The use of weighted GEKS for the calculation of consumer price indices: an experimental application to Italian scanner data

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Outline

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The purpose of the work

The aim of this work is analyse the implication of the use of weights to calculate GEKS indices.

Well-known rationale for the use of weights:

The weighted version of rolling windows GEKS allows to deal with the issue of different reliability of bilateral indices: comparisons between two periods that are distant in time (for example at the ends of the time-window) may be considered less reliable than between adjacent periods.

The idea is that separation between months may affect the amount of overlapping items, which contribute to the calculation of the bilateral indices underlying GEKS (especially when 25 months windows are used).

On the empirical ground, we are interested in assessing the size of the impact of the use of weights and in the evaluation of the performace of the weighted version of GEKS against the GEKS, using different splicing options.

In our analysis we will limit to the case of Törnqvist GEKS.

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Main results

Main results of the experimentation:

- the use of weights in the calculation of GEKS proved to have limited impact on the final indices (in terms of both levels and yoy rates).
- It tends to reduce the distance between the rolling windows GEKS and their «full» window counterpart. In other words, the use of weights seems to reduce the effect of the constraint of non-revising the indices.
- Moreover, the preliminary results of an analysis based on the calculation of a target “true” cost of living index, under the hypothesis of Constant Elasticity of Substitution preferences, seems to confirm that the weighted GEKS tend to perform better than its unweighted version.

The methodology

Following Rao (2001), the weighted version of GEKS indices are calculated as the weighted least squares estimators of Π in the model specification:

$$lnI_{rt}^{T} = \Pi_r - \Pi_t + u_{rt}$$

With $E(u_{rt}) = 0$ and $Var(u_{rt}) = \frac{\sigma^2}{w_{rt}}$

That is: $$WGEKS_{rt} = \frac{\exp(\widehat{\Pi_r})}{\exp(\widehat{\Pi_t})} = exp(\widehat{\Pi_r} - \widehat{\Pi_t})$$

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The methodology

To make the method operational it is necessary to specify the matrix weights. Two measures used by Melser (2018) and Melser and Webster (2021) are considered:

- the average matched expenditure share (AMES) weights:

$$w_{rt}^{AMES} = \frac{1}{2} \cdot \left[\left(\frac{\sum_{i \in U_{rt}} p_{i,t} \cdot q_{i,t}}{\sum_{i \in U_t} p_{i,t} \cdot q_{i,t}} \right) + \left(\frac{\sum_{i \in U_{rt}} p_{i,r} \cdot q_{i,r}}{\sum_{i \in U_r} p_{i,r} \cdot q_{i,r}} \right) \right]$$

- the average matched product shares (AMPS) weights:

$$w_{rt}^{AMPS} = \frac{1}{2} \cdot \left[\frac{N_{rt}}{N_t} + \frac{N_{rt}}{N_r} \right]$$

where $U_{rt} = U_r \cap U_t$ is the set of items i available in both periods and $N_{rt} = |U_{rt}|$ is the number of the matching items.

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The case study

For the experimental use of weighted GEKS we have considered five product aggregates: Chocolate, Packaged ice cream, Olive oil, Body hygiene products and Cosmetic products.

Data come from the sample of shops of the large-scale retail distribution of Rome for the period Dec-2018 : Feb-2022.

Average number of GTINs by retail trade channels (province of Rome)

	Product aggregates				
Retail trade channels	*Chocolate*	*Packaged ice cream*	*Olive oil*	*Body hygiene products*	*Cosmetic products*
Hypermarkets (11)	1.666	887	472	2.042	1.869
Supermarkets (52)	1.663	963	443	1.726	763
Discounts (20)	599	332	67	413	92
Small sales areas (25)	857	585	242	619	48
Specialist drug (15)				1.469	3.589
Total (123)	**2.381**	**1.371**	**627**	**3.354**	**4.927**

Dic. 2018 - Feb. 2022

The case study

Main features of the five products used for the analysis:

- Unlike Olive oil, Chocolates and Packaged ice creams exhibit some seasonality in sales.
- Body hygiene products and Cosmetic products are characterized by the presence of many distinct groups of items with a large number of very similar GTINs whose sales vary significantly from period to period.

Considering the matrices of weights, generally, as the distance between two periods increases, AMPS tends to decrease much faster than AMES. Consequently, the AMES usually remains on relatively higher levels than the AMPS.

For seasonal products, the declines of the AMPS and AMES are less regular as compared to non-season products.

Finally, AMES and AMPS weights decrease much faster for products with a higher attrition, such as Cosmetic products.

The results of the first set of empirical analyses

For our experimental use of weighted GEKS, we have calculated the indices for rolling windows of 25 periods length with (4) different splicing options, together with the full window (39 periods) indices.

We used both AMES and AMPS weights.

Indices are calculated for each retail trade channel: the items are defined as the combination of barcode and retailers' channel (the average price of the each item is defined as the ratio of the total turnover and quantities sold by the shops of the same type).

The results of these calculations have then been compared with those obtained with the "standard" GEKS method.

In total, for food products and non-food products, we have estimated 180 and 150 indices, respectively.

The results of the first set of empirical analyses

The behaviour of the indices is assessed through the comparison of the their annual rates of change.

The table shows the number of positive and negative differences, max and min (in p.p.) between the yoy rates of change of the full window WGEKS and GEKS indices, by retailer trade channel and system of weights.

Product		chocolate				ice cream				olive oil				cosmetics					body hygiene products				
Retail channel		s.s.a	hyper	super	discoun	s.s.a	hyper	super	discoun	s.s.a	hyper	super	discoun	s.s.a	hyper	super	discoun	s.d.	s.s.a	hyper	super	discoun	s.d.
AMES	pos	18	13	13	11	14	15	15	12	16	14	13	17	8	10	12	9	11	13	11	8	19	10
	neg	9	14	14	16	13	12	12	15	11	13	14	10	19	17	15	18	16	14	16	19	8	17
	max	0,11	0,14	0,16	0,05	0,14	0,27	0,12	0,03	0,06	0,08	0,10	0,67	0,06	0,24	0,10	0,16	0,05	0,09	0,16	0,08	0,10	0,10
	min	-0,08	-0,23	-0,18	-0,04	-0,08	-0,21	-0,09	-0,04	-0,05	-0,07	-0,09	-0,37	-0,27	-0,36	-0,13	-0,12	-0,09	-0,13	-0,14	-0,09	-0,06	-0,22
AMPS	pos	21	11	11	10	15	15	14	12	19	15	16	12	9	10	12	11	10	13	11	11	18	11
	neg	6	16	16	17	12	12	13	15	8	12	11	15	18	17	15	16	17	14	16	16	9	16
	max	0,07	0,14	0,18	0,06	0,13	0,34	0,18	0,06	0,10	0,14	0,17	0,62	0,23	0,28	0,10	0,08	0,04	0,10	0,19	0,09	0,11	0,11
	min	-0,11	-0,21	-0,17	-0,04	-0,09	-0,25	-0,11	-0,05	-0,05	-0,13	-0,16	-0,42	-0,39	-0,32	-0,14	-0,13	-0,10	-0,13	-0,16	-0,09	-0,07	-0,22

The results of the first set of empirical analyses

The differences in the inflation rates are generally limited. There is no evidence that the introduction of the weights bring to a systematic under or overestimation of the dynamic of the prices of the products considered (but in some cases the discrepancies are not so small).

Moreover, the use of AMPS weights seems to have mostly a bigger impact on the indices than AMES weights have.

Product		chocolate				ice cream				olive oil				cosmetics					body hygiene products				
Retail channel		s.s.a	hyper	super	discount	s.s.a	hyper	super	discount	s.s.a	hyper	super	discount	s.s.a	hyper	super	discount	s.d.	s.s.a	hyper	super	discount	s.d.
AMES	pos	18	13	13	11	14	15	15	12	16	14	13	17	8	10	12	9	11	13	11	8	19	10
	neg	9	14	14	16	13	12	12	15	11	13	14	10	19	17	15	18	16	14	16	19	8	17
	max	0,11	0,14	0,16	0,05	0,14	0,27	0,12	0,03	0,06	0,08	0,10	0,67	0,06	0,24	0,10	0,16	0,05	0,09	0,16	0,08	0,10	0,10
	min	-0,08	-0,23	-0,18	-0,04	-0,08	-0,21	-0,09	-0,04	-0,05	-0,07	-0,09	-0,37	-0,27	-0,36	-0,13	-0,12	-0,09	-0,13	-0,14	-0,09	-0,06	-0,22
AMPS	pos	21	11	11	10	15	15	14	12	19	15	16	12	9	10	12	11	10	13	11	11	18	11
	neg	6	16	16	17	12	12	13	15	8	12	11	15	18	17	15	16	17	14	16	16	9	16
	max	0,07	0,14	0,18	0,06	0,13	0,34	0,18	0,06	0,10	0,14	0,17	0,62	0,23	0,28	0,10	0,08	0,04	0,10	0,19	0,09	0,11	0,11
	min	-0,11	-0,21	-0,17	-0,04	-0,09	-0,25	-0,11	-0,05	-0,05	-0,13	-0,16	-0,42	-0,39	-0,32	-0,14	-0,13	-0,10	-0,13	-0,16	-0,09	-0,07	-0,22

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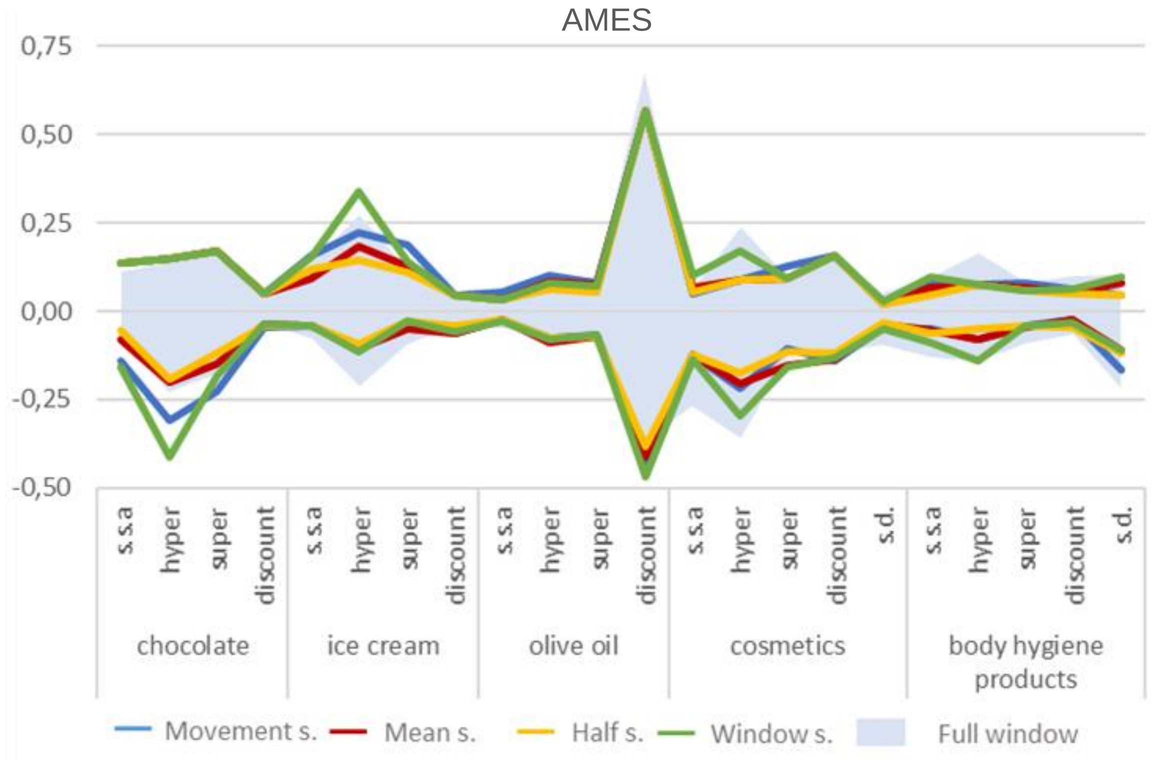
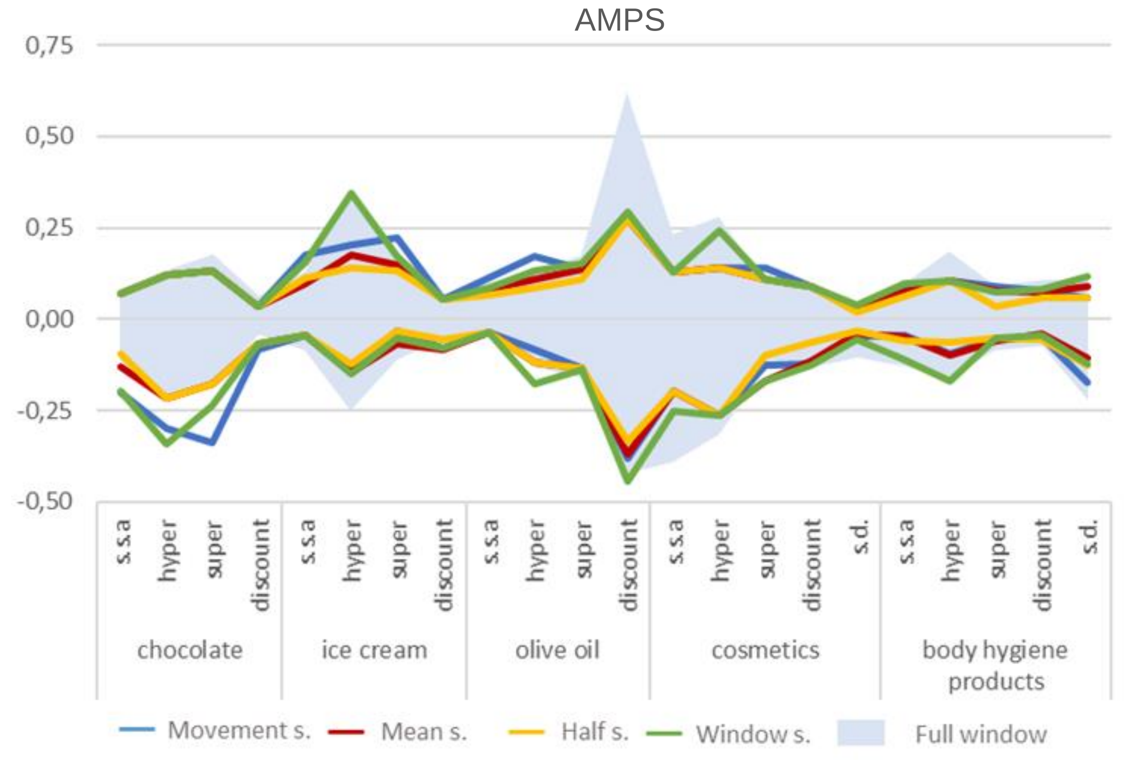
The results of the first set of empirical analyses

Concerning the rolling windows, the impact on the annual rates of change is modest as well (with few exceptions), but seems to be affected by the splicing method adopted to link the indices of the rolling windows: the adoption of the half and mean splicing seems to have the smaller impact's range in almost all the cases as compared to the other splicing options.





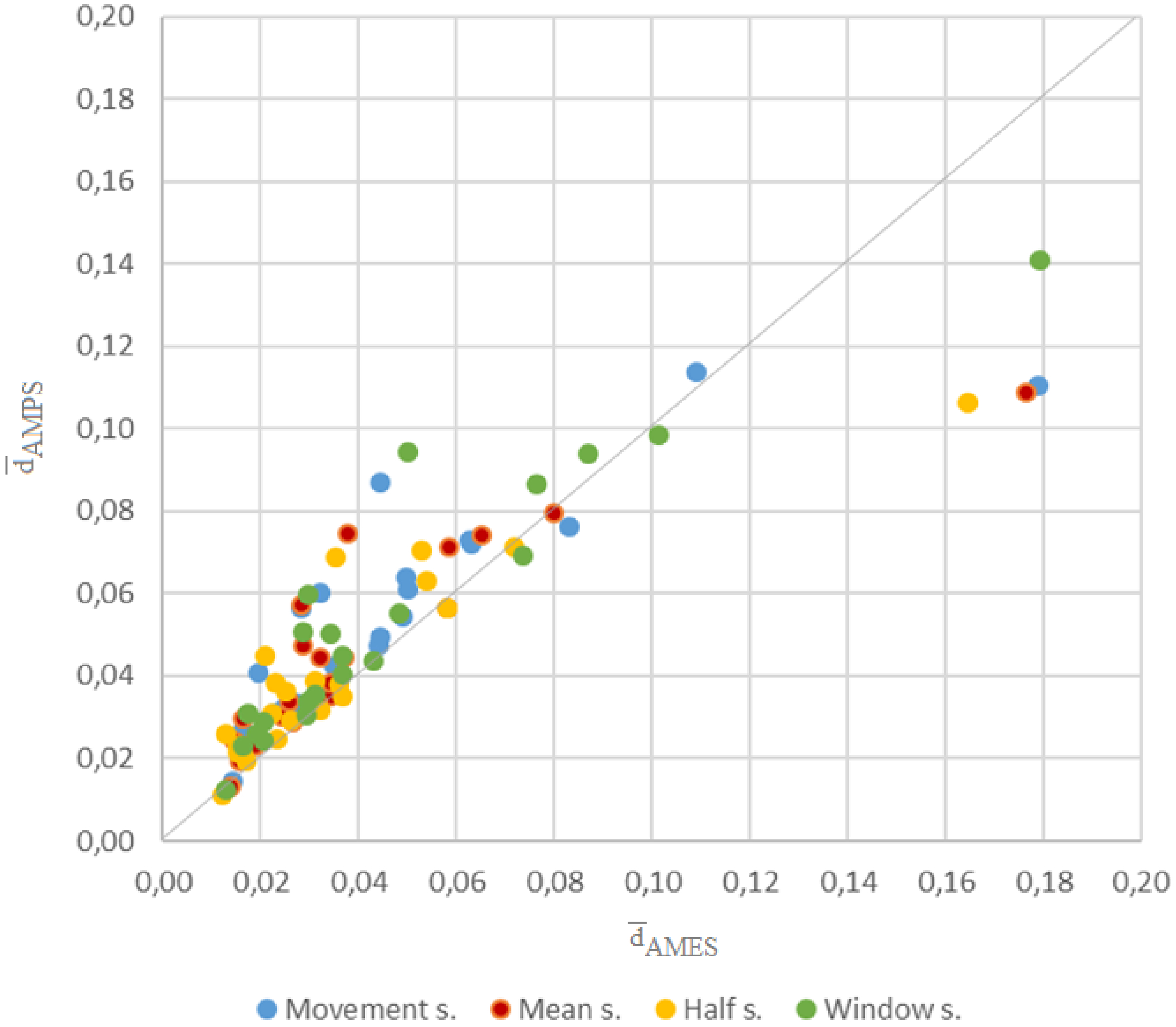
The results of the first set of empirical analyses

As in the case of full window GEKS, the choice of AMPS weights produces generally wider deviations of the weighted version of RGEKS from the standard counterpart, than AMES weights.

This is made clear by comparing the arithmetic mean of absolute differences between the yoy rates of change of the weighted ($x_{w,t}^{25}$) and standard (x_t^{25}) version of RGEKS for both the systems of weights.

$$\bar{d}_w = \frac{1}{T}\sum_t d_{w,t} = \frac{1}{T}\sum_t \left|x_{w,t}^{25} - x_t^{25}\right|$$

$$w = AMES, AMPS$$



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The results of the first set of empirical analyses

Finally, we have analysed the implication of the use of weights in terms of the distance between the full window GEKS and the corresponding rolling window indices.

To this aim, we have calculated the arithmetic mean of the absolute differences between the inflation rates x of the full window and of the 25 periods rolling windows weighted GEKS calculated using AMES:

$$\bar{D}_{AMES} = \frac{1}{T}\sum_t D_{AMES,t} = \frac{1}{T}\sum_t \left| x_{AMES,t}^{full} - x_{AMES,t}^{25} \right|$$

and the standard deviation of those differences: $s(D_{AMES,t})$

The same was done for weighted GEKS using AMPS and also for standard GEKS (with analogous symbols).

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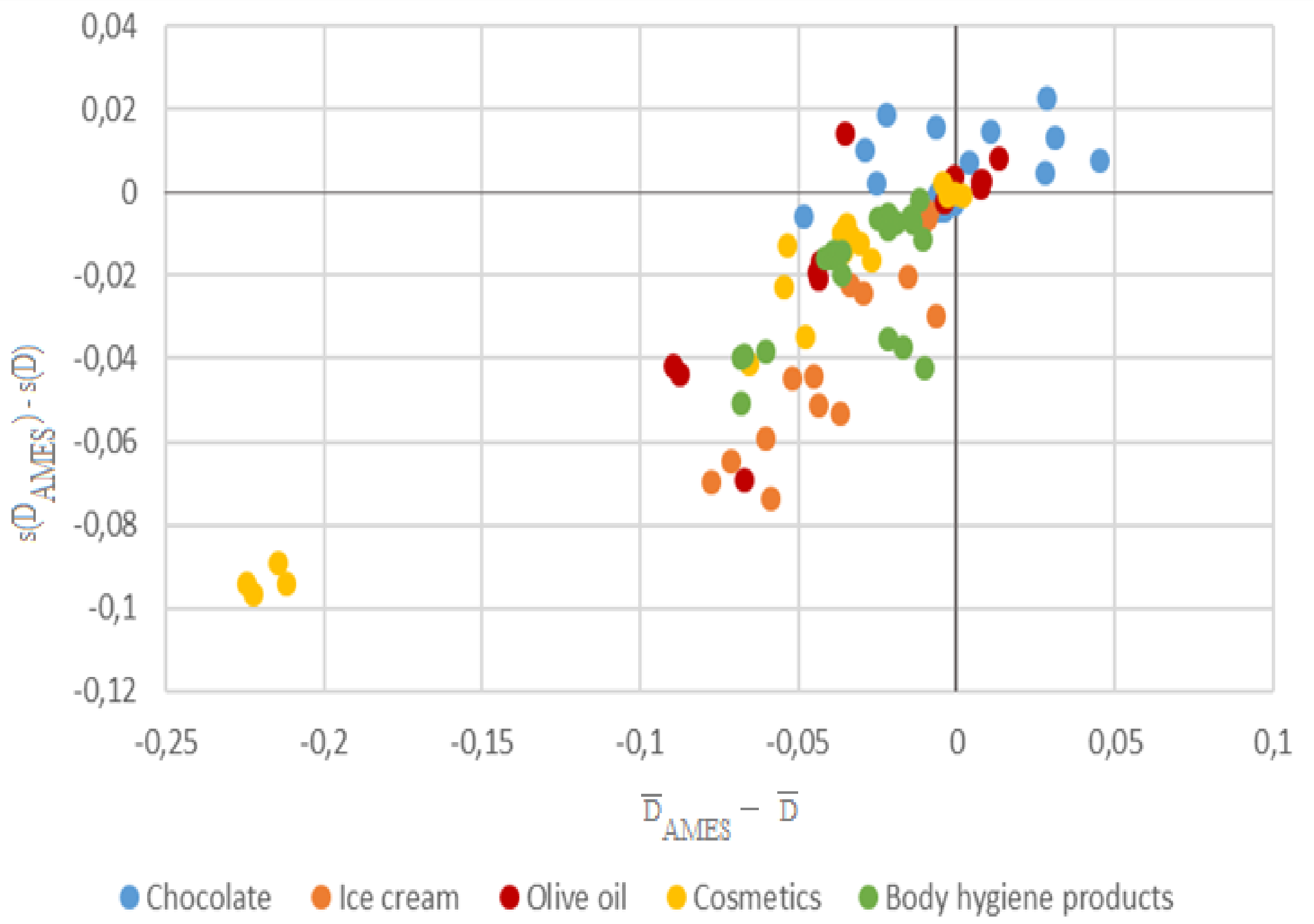
The results of the first set of empirical analyses

The scatter graph shows that the average of the absolute differences of inflation rates of the full window and of the 25 periods rolling windows of the standard GEKS (i.e. $\overline{D}$) is mostly greater than $\overline{D}_{AMES}$.

That is, the absolute distance between the rolling windows GEKS and the transitive corresponding index tends to increase when no explicit weights are used for the calculation of the multilateral indices.

Moreover, the absolute differences $D_{AMES,t}$ tend to stay closer to their average than D_t do



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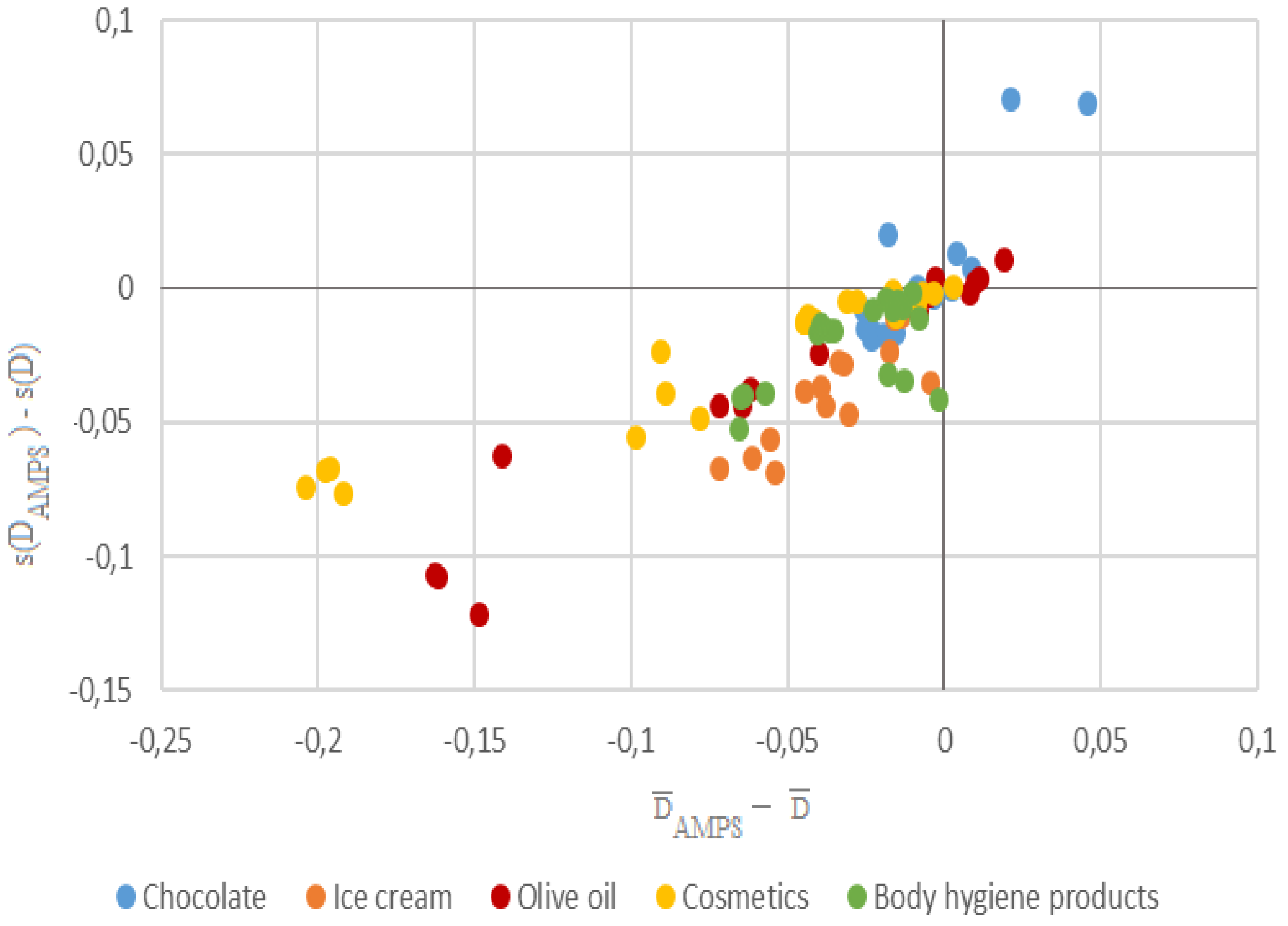
The results of the first set of empirical analyses

These results are even more evident in the case of AMPS weighted GEKS.

From this point of view, the impact of imposing the non-revisability constraint on the multilateral indices seems to be, generally, less severe when the weighted version (and in particular AMPS weighted version) of GEKS are considered.



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The Simulation framework: CES and Sato-Vartia index

- Another approach based on the definition of a "true" cost of living index has been explored in order to determine how well weighted and unweighted GEKS approximate the "true" indexes.

- We follow previous studies (Diewert and Fox, 2018; Melser and Webster, 2021) in supposing that consumers have Constant Elasticity of Substitution CES preferences over products:

$$C(p_t) = \left(\sum_{i \in U_t} a_i \, p_{it}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

where $\sigma \geq 0$ is the elasticity of substitution and the a_i are quality or taste (positive) parameters

- By Shepard's Lemma the optimal expenditure shares are:

$$s_{it} = \frac{a_i p_{it}^{1-\sigma}}{\sum_{i \in U_t} a_i \, p_{it}^{1-\sigma}}$$

where i = 1,...,N; t = 1,...,T. Therefore given the price vectors $\mathbf{p}_t$, the vector of positive parameters a and the nonnegative parameter σ where σ ≠ 1, then the share vectors s can be computed for t = 1,...,T.

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The Simulation framework: CES and Sato-Vartia index

- The Sato—Vartia index equals the cost of living index of a consumer with a CES utility function:

$$I_{0t}^{ST} = \prod_{i \in U_{0t}} \left(\frac{p_{it}}{p_{i0}}\right)^{w_{i0t}} \quad , w_{i0t} = \frac{\left(\frac{\tilde{s}_{it} - \tilde{s}_{io}}{ln\tilde{s}_{it} - ln\tilde{s}_{io}}\right)}{\sum_{i \in U_{0t}} \left(\frac{\tilde{s}_{it} - \tilde{s}_{io}}{ln\tilde{s}_{it} - ln\tilde{s}_{io}}\right)} \quad , \tilde{s}_{it} = \frac{p_{it} q_{it}}{\sum_{i \in U_{0t}} p_{it} q_{it}}$$

- Simulations using real data from 3 product aggregates in the food sector (Chocolate, Packaged ice cream and Olive oil) for the province of Rome in the period Dec-2018 : Feb-2022

- Given the price vectors $\mathbf{p}_t$, from actual scanner data the share vectors st and $\tilde{s}_{it}$ are computed by specifying the elasticity σ equal to 0, 2, 3 and 5.

- We estimated the unweighted Törnqvist GEKS and weighted Törnqvist GEKS using AMES as weights over a 25-month window and using as linking method the movement splice

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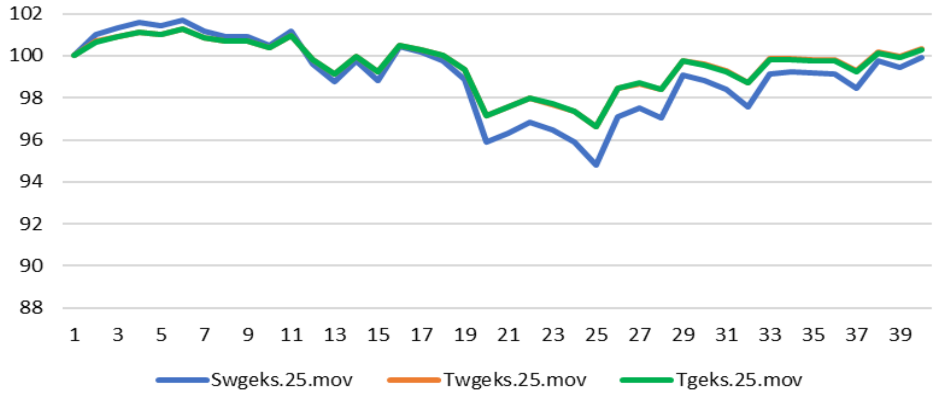
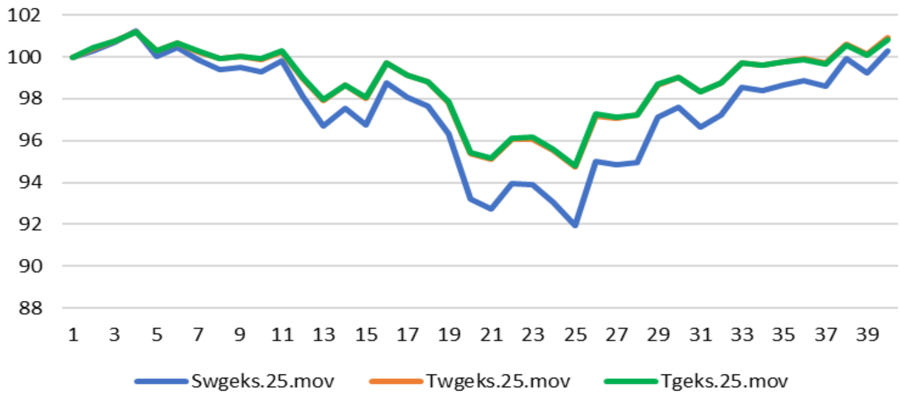
The Simulation framework: empirical results

Alternative price levels for different methods and elasticities of substitution- Olive Oil- All outlets

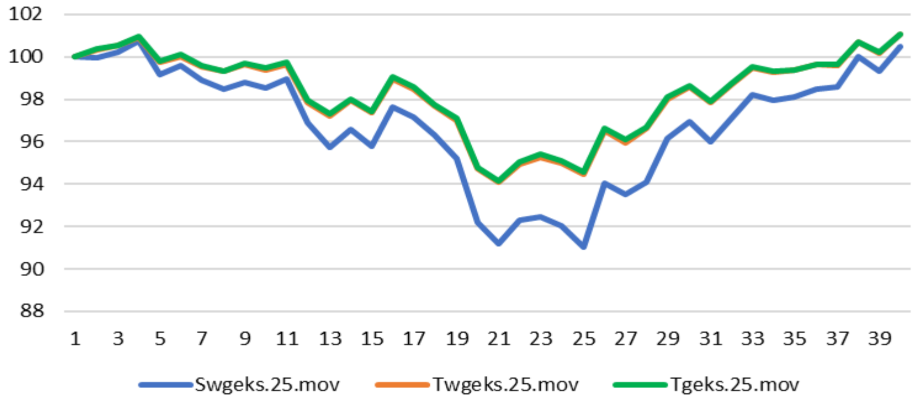
σ=0



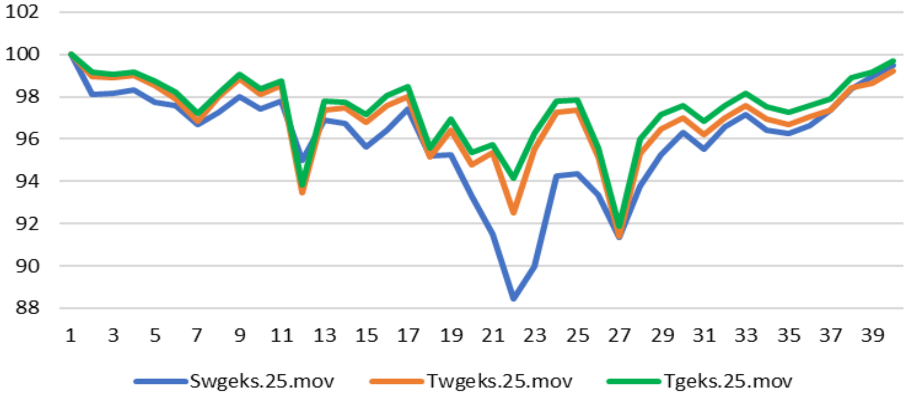
σ=2



σ=3



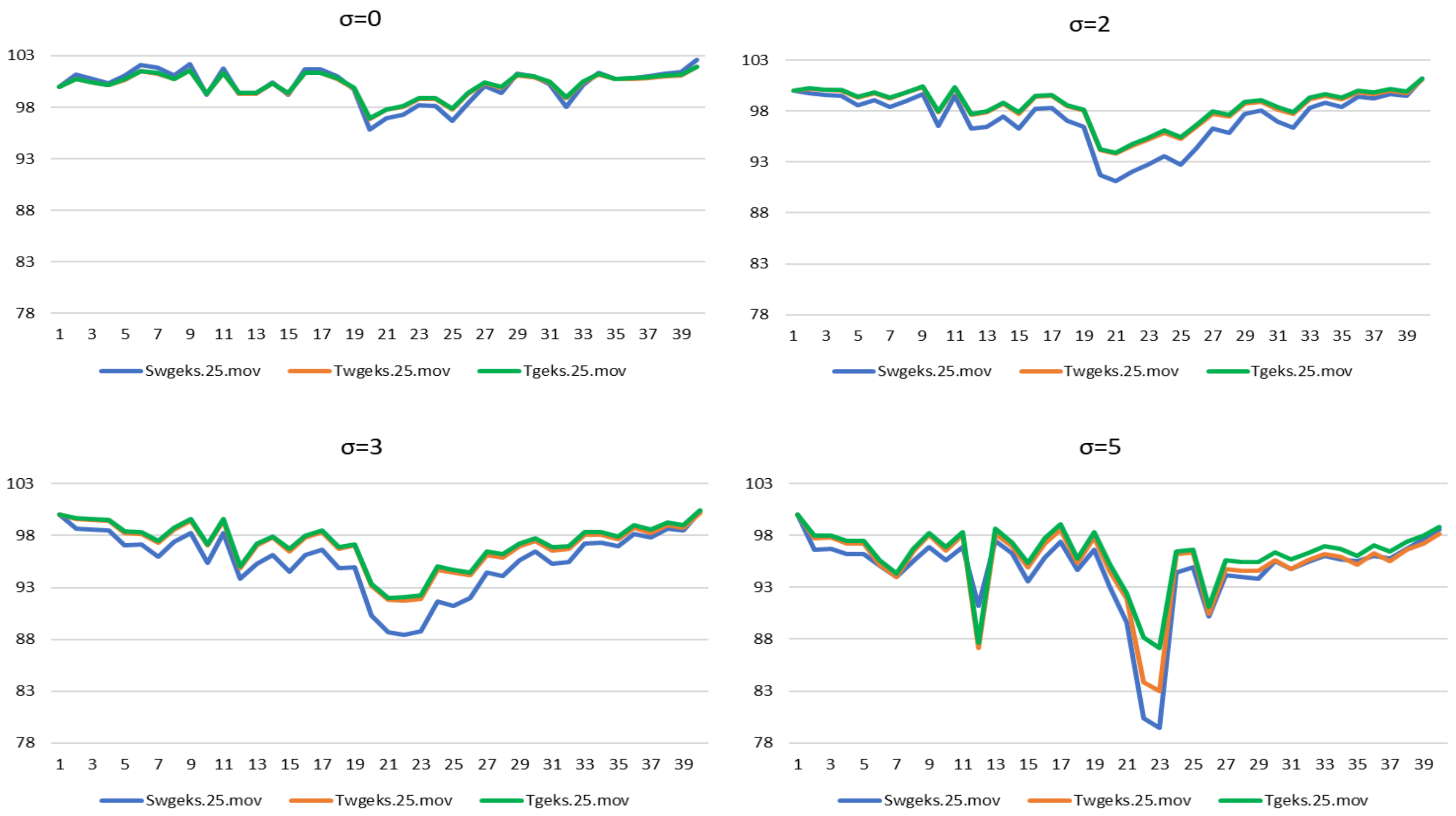
σ=5



Törnqvist WGEKS performs better than the unweighted Törnqvist GEKS with regard to the CES price levels

The Simulation framework: empirical results

Alternative price levels for different methods and elasticities of substitution- Olive Oil- Hypermarkets



The weighted GEKS seems to approximate the “true” indexes better than the unweighted GEKS

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Concluding remarks

- The generalization of the standard GEKS method, based on Törnqvist indices, used for the compilation of transitive consumer price indices, was tested on real data collected in the province of Rome in the period 2018, December – 2022, Febraury.

- The different systems of weights considered for introducing information on the reliability of the underlying binaries for the weighting matrix proved to have moderate effect on the dynamic of GEKS.

- However our findings suggest that the weighted version of rolling windows GEKS, under different splicing options, tend to be slightly closer to the full window counterpart, as compared to standard GEKS.

- The better performance of the Weighted GEKS is confirmed when following another approach based on the CES preferences and Sata-Vartia index.

- The results obtained are interesting but this line of research needs to be further developed.

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Thank you!