthly chained Törnqvist this appears to be a coincidental cancelling out of biases in two opposite directions – chain drift and quality-change bias – for televisions, which have by far the most significant weight of all the eight products.

Further work is required in two areas. First, the ITRYGEKS approach with explicit imputation should be empirically tested with scanner data that has numeric characteristics and/or categorical characteristics where no new categories appear or disappear over the period investigated. Second, empirical and theoretical investigation into the differences between the ITRYGEKS(TD) and the RYTPD can clarify whether the latter is an effective method in situations where there is likely to be quality change due to new and disappearing items but where no (or little) information on characteristics is present in the data.

Appendix 1: Derivation of decomposition (10)

In this appendix we derive decomposition (10) of the single imputation Törnqvist index (9). For convenience we write the index as

$$P_{SIT}^{0t} = \left[\prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^0} \prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^t} \prod_{i \in U_{D(t)}^0} \left(\frac{\hat{p}_i^t}{p_i^0} \right)^{s_i^0} \prod_{i \in U_{N(0)}^t} \left(\frac{p_i^t}{\hat{p}_i^0} \right)^{s_i^t} \right]^{\frac{1}{2}}, \tag{A.1}$$

where $\hat{p}_i^0$ and $\hat{p}_i^t$ are the imputed prices in periods 0 and *t*, and s_i^0 and s_i^t are the expenditure shares (with respect to the total set of items in periods 0 and *t*). As in the main text, we will denote the expenditure shares with respect to the set U^{0t} of matched items in periods 0 and *t* by $s_{i(0t)}^0$ and $s_{i(0t)}^t$; $s_{iD(t)}^0$ and $s_{iN(0)}^t$ are the expenditure shares of *i* with respect to the sets $U_{D(t)}^0$ and $U_{N(0)}^t$ of disappearing and new items; $s_{D(t)}^0$ and $s_{N(0)}^t$ are the period 0 and *t* aggregate expenditure shares of the disappearing and new items; s_M^0 and s_M^t are the period 0 and period *t* aggregate expenditure shares of the matched items.

Since $s_i^0 = s_{i(0t)}^0 s_M^0$ and $s_i^t = s_{i(0t)}^t s_M^t$ for $i \in U^{0t}$, $s_i^0 = s_{iD(t)}^0 s_{D(t)}^0$ for $i \in U_{D(t)}^0$ and $s_i^t = s_{iN(0)}^t s_{N(0)}^t$ for $i \in U_{N(0)}^t$, equation (A.1) can be written as

$$P_{SIT}^{0t} = \left[\prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_{i(0t)}^0 s_M^0} \prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_{i(0t)}^t s_M^t} \prod_{i \in U_{D(t)}^0} \left(\frac{\hat{p}_i^t}{p_i^0} \right)^{s_{iD(t)}^0 s_{D(t)}^0} \prod_{i \in U_{N(0)}^t} \left(\frac{p_i^t}{\hat{p}_i^0} \right)^{s_{iN(0)}^t s_{N(0)}^t} \right]^{\frac{1}{2}}$$

$$= \left[\prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_{i(0t)}^0 (1 - s_{D(t)}^0)} \prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_{i(0t)}^t (1 - s_{N(0)}^t)} \prod_{i \in U_{D(t)}^0} \left(\frac{\hat{p}_i^t}{p_i^0} \right)^{s_{iD(t)}^0 s_{D(t)}^0} \prod_{i \in U_{N(0)}^t} \left(\frac{p_i^t}{\hat{p}_i^0} \right)^{s_{iN(0)}^t s_{N(0)}^t} \right]^{\frac{1}{2}}, \quad \text{(A.2)}$$

using $s_M^0 = 1 - s_D^0$ and $s_M^t = 1 - s_N^t$. Rearranging terms gives

$$P_{SIT}^{0t} = \left[\prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_{i(0t)}^0} \prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_{i(0t)}^t} \right]^{\frac{1}{2}} \left[\frac{\prod_{i \in U_{D(t)}^0} \left(\frac{\hat{p}_i^t}{p_i^0} \right)^{s_{iD(t)}^0 s_{D(t)}^0}}{\prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_{i(0t)}^0 s_{D(t)}^0}} \right]^{\frac{1}{2}} \left[\frac{\prod_{i \in U_{N(0)}^t} \left(\frac{p_i^t}{\hat{p}_i^0} \right)^{s_{iN(0)}^t s_{N(0)}^t}}{\prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_{i(0t)}^t s_{N(0)}^t}} \right]^{\frac{1}{2}}$$

$$= \prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{\frac{s_{i(0t)}^0 + s_{i(0t)}^t}{2}} \left[\frac{\prod_{i \in U_{D(t)}^0} \left(\frac{\hat{p}_i^t}{p_i^0} \right)^{s_{iD(t)}^0}}{\prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_{i(0t)}^0}} \right]^{\frac{s_{D(t)}^0}{2}} \left[\frac{\prod_{i \in U_{N(0)}^t} \left(\frac{p_i^t}{\hat{p}_i^0} \right)^{s_{iN(0)}^t}}{\prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{s_{i(0t)}^t}} \right]^{\frac{s_{N(0)}^t}{2}}, \quad \text{(A.3)}$$

which is equation (18) in the main text.

Appendix 2: Derivation of equation (14)

Following de Haan (2004), in this appendix we derive expression (14) for the bilateral time dummy hedonic index. Because an intercept term is included in model (13), the weighted sum of the regression residuals $e_i^t = \ln(p_i^t) - \ln(\hat{p}_i^t) = \ln(p_i^t / \hat{p}_i^t)$ is equal to 0 in each period, hence

$$\sum_{i \in U^t} w_i^t \ln(p_i^t / \hat{p}_i^t) = \ln \left[\prod_{i \in U^t} \left(\frac{p_i^t}{\hat{p}_i^t} \right)^{w_i^t} \right] = 0; \qquad t = 0, \ldots, T, \quad \text{(A.4)}$$

where w_i^t denotes the weight for item i in period t in a WLS regression. If we separate the base period 0 from the comparison periods t, the second expression of (A.4) can be written as

$$\ln\left[\prod_{i\in U^0}\left(\frac{p_i^0}{\hat{p}_i^0}\right)^{w_i^0}\right] = \ln\left[\prod_{i\in U^t}\left(\frac{p_i^t}{\hat{p}_i^t}\right)^{w_i^t}\right] = 0\,; \qquad t = 1,\ldots,T\,. \tag{A.5}$$

Exponentiating (A.5) yields the following relation:

$$\prod_{i\in U^0}\left(\frac{p_i^0}{\hat{p}_i^0}\right)^{w_i^0} = \prod_{i\in U^t}\left(\frac{p_i^t}{\hat{p}_i^t}\right)^{w_i^t} = 1; \qquad t = 1,\ldots,T\,. \tag{A.6}$$

Next, we rewrite (A.6) as

$$\prod_{i\in U^{0t}}\left(\frac{p_i^0}{\hat{p}_i^0}\right)^{w_i^0}\prod_{i\in U^0_{D(t)}}\left(\frac{p_i^0}{\hat{p}_i^0}\right)^{w_i^0} = \prod_{i\in U^{0t}}\left(\frac{p_i^t}{\hat{p}_i^t}\right)^{w_i^t}\prod_{i\in U^t_{N(0)}}\left(\frac{p_i^t}{\hat{p}_i^t}\right)^{w_i^t} (=1)\,, \tag{A.7}$$

where, as before, U^{0t} denotes the set of matched items (with respect to periods 0 and *t*), $U^0_{D(t)}$ is the set of disappearing items, and $U^t_{N(0)}$ is the set of new items.

We now assume that the regression weights for the matched items are the same in periods 0 and *t*, i.e., $w_i^0 = w_i^t = w_i^{0t}$ for $i \in U^{0t}$. In that case, multiplying the right and left hand side of equation (A.7) by $\prod_{i\in U^{0t}}(\hat{p}_i^t / p_i^0)^{w_i^{0t}}$ gives

$$\prod_{i\in U^{0t}}\left(\frac{\hat{p}_i^t}{\hat{p}_i^0}\right)^{w_i^{0t}}\prod_{i\in U^0_{D(t)}}\left(\frac{p_i^0}{\hat{p}_i^0}\right)^{w_i^0} = \prod_{i\in U^{0t}}\left(\frac{p_i^t}{p_i^0}\right)^{w_i^{0t}}\prod_{i\in U^t_{N(0)}}\left(\frac{p_i^t}{\hat{p}_i^t}\right)^{w_i^t}\,. \tag{A.8}$$

Multiplying both sides of (A.8) by $\prod_{i\in U^0_{D(t)}}(\hat{p}_i^t / p_i^0)^{w_i^0}$ yields

$$\prod_{i\in U^{0t}}\left(\frac{\hat{p}_i^t}{\hat{p}_i^0}\right)^{w_i^{0t}}\prod_{i\in U^0_{D(t)}}\left(\frac{\hat{p}_i^t}{\hat{p}_i^0}\right)^{w_i^0} = \prod_{i\in U^{0t}}\left(\frac{p_i^t}{p_i^0}\right)^{w_i^{0t}}\prod_{i\in U^0_{D(t)}}\left(\frac{\hat{p}_i^t}{p_i^0}\right)^{w_i^0}\prod_{i\in U^t_{N(0)}}\left(\frac{p_i^t}{\hat{p}_i^t}\right)^{w_i^t}\,. \tag{A.9}$$

Next, multiplying both sides of (A.9) by $\prod_{i\in U^t_{N(0)}}(\hat{p}_i^t / \hat{p}_i^0)^{w_i^t}$ gives

$$\prod_{i\in U^{0t}}\left(\frac{\hat{p}_i^t}{\hat{p}_i^0}\right)^{w_i^{0t}}\prod_{i\in U^0_{D(t)}}\left(\frac{\hat{p}_i^t}{\hat{p}_i^0}\right)^{w_i^0}\prod_{i\in U^t_{N(0)}}\left(\frac{\hat{p}_i^t}{\hat{p}_i^0}\right)^{w_i^t} = \prod_{i\in U^{0t}}\left(\frac{p_i^t}{p_i^0}\right)^{w_i^{0t}}\prod_{i\in U^0_{D(t)}}\left(\frac{\hat{p}_i^t}{p_i^0}\right)^{w_i^0}\prod_{i\in U^t_{N(0)}}\left(\frac{p_i^t}{\hat{p}_i^0}\right)^{w_i^t}\,. \tag{A.10}$$

Using $\hat{p}_i^t / \hat{p}_i^0 = \exp\hat{\delta}^t$ for all *i*, equation (A.10) can be written as

$$\left[\exp\hat{\delta}^t\right]^{w^{0t}+w^0_{D(t)}+w^t_{N(0)}} = \prod_{i\in U^{0t}}\left(\frac{p_i^t}{p_i^0}\right)^{w_i^{0t}}\prod_{i\in U^0_{D(t)}}\left(\frac{\hat{p}_i^t}{p_i^0}\right)^{w_i^0}\prod_{i\in U^t_{N(0)}}\left(\frac{p_i^t}{\hat{p}_i^0}\right)^{w_i^t}\,, \tag{A.11}$$

where $w^{0t} = \sum_{i\in U^{0t}} w_i^{0t}$, $w^0_{D(t)} = \sum_{i\in U^0_{D(t)}} w_i^0$ and $w^t_{N(0)} = \sum_{i\in U^t_{N(0)}} w_i^t$. It follows that

$$\exp\hat{\delta}^t = P_{TD}^{0t} = \left[\prod_{i \in U^{0t}} \left(\frac{p_i^t}{p_i^0} \right)^{w_i^{0t}} \prod_{i \in U_{D(t)}^0} \left(\frac{\hat{p}_i^t}{p_i^0} \right)^{w_i^0} \prod_{i \in U_{N(0)}^t} \left(\frac{p_i^t}{\hat{p}_i^0} \right)^{w_i^t} \right]^{\frac{1}{w^{0t}+w_{D(t)}^0+w_{N(0)}^t}}, \qquad (A.12)$$

which is equation (14) in the main text.

Appendix 3: Characteristics available for each product

Camcorders		**Desktop computers**		**Digital Cameras**		**DVD players and recorders**	
Characteristic	Levels	Characteristic	Levels	Characteristic	Levels	Characteristic	Levels
16:9 REC#FORMAT	2	3D	3	ANZ CAMERA TYPE	3	1080P UPSCALING	3
3 CHIP	2	APPLE OS	3	AUTOFOCUS	2	3D CONVERSION	2
BRAND	6	ASPECT RATIO	3	AVCHD / LITE	4	APP STORE(IPTV)	3
CF CARD	3	BLUETOOTH	2	BATTERY TYPE	3	BD LIVE	2
CHIPTYPE	2	BRAND	9	BAYONET MOUNT	7	BRAND	13
DIGITAL INDEX	2	BRIGHTNESS	2	BLUETOOTH	3	CF CARD	3
DIGITAL INPUT	2	CHIPSET BRAND	5	BRAND	15	COMMON INTER CI	3
DSC CAMC RES#	3	CHIPSET TYPE	43	BRIDGE CAMERA	2	CONSUMP STANDBY	29
HANDHELD POSIT#	2	CSP-MHZ	27	CHIPTYPE	5	CONSUMPTION W	43
HD Formats	5	DEPTH IN CM	6	COMPACT FLASH	2	DIVX	2
HDD CAPACITY	8	DESIGN	4	DEPTH IN MM	182	DLNA	2
HDMI	3	DVD WRITER	4	DIGITAL ZOOM	12	DTS DECODER	4
IEEE-1394	2	GPU BRAND	5	DISPL#RES# PIX#	30	DVB-T	3
IMAGE STABIL#	3	GPU MODEL	66	DSC CAMC RES#	4	DVD AUDIO	3
IMAGE STABILIZ#	4	HEIGHT IN CM	6	DUAL CAM	4	DVD Categories	10
LCD SCREEN SIZE	8	LED	3	EFFECTIVE PIXEL	50	DVD REC# FORMAT	9
MEMORY CAPACITY	15	MC-EDITION	2	FACE DETECTION	4	EPG	3
MEMORY CARD	2	MEMORY TECHNO	3	FOLDABLE LCD	3	GOOGLE TV	2
MEMORY STICK	3	MICROSOFT OS	10	FRAMES PER SEC	12	HDD CAPACITY	10
MODEL	78	MODEL	82	GPS	2	HDMI	2
MPEG STANDARD	3	NUMBER OF CORES	5	HDMI PORT	2	HDTV	3
MULTIMEDIA CARD	3	OFFICE SUITE	2	HEIGHT IN MM	144	LAYER	3
OPTICAL ZOOM	21	ONBOARD GRAPHIC	3	IMAGE STABIL#	3	LCD SCREEN SIZE	10
OUTDOOR FUNCTIO	3	PLATFORM	2	INNER ZOOM	4	MEDIAPLAYER	4
P2	2	PROCESSOR	23	KID CAMERA	3	MEMORY CARD	2
PIXEL TOTAL	38	PROCESSOR NUMBE	91	LCD MONITOR SIZ	15	MEMORY STICK	3
RESOLUTION	3	RAM-MB	9	LIVE VIEW	2	MODEL	105
SD CARD	3	REMOTE-CONTROL	2	MAX VIDEO HT RE	10	MP3 ENCODING	2
SD Format	7	SCREEN SIZE	2	MAX VIDEO WI RE	12	MP3 PLAYBACK	3
SD/HD Format	10	SECURITY SW	2	MAX# ISO SENSIT	15	MPEG-4 RECORD#	2
SMART MEDIA	2	SIZE/INCH	9	MEMORY TYPE	4	NO# OF DISC	3
TOUCHSCREEN	3	STORAGE CAPAC#	13	MICRO DRIVE	2	PC STREAMING	3
TYPE	2	TV OUT	2	MODEL	211	REAL DOLBY DEC#	6
Type Memory Cards	7	VIDEOCARD	2	Multiple Slots	3	SD CARD	3
USB CONNECTION	2	Vision	3	OPTICAL ZOOM	16	SUPER AUDIO CD	3
VIDEO FORMAT	8	V-RAM	12	OUTDOOR FUNCTIO	5	USB RECORDING	3
VIDEOTO MC	3	WEIGHT IN KG	5	PHOTOS PER SEC#	55	WEB CONTENT ACC	3
VIEWFINDER	3			PICTBRIDGE	2	WIRELESS INTERF	3
WEIGHT IN GRAM	62						
WIRELESS INTERF	2						
XD-PICTURE CARD	3						

Laptop computers		Microwaves		Televisions		Portable media players	
Characteristic	Levels	Characteristic	Levels	Characteristic	Levels	Characteristic	Levels
3G/BB BUNDLE	2	ARTICLE FLAG	3	3D	2	AIRPLAY	2
APPLE OS	4	BRAND	14	3D CONVERSION	2	BLUETOOTH	3
ASPECT RATIO	4	CAVITY MATERIAL	6	BRAND	14	BRAND	13
BLUETOOTH	3	CONSTR#2	2	BRIGHTNESS	20	BUILT IN CAMERA	3
BRAND	13	CONSTRUCTION	3	CONSUMP STANDBY	28	CE FUSION REPORTIN	5
BRIGHTNESS	3	COOKING PLATE	4	CONSUMPTION W	128	CF CARD	3
CE FUSION REPORTII	2	COOKINGFUNCTION	4	CONTRAST RATIO	50	COLOUR DISPLAY	2
CENTRINO	2	DEPTH IN CM	34	DISPLAY HD DC	3	DAB	2
CHIPSET BRAND	5	DETAILED MAIN TYPES	7	DIVX	3	Dictation Machine total	2
CHIPSET TYPE	52	DIGITAL CLOCK	4	DLNA	3	DIGITAL SPEECH	2
DVD WRITER	4	DOOR OPENING S#	4	DVB-C	3	DIGITAL SYSTEM	2
FINGERPRINT-ID	2	DOOR TYPE	3	DVB-S	2	DISPLAY-TECHNO	6
FORM FACTOR	7	FRONT DECORAT#	8	DVB-T	3	DIVX	3
GPU BRAND	3	GROUPS AFFILIATION	7	DVD PLAYER/REC#	2	DRM	5
GPU MODEL	88	HEIGHT	40	ENERGY STAR AU#	11	GAMING	3
HARD DRIVE TYPE	2	HUMIDITY SENSOR	3	FRAME REFRESH	9	H264 PLAYBACK	3
HEIGHT IN CM	116	INFRARED SENSOR	3	FREQUENCY	4	MEMORY CARD	3
HYPER THREADING	2	INVERTER	3	HD READY	3	MEMORY STICK	2
LED	2	KEYFEATURES	3	HD Types	3	MODEL	75
LED TECHNOLOGY	2	LIMIT_HOURS	3	HDD CAPACITY	3	MODELTEXT	209
MC-EDITION	2	LIMIT_MINUTES	5	ISDB-T	3	MP3 ENCODING	3
MEMORY TECHNO	3	MAIN TYPES - IL-	4	LED TECHNOLOGY	4	MPEG 2	3
MICROSOFT OS	15	MODEL	88	MEDIAPLAYER	4	MPEG 4	3
MODEL	278	MODELTEXT	187	MODEL	245	MULTIMEDIA CARD	3
Next Gen Core	25	NUMBER OF TRAYS	3	NO OF HDMI	6	OPERATING SYST#	4
OFFICE SUITE	5	POWERSTEPS	10	PC STREAMING	4	PC CONNECTION	3
ONBOARD GRAPHIC	3	PROGRAMMER	4	REFLECTION ANGL	12	PHOTO VIEWER	2
PROCESSOR NUMBE	124	QUARTZ GRILL	4	RESOLUTION HIG#	9	RADIO BUILT-IN	2
REMOTE-CONTROL	2	SKEWER	3	RESOLUTION WID#	11	SD CARD	3
SCREEN SIZE	5	STEAM FUNCT#INT	4	RESPONSE TIME	18	SIZE/INCH	21
TRUE RESOLUTION	11	THERMO SENSOR	4	SCREEN FORMAT	3	STOR# CAPACITY	20
TV CARD	3	TIME CONTROL	3	SD CARD	3	TOUCHSCREEN	3
TV OUT	2	TOUCH CONTROL	3	SIZE/CM	38	TV OUT	3
USB PORTS	5	TURNTABLE	3	SIZE/INCH	26	VIDEORECORD	3
V-RAM	7	TYPE OF MICROW#	6	TELE-TEXT	3	WEIGHT IN GRAM	70
WIMAX	2	VOLUME LTRS	20	USB	3	XHTML BROWSER	3
WIRELESS LAN	2	WATTAGE	11	USB RECORDING	3		
		WEIGHT SENSOR	3	WIFI	4		
		WIDTH IN CM	42				
		WITH FAN/CONVEC	2				
		WITH GRILL	2				
		WITH TOP/BOTTOM	2				
		YEAR OF INTRO	6				

Appendix 4: Regression modelling

The approach taken to the modelling of time dummy hedonic models – on both the bilateral pooled data for the ITRYGEKS(TD) and the rolling 13-month windows of pooled data for the RYTD – was to include all the characteristics we had available in the data. We are ultimately interested in methods that can be incorporated into production with as little manual intervention as possible.[36]

[36] This goes in particular for Statistics New Zealand, where the plan is to put scanner data into production for the Consumers Price Index for consumer electronics, and then supermarkets, in the near future.

All the characteristics, including the few numeric characteristics were modelled as categorical, so we are not imposing any parametric form on the numeric characteristics. Given the quantity of data, we do not need to worry about degrees of freedom.

Table 2 below summarises the adjusted R-squares for the regression models.[37] Note that, because we are using monthly averages of prices (unit values), the R-squares are significantly higher than if we were applying the same models to the underlying prices. What this means is that it is not valid to compare these fit estimates to models on actual prices or higher-frequency averages, for example weekly averages.

Table 2. Average[38] adjusted R-squares

	ITRYGEKS(TD)	RYTD	RYTPD
Camcorders	0.970	0.972	0.933
Desktop computers	0.979	0.983	0.969
Digital cameras	0.983	0.988	0.974
DVD players and recorders	0.973	0.977	0.980
Laptop computers	0.980	0.980	0.950
Microwaves	0.982	0.987	0.980
Televisions	0.988	0.989	0.979
Portable media players	0.988	0.991	0.990

[37] We do not include results for the ITRYGEKS(TPD) because the relative goodness of fit compared to the ITRYGEKS(TD) can be approximately inferred by comparing the RYTD and the RYTPD, and also because it is not a viable method for quality adjusting this data.

[38] The ITRYGEKS(TD) incorporates 630 regressions for each product, corresponding to all the distinct pairs of months within the three year period. The rolling year methods – both the RYTD and the RYTPD – are each based on 24 regressions for each product, corresponding to each of the 13-month rolling windows over the three year period.

References

Aizcorbe, A., C. Corrado and M. Doms (2003), When Do Matched-Model and Hedonic Techniques Yield Similar Price Measures?, Working Paper no. 2003-14, Federal Reserve Bank of San Francisco.

Balk, B.M. (1981), A Simple Method for Constructing Price Indices for Seasonal Commodities, *Statistische Hefte* 22, 1-8.

Balk, B.M. (2001), Aggregation Methods in International Comparisons: What Have We Learned? ERIM Report, Erasmus Research Institute of Management, Erasmus University Rotterdam.

Balk, B.M. (1981), A Simple Method for Constructing Price Indices for Seasonal Commodities, *Statistische Hefte* 22, 1-8.

Caves, D.W., L.R. Christensen and W.E. Diewert (1982), Multilateral Comparisons of Output, Input and Productivity Using Superlative Index Numbers, *The Economic Journal* 92, 73-86.

Destatis (2009), *Handbook on the Application of Quality Adjustment Methods in the Harmonised Index of Consumer Prices*, Statistics and Science, Vol. 13. Federal Statistical Office of Germany, Wiesbaden.

Diewert, W.E. (1999), Axiomatic and Economic Approaches to International Comparisons, pp. 13-87 in A. Heston and R.E. Lipsey (eds.), *International and Interarea Comparisons of Income, Output and Prices*, Studies in Income and wealth, Vol. 61. University of Chicago Press, Chicago.

Diewert, W.E. (2003), Hedonic Regressions: A Review of Some Unresolved Issues, Paper presented at the Seventh Meeting of the International Working Group on Price Indices (Ottawa Group), 27-29 May 2003, INSEE, Paris.

Diewert, W.E. (2004), On the Stochastic Approach to Linking the Regions in the ICP, Discussion Paper no. 04-16, Department of Economics, The University of British Columbia, Vancouver, Canada.

Diewert, W.E., S. Heravi and M. Silver (2009), Hedonic Imputation versus Time Dummy Hedonic Indexes, pp. 87-116 in W.E. Diewert, J. Greenlees and C. Hulten (eds.), *Price Index Concepts and Mesurement*, Studies in Income and Wealth, Vol. 70. University of Chicago Press, Chicago.

Eltetö, O. and P. Köves (1964), On a Problem of Index Number Computation Relating to International Comparisons, *Statisztikai Szemle* 42, 507-518 (in Hungarian).

Eurostat and OECD (2006), *Methodological manual on PPPs*. Available at www.oecd.org/std/pricesandpurchasingpowerparitiesppp.

Feenstra, R.C. and M.D. Shapiro (2003), High Frequency Substitution and the Measurement of Price Indexes, pp. 123-146 in R.C. Feenstra and M.D. Shapiro (eds.), *Scanner Data and Price Indexes*, Studies in Income and Wealth, Vol. 61. University of Chicago Press, Chicago.

Gini, C. (1931), On the Circular Test of Index Numbers, *Metron* 9, 3-24.

Greenlees, J. and R. McClelland (2010), Superlative and Regression-Based Consumer Price Indexes for Apparel Using U.S. Scanner Data, Paper presented at the Conference of the International Association for Research in Income and Wealth, St. Gallen, Switzerland, August 27, 2010.

van der Grient, H.A. and J. de Haan (2010), The Use of Supermarket Scanner Data in the Dutch CPI, Paper presented at the Joint ECE/ILO Workshop on Scanner Data, 10 May 2010, Geneva. Available at www.cbs.nl.

van der Grient, H.A. and J. de Haan (2011), Scanner Data Price Indexes: The ’Dutch Method’ versus RYGEKS, Paper presented at the Twelfth Meeting of the International Working Group on Price Indices (Ottawa Group), 4-6 May 2011, Wellington, New Zealand.

de Haan, J. (2004), The Time Dummy Index as a Special Case of the Imputation Törnqvist Index, Paper presented at the Eighth Meeting of the International Working Group on Price Indices (Ottawa Group), 23-25 August 2004, Helsinki, Finland.

de Haan, J. (2010), Hedonic Price Indexes: A Comparison of Imputation, Time Dummy and ‘Re-pricing’ Methods, *Journal of Economics and Statistics* (*Jahrbücher fur Nationalökonomie und Statistik)* 230, 772-791.

de Haan, J. and H.A. van der Grient (2011), Eliminating Chain drift in Price Indexes Based on Scanner Data, *Journal of Econometrics*, Vol. 161, Issue 1, 36-46.

Hill, R.J. and D. Melser (2008), Hedonic Imputation and the Price Index Problem: An Application to Housing, *Economic Inquiry* 46, 593-609.

ILO/IMF/OECD/UNECE/Eurostat/The World Bank (2004), *Consumer Price Index Manual: Theory and Practice*. ILO Publications, Geneva.

Ivancic, L., W.E. Diewert and K.J. Fox (2009), Scanner Data, Time Aggregation and the Construction of Price Indexes, Discussion Paper 09-09, Department of Economics, University of British Columbia, Vancouver, Canada.

Ivancic, L., W.E. Diewert and K.J. Fox (2011), Scanner Data, Time Aggregation and the Construction of Price Indexes, *Journal of Econometrics*, Vol. 161, Issue 1, 24-35.

Johansen, I. and R. Nygaard (2011), Dealing with Bias in the Norwegian Superlative Price Index of Food and Non-alcoholic Beverages, Room document at the twelfth meeting of the International Working Group on Price Indices (Ottawa Group), 4-6 May 2011, Wellington, New Zealand.

Kokoski, M.F., Moulton, B.R. and K.D. Zieschang (1999), Interarea Price Comparisons for Heterogeneous Goods and Several Levels of Commodity Aggregation, pp. 123-169 in A. Heston and R.E. Lipsey (eds.), *International and Interarea Comparisons of Income, Output and Prices*, Studies in Income and Wealth, Vol. 61. University of Chicago Press, Chicago.

Krsinich, F. (2011), Price Indexes from Scanner Data: A Comparison of Different Methods, Paper presented at the twelfth meeting of the International Working Group on Price Indices (Ottawa Group), 4-6 May 2011, Wellington, New Zealand.

Krsinich, F. (2012), A Fresh Look at Patterns in Gadget Sales. *Economic News*, Statistics New Zealand, April 2012.

Ribe, M. (2012), Some Properties of the RGEKS Index for Scanner Data, Paper presented at Statistics Sweden's scanner data workshop, 7-8 June 2012, Stockholm, Sweden.

Silver, M. (2003), The Use of Weights in Hedonic Regressions: The Measurement of Quality-Adjusted Price Changes, pp. 135-147 in Th. Lacroix (ed.), *Proceedings of the Seventh Meeting of the Ottawa Group*. INSEE, Paris.

Silver, M. and S. Heravi (2005), A Failure in the Measurement of Inflation: Results from a Hedonic and Matched Experiment Using Scanner Data, *Journal of Business & Economic Statistics* 23, 269-281.

Silver, M. and S. Heravi (2007), The Difference Between Hedonic Imputation Indexes and Time Dummy Hedonic Indexes, *Journal of Business & Economic Statistics* 25, 239-246.

Summers, R. (1973), International Price Comparisons Based Upon Incomplete Data, *Review of Income and Wealth* 19, 1-16.

Syed, I. (2010), Consistency of Hedonic Price Indexes in the Presence of Unobserved Characteristics, Discussion Paper, School of Economics, University of New South Wales, Sydney, Australia.

Szulc, B. (1964), Indices for Multiregional Comparisons, *Przeglad Statystycny* 3, 239-254 (in Polish).

Triplett, J.E. (2006), Handbook on Hedonic Indexes and Quality Adjustments in Price Indexes. Organization for Economic Co-operation and Development, Paris.