An Examination of the Differences between the Cobb-Douglas Price Index and the Geometric Lowe Price Index

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Abstract

Consumer Price Indexes are compiled in two stages: lower level using unweighted averages of price changes and the higher level using share weighted averages. The *Consumer Price Index Manual* (*CPI Manual*) 2004 strongly supports the use of geometric means at the lower level, an innovation taken up by a majority of statistical offices. The *CPI Manual* provides support for including geometric averaging in higher level formula, something largely not yet adopted, but a logical next major innovation in CPI methodology. This paper presents the case for using geometric averaging at the aggregate levels and explores the relationship between the Cobb-Douglas and the Geometric Lowe indexes, the two geometric formulations most readily available for index compilers. A formal exact decomposition of the difference between them is derived. Empirical results on the difference between these formulas are provided using United States CPI data.

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I. INTRODUCTION

Consumer Price Indexes are compiled in two stages, at the lower "elementary" level as unweighted averages of price changes and at the higher level as expenditure-share weighted averages of the lower level indexes. The *Consumer Price Index Manual* (*CPI Manual*) 2004 strongly supports the use of geometric means at the lower level, an innovation taken up by a majority of statistical offices.

The *CPI Manual* supports, somewhat less strongly, geometric averaging in higher level formula, something largely not yet adopted.[2] For this purpose the *CPI Manual* recommends the ideal indexes—the Fisher, Törnqvist, and Walsh price indexes—as the target formulas for the higher-level indexes. These formulas all use geometric averaging; they also use symmetric weights based on quantity or expenditure information from both the reference and current periods.[3]

Currently, most national statistical offices (NSOs) use what they often describe as "Laspeyres-type" indexes for the higher level CPI formula, i.e., they are targeting the Laspeyres, which uses arithmetic averaging and reference-period weights, rather than one of the ideal indexes with geometric averaging and weights from both the reference and current periods.

The Laspeyres index can be expressed (see equation (1) below) as an arithmetic average of price ratios (current-period prices divided by reference period prices) with reference-period share weights. NSOs approximate a Laspeyres index with either a Lowe or a Young index. Like Laspeyres, both Lowe and Young indexes are arithmetic averages of price ratios with expenditure share weights; the difference is that these weights are not from the reference period but are derived from a survey period that is earlier than the reference period. The Lowe index weights are shares of survey-period expenditures that have been price-updated to the price-reference period and the Young index uses the survey period shares without any price updating. Whether Lowe or Young better approximates Laspeyres depends on whether shares of price-updating survey-period expenditures or those of the survey-period expenditures without price updating are closer to the unknown (in real time at least) reference-period expenditure shares.

This paper advocates that NSOs use a geometric formula for higher level CPI aggregation. Since the NSOs cannot get (in real time) reference-period weights, let alone current-period weights, indexes bases on symmetric weights are not feasible. What is feasible for the NSOs is to use geometric averaging, which requires no more data than Lowe or Young indexes.

The advantages of using geometric averaging at the higher level are well documented, and their adoption as a feature of CPI compilation is the logical next innovation in CPI methodology.

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[2] Notable exceptions are the United States chained urban CPI compiled retrospectively as a chained Törnqvist index (Greenlees and Williams, 2009) and the Swedish CPI that uses a chained Walsh price index (Ribe, 2005).

[3] The Fisher index uses a geometric mean of Laspeyres and Paasche price indexes; Törnqvist is the geometric mean of price changes whose weights are an arithmetic mean of reference and current period weights; and Walsh is an arithmetic fixed quantity basket price index for which the quantities are fixed at the geometric mean of reference and current period quantities.

Geometric Lowe and Geometric Young indexes (geometric versions of the arithmetic Lowe and Young indexes) can be produced with the same data as the arithmetic versions. The question is: should the weights (the survey-period shares) be price-updated, as in the arithmetic Lowe, to form the geometric Lowe, or should they be used directly to form the geometric version of the Young index? We favor the geometric Young index, which is known as the Cobb-Douglas index, for higher level aggregation arguing that statistical offices should abandon price–updating when moving to a geometric formulation. There is, to the authors' knowledge, no formal examination as to the nature and extent of how the geometric Lowe and geometric Young price indexes differ.

This paper, in section II, outlines the features of the arithmetically-based aggregation of the Laspeyres, Lowe, and Young price indexes and compares them to geometric formula alternatives available to national statistical offices. In section III, it presents the case for using geometric averaging at the aggregate levels and briefly reviews research in the U.S. on this issue. Section IV considers their geometric counterparts and derives a formal exact decomposition for the difference between the geometric Lowe and Young price indexes. This decomposition identifies a bias on the part of the geometric Lowe that has the potential to lead to excess drift and volatility. Section V provides simulations on the extent of the differences between the indexes, and section VI examines the relationship between the Cobb-Douglas and Törnqvist indexes. Section VII concludes.

II. ARITHMETIC HIGHER LEVEL PRICE INDEX NUMBER FORMULAS USED IN PRACTICE

The Laspeyres price index is given by:

$$I_L^t = \sum_{i=1}^{n} \frac{p_i^t q_i^0}{p_i^0 q_i^0} = \sum_{i=1}^{n} \frac{p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}\left(\frac{p_i^t}{p_i^0}\right) = \sum_{i=1}^{n} s_i^0\left(\frac{p_i^t}{p_i^0}\right), \text{ where } \quad s_i^0 = \frac{p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} \tag{1}$$

The first term of equation (1) is the standard Laspeyres formula: a fixed quantity basket index with p_i^0 and q_i^0 denoting, respectively, prices and quantities in period 0 for $i = 1,\ldots, n$ products (elementary aggregates when the formula is being used at the higher level); period t prices, p_i^t, are weighted by the period 0 quantities. The Laspeyres price index measures the change in cost of a fixed quantity basket from period 0 to period t. It does this by keeping the quantities fixed in both periods compared; only the prices change. The fixed quantities are those of period 0, which is the base price reference period; this is also when the index equals 100.

NSOs do not usually use the standard Laspeyres formula as their target for calculating their price indexes. Instead, they use a weighted average of price relatives, given by the second term in equation (1), where the weights are the expenditure shares, s_i^0, from period 0.

In practice, there is a time lag between the survey period for the expenditures shares, b, and their first use in the index because it takes time to compile and process expenditure survey data. The CPI in many countries uses expenditure shares from the earlier period (b) as weights to average price relatives from the current period prices (t) compared to those in the price reference period

(0). This price index is known in the literature as a Young index and uses the following formulation:

$$I_Y^t = \sum_{i=1}^{n} s_i^b \left(\frac{p_i^t}{p_i^0} \right), \text{ where } \quad s_i^b = \frac{p_i^b q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b} \tag{2}$$

The Young index keeps the expenditure shares fixed in the expenditure survey period *b*, unlike the Laspeyres price index that keeps the shares fixed in the price reference period 0 and holds the quantities purchased fixed at period 0 levels.

Other countries' CPIs keep the period-*b* quantities fixed and update them for price changes between period *b* and the price reference period 0. This index measure, which is known in the literature as a Lowe index, has the following formula:

$$I_{Lo}^t = \frac{\sum_{i=1}^{I} \left[p_i^b q_i^b \frac{p_i^0}{p_i^b} \right] \frac{p_i^t}{p_i^0}}{\sum_{i=1}^{I} p_i^b q_i^b \frac{p_i^0}{p_i^b}} = \frac{\sum_{i=1}^{I} p_i^t q_i^b}{\sum_{i=1}^{I} p_i^0 q_i^b} = \frac{\sum_{i=1}^{I} p_i^0 q_i^b \frac{p_i^t}{p_i^0}}{\sum_{i=1}^{I} p_i^0 q_i^b} \tag{3}$$

The expression in brackets in the first term are the *b*-period expenditures, $p_i^b q_i^b$, price-updated to period 0. The second term shows the Lowe index to be a period-*b* fixed-basket index and the third to be a weighted average of price changes where the weights are hybrid weights with little economic meaning. Updating the expenditure shares for price changes between these periods is not to update the weights, but to transform the index from a fixed expenditure share index to a fixed quantity basket index. Fixed basket indexes benefit from an easy interpretation. They indicate what it costs at today's prices to purchase the same fixed quantities from the reference period.

III. PROBLEMS WITH THE ARITHMETIC LOWE AND YOUNG AND THE CASE FOR A GEOMETRIC HIGHER-LEVEL FORMULA

As Chapter 15 of the *CPI Manual* (2004) notes, both the Young and Lowe indexes are biased approximations of the Laspeyres index due to the use of the older weights from period *b*. Chapter 15 also shows that the Laspeyres index is biased in relation to the target ideal indexes—Fisher, Törnqvist, and Walsh. However, at this time NSOs are only able to produce the ideal indexes belatedly because they do not have a source for deriving the current period weights that the ideal indexes require until several years after the current period. The NSOs can, however, produce geometric versions of the Young or Lowe indices, which likely have less bias with respect to the ideal indexes than the traditional arithmetic ones.

There have been discussions among practitioners about which arithmetic index they should use for their price indices: Lowe or Young. For example, the CPI Manual (ILO, 2004) suggests in Chapter 16 that the Lowe index might be preferred over the Young index because it has better axiomatic properties. Hansen (2006, 2007), views both the Lowe and the Young as approximations of a Laspeyres index that, in turn, is an approximation of the ideal indexes. He argues that the question is whether the original, not-price-updated expenditure shares from period *b* (the period of the expenditure or budget survey) , or the shares of the expenditures updated for price change between period *b* and period 0 (the price reference period) is likely to be closer to the true, but not-yet-known, period 0 shares.

Price updating will raise the shares of items with relatively high price change between periods *b* and 0 and reduce the shares of the others. Hanson notes that the determining factor is the elasticity of demand. If, on one hand, demand is inelastic (i.e., close to zero), the shares of the price-updated expenditures will be closer to the period 0 shares and the Lowe index is more appropriate. On the hand, if it is relatively elastic (i.e., close to one), consumers will reduce the quantity of the items with the largest price increases, leaving the shares relatively unchanged, and the Young index is more appropriate.

Hanson makes a further point; an arithmetic Young index implies—inconsistently—that the elasticity of substitution among elementary aggregates is equal to one during the time between periods *b* and 0 and then is zero between periods 0 and *t*. The geometric Young (or Cobb Douglas) is consistent, making the same elasticity assumption for both time intervals. The same consistency argument, of course, can be made in favor of the arithmetic Lowe as well as against the geometric Lowe.

Empirical evidence using data from the Denmark CPI indicates that there is no clear advantage of one index over the other. In some periods the Young version exceeds the Lowe while in others, the reverse holds. Both diverge over different periods from the target indexes.

Greenlees and Williams (2009) examine alternative index measures using data from the U.S. CPI in a retrospective analysis. The headline US CPI, which is called the Consumer Price Index for All Urban Consumers or CPI-U, is a Lowe index. Its weights are revised every two years, with first use in the US CPI for January of each even-numbered year. The weights, which are from two years of the US Consumer Expenditure Survey (CES), are mean-annual expenditures for a two-year period ending 13 months before their first use. They are updated for price change from the mid-point of the weight period to the December before their fist use. For example, the weights of US CPI in January 2010 (period $t = 1$) uses expenditures from 2007-2008 (period *b*) that were price-updated to December 2009 (period 0). There is approximately a two-year lag from the midpoint of the survey period, *b*, to price reference period 0.

The United States also produces an alternative index, the Chained Consumer Price Index for All Urban Consumers or C-CPI-U, which unlike the CPI-U, is subsequently revised. The final version is a Törnqvist index that is released two-years after the corresponding CPI-U. The initial versions are geometric Young (Cobb-Douglas) indexes. In February 2011, Törnqvist index for each of the 12 months of 2008 were released, replacing the previously published versions.

Table 1 Alternative index measures for the U.S. CPI

	Initial C-CPI-U	**Final C-CPI-U**	**CPI-U**
Higher-level formula	Young (Cobb-Douglas)	Törnqvist	Lowe
Month-Year			

December 1999	100.0	100.0	168.3
December 2008	120.661	121.557	210.228
Percentage Change	20.7	21.6	24.9
Annual Growth	2.4	2.5	2.8

Source: U.S. Bureau of Labor Statistics

Greenlees and Williams compiled a number of indexes using weight data derived from the U.S. CES for 1999 to 2007.They derive selected fixed basket and superlative indexes. Among the fixed basket indexes, they derive a Young index with biannual weight updates— the same update cycle as the headline U.S. CPI-U, which is a Lowe index. These two indexes show a very similar trend over the 2001 to 2007 period.

Greenlees and Williams also derived annual chained indexes for five different price index measures—Laspeyres, Paasche, Fisher, Törnqvist, and Satio-Vartia. There is a two year lag in finalizing the CES estimates, so the measures could only be calculated with a similar lag. They compare these indexes with the CPI-U and the C-CPI-U, which is an approximation to a monthly chained Törnqvist index. Their analysis indicates that the annual chained Fisher, Törnqvist and Satio-Vartia indexes, all geometric aggregate indexes, yield results that are very similar for the eight year period. These indexes, along with the chained Laspeyres, show lower growth rates than the CPI-U and the Young index.

The C-CPI-U index, as a monthly-chained Törnqvist approximation,[4] is close to the annual chained Törnqvist index, but it showed slightly lower growth. The authors note that this was most likely due to the annual vs. monthly chaining procedures of the two as the monthly chained C-CPI-U was affected more by large increases and subsequent large declines in fuel prices during the 2005 and 2006 period.

The preliminary C-CPI-U, which is published with the headline CPI-U, is a Cobb-Douglas index with weights that are actual expenditure shares (not price updated) from the (Consumer Expenditure) Survey two years previous. Thus, we can track the movement of a bi-annual chain Cobb-Douglas index for the U.S. over the period December 1990 through December 2008, the last month for which a revision has been made. As shown in Table 1, the Cobb-Douglas index grows by 20.7% compared to 21.6% for the approximated Törnqvist and 24.9% for the CPI-U (Lowe). These translate to annual growth rates of 2.4, 2.5, and 2.8 percent, respectively. Why would there be such differences?

[4] We use the term *approximation* to a Törnqvist. A true monthly-chained Törnqvist needs true monthly shares that reflect consumers' responses to relative monthly price change. The C-CPI-U compilation is based on 211 item strata for 38 area strata and does have sufficiently rich monthly data to populate the 8018 monthly cells. The C-CPI-U uses some kind of rolling allocation that apportions the national monthly expenditure for each item stratum (of the 211 item strata) among the 38 index areas according to their shares for that stratum in the previous 12 months. [The author's acknowledge the help of Walter Lane with this detail.]

Greenlees and Williams also estimate the demand elasticity for U.S. consumers and find that it ranged between 0.52 and 0.73 between 2000 and 2006, indicating that demand is somewhat inelastic. The Cobb-Douglas index, which is consistent with unitary demand elasticity, assumes more substitution than is apparent in the U.S. data. Hence, the index should be expected to grow more slowly than indexes such as the Törnqvist that reflect the actual elasticity of substitution among the elementary aggregates. The growth in the CPI-U over the same period is larger because the Lowe index holds quantities fixed at those during the weight period consistent with demand elasticity of zero, i.e., no substitution by consumers. Thus, it would show a higher rate of growth than Törnqvist and Cobb-Douglas indexes.

From the U.S. data, it is clear that the Young and Lowe indexes overstate the inflation measures provided by the target indexes. It is also clear that the Cobb-Douglas index is closer to the target indexes than these alternatives.

Greenlees (2010) notes that when the new weights are available and the C-CPI-U is revised to be a Törnqvist index, it tends to be revised upward. He suggests using the Lloyd-Moulton index (Constant elasticity of substitution) which has the following form:

$$I_{LM}^{t} = \left[\sum_{i=1}^{n} s_i^b \left(p_i^t \big/ p_i^b \right)^{1-\eta} \right]^{1/(1-\eta)}$$

Greenlees estimates the demand elasticity (η) based on historical estimates from the U.S. Consumer Expenditure Survey. He uses a pooled regression approach to estimate η from 1999 to 2008 and uses these values in the Lloyd Moulton index (with a two year lag) for calculating the preliminary C-CPI-U. The results show that the revisions between the preliminary and final C-CPI-U are much smaller when the Lloyd-Moulton index is used to for the preliminary measure vs. the Cobb Douglas index.

Lent and Dorfman (2009) take a slightly different approach for approximating an ideal index result. They find that a weighted average of arithmetic and geometric Laspeyres indexes (called an AG Mean index) can also approximate a superlative target index. In their research the weights are estimates of demand elasticity from previous periods and has the following form:

$$I_{AG}^{t} = \eta \prod_{i=1}^{n} \left(p_i^t \big/ p_i^0 \right)^{s_i^0} + (1-\eta) \left(\sum_{i=1}^{n} s_i^0 \, p_i^t \big/ p_i^0 \right)$$

They demonstrate that the AG Mean provides a close approximation to the superlative Fisher index under normal conditions when $0 \leq \eta \leq 1$.

In a recent paper, Balk (2009) has suggested that the Cobb-Douglas price index (called a geometric Young index in the *CPI Manual*), is a better choice for NSOs than the Lowe index because its substitution bias is likely to be less. This is the condition demonstrated by the U.S. CPI analysis.

For elementary-level indexes, Chapter 20 of the CPI Manual recommends the use of the geometric Jevons index if weights are not available for individual varieties in the sample. The use of a geometric formula at the aggregate level is compatible with the Jevons index at the lower level so that consistency in aggregation is maintained. Hence, the CPI Manual supports the use of geometric formulas at both the elementary and the aggregate levels of index compilation.[5]

Since the NSOs cannot produce the target indexes in current periods, they should consider compiling aggregate indexes using a geometric index formula, of which there are two easily available forms they could compile: the Cobb-Douglas or the Geometric Lowe.[6]

The Cobb-Douglas price index has the following form:

$$I_{CD}^{t} = \prod_{i=1}^{n} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^b}, \quad \text{where} \quad s_i^b = \frac{p_i^b q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b} \tag{4}$$

The other alternative that NSOs could easily adopt is the geometric version of the Lowe price index that uses their conventional price-updated weights:

$$I_{GLo}^{t} = \prod_{i=1}^{n} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^{b0}}, \quad \text{where} \quad s_i^{b0} = \frac{p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} \tag{5}$$

Which of these alternatives would be better for NSOs to use? We explore this issue in the next two sections by examining the differences between the two indexes and their sensitivity to price change over critical periods.

IV. A DECOMPOSITION OF THE DIFFERENCE BETWEEN THE COBB DOUGLAS AND GEOMETRIC LOWE INDEXES

Following on from equation (4) we first define a Cobb Douglas price index as:

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[5] In this regard, Chapter 1, paragraph 1.40, the *CPI Manual* states:

> "The geometric Young and Laspeyres indices have the same information requirements as their ordinary arithmetic counterparts. They can be produced on a timely basis. Thus, these geometric indices must be treated as serious practical possibilities for purposes of the CPI calculation."

[6] In most countries, producing a Laspeyres index or its geometric counterpart are not practical because of the length of time required to process the expenditure survey data and select a new basket of items (about two-three years). Most countries do not have dedicated resources for conducting an expenditure survey at regular intervals such as every three, or even every five, years. Obtaining accurate prices retrospectively for the new items during the weight reference period is also of concern.

$$I_{CD}^{t} = \prod_{i=1}^{n} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^b} \text{ and } \ln\left(I_{CD}^t\right) = \sum_{i=1}^{n} s_i^b \ln \frac{p_i^t}{p_i^0} = \sum_{i=1}^{n} s_i^b y_i^t \qquad (6)$$

where $s_i^b = \frac{w_i^b}{\sum_{i=1}^{n} w_i^b} = \frac{p_i^b q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b}$ are period b expenditure shares and $y_i^t \equiv \ln\left(\frac{p_i^t}{p_i^0}\right)$ is the logarithm of the ith price relative.

The difference between the logarithms of a Geometric Lowe and a Cobb Douglas price index is given by:

$$\ln(I_{GLo}^t) - \ln(I_{CD}^t) = \frac{\sum_{i=1}^{n} p_i^b q_i^b \left(\frac{p_i^0}{p_i^b}\right) \ln\left(\frac{p_i^t}{p_i^0}\right)}{\sum_{i=1}^{n} p_i^b q_i^b \left(\frac{p_i^0}{p_i^b}\right)} - \frac{\sum_{i=1}^{n} p_i^b q_i^b \ln\left(\frac{p_i^t}{p_i^0}\right)}{\sum_{i=1}^{n} p_i^b q_i^b} = \frac{\sum_{i=1}^{n} w_i^b x_i y_i}{\sum_{i=1}^{n} w_i^b x_i} - \frac{\sum_{i=1}^{n} w_i^b y_i}{\sum_{i=1}^{n} w_i^b} \qquad (7)$$

where $x_i^t \equiv \frac{p_i^0}{p_i^b}$.

Adopting a Bortkiewicz (1923) decomposition[7] it can be shown that:

$$\ln(I_{GLo}^t) - \ln(I_{CD}^t) = \frac{\sum_{i=1}^{n} w_i^b x_i y_i}{\sum_{i=1}^{n} w_i^b x_i} - \overline{y}_i = \left(\rho_{x,y}^{w^b} cv_x^{w^b} \sigma_y^{w^b}\right) \qquad (8)$$

and

$$\frac{I_{GLo}^t}{I_{CD}^t} = \exp\left(\rho_{x,y}^{w^b} cv_x^{w^b} \sigma_y^{w^b}\right) \qquad (9)$$

where $\rho_{x,y}^{w^b}$ is the w_i^b-weighted correlation coefficient between price relatives x_i and y_i, $cv_x^{w^b} = \sigma_x^{w^b} / \overline{x}^{w^b}$ is the w_i^b-weighted coefficient of variation for x, and $\sigma_y^{w^b}$ the standard deviation for y.

[7] See Bortkiewicz (1923; 374-375) for the first application of this correlation coefficient decomposition technique: we define a correlation coefficient between u and v as $\rho_{u,v} = \left(\sum uv - m\overline{u}\overline{v}\right)/m\sigma_u\sigma_v$. Then $\sum uv / \sum u = \sigma_u \sigma_v \rho_{u,v} / \overline{u} + \overline{v} = \mathrm{cov}(u,v)/\overline{u} + \overline{v}$ and $\sum wuv / \sum wu$ yield w-weighted terms for the decomposition.

First, it is apparent from equation (9) that for the Geometric Lowe to equal a Cobb Douglas price index it is necessary that: 1) for all *i*, period *b* to 0 price changes are the same OR the logarithms of all *i* period 0 to *t* price changes are the same, or 2) there is no (weighted) correlation between period *b* to 0 and period 0 to *t* price changes. These are extreme conditions. Having no dispersion in price changes is a negation of the index number problem. In addition, we cannot dismiss the possibility of, say, long-run relative price trends such as decreases for consumer electronics (positive correlation) or relative price changes in a given direction returning to equilibrium after a shock or seasonal fluctuation (negative correlation).

Second, equation (9) indicates there is a *potential* drift in the difference between the results of the indexes. It is well established in theory and empirical work that the dispersion in relative prices increases with increases in inflation.[8] Thus as inflation increases, the Geometric Lowe will drift from the Geometric Young as $cv_x^{w^b}$ and, more particularly, $\sigma_y^{w^b}$ (most likely) increase.[9] Note that $\sigma_y^{w^b}$ is likely to be the most potent driver of the drift since it is not corrected, as is the coefficient of variation, $cv_x^{w^b}$, for changes in the mean is concerned with the (larger) index changes between period 0 to *t*, than *b* to 0 for $cv_x^{w^b}$. Drift between the two indexes also requires that $\rho_{x,y}^{w^b} \neq 0$, some correlation between period *b* to 0 and period 0 to *t* price changes. The nature of the correlation dictates the direction of the drift. A positive correlation arising from long-run trends, for example, with consumer electronics, leads to the geometric Lowe drifting upward, and a negative correlation, as with seasonal goods and their prices returning to equilibrium after shocks, leading to a downward drift. However, the drift is *potential*: since $\rho_{x,y}^{w^b}$, and the other components, are multiplicative in equation (9), any chance lowering of $\rho_{x,y}^{w^b}$ to near zero in a month will lead to the two formula being very similar in spite of increasing $cv_x^{w^b}$ and $\sigma_y^{w^b}$. Not only can drift in the difference be expected but excessive volatility in the form of spikes if in some moths any component of the right-hand-side of (9) is negative while in others it is not. The direction and extent of drift is an empirical matter as demonstrated earlier in the U.S. CPI data.

Third, note that $cv_x^{w^b}$ is a constant, at least between rebasing. If there is relatively low variation in price changes between periods *b* and 0*,* then all subsequent differences between the two formula, given the multiplicative nature of the relationship in equation (9), will be relatively small. Indeed, the shorter the time lag between periods *b* and 0, other things being equal, the less the expected difference between the two formulas.

[8] Early empirical research in this area includes Glejser (1965), Vining and Elwertowski (1976), and Parks (1978). Most of the evidence on this relationship relies on regressions of relative price dispersion on inflation with a common finding of a positive relationship, although this finding is not universal. The main two theoretical models to explain the relationship are signal extraction models in which inflation which is not correctly anticipated by economic agents leading to erroneous output levels inflation— Hercowitz (1982), Friedman (1977) and Lastrapes (2006) —and models with price-setting behavior and price-rigidities that vary across markets—see Ball and Mankiw (1995). Other models include search cost theory—see Van Hoomissen (1988).

[9] A finding of an association between the dispersion in relative prices and their mean also applies to the coefficient of variation as a measure of dispersion (Reinsdorf, 1983 and Silver and Ioannidis, 2001).

Fourth, we depict the difference between the formulas as formula bias, as distinct from substitution bias. Balk (2009) indicates that under normal assumptions about consumer behavior, the Cobb-Douglas price index would have an upward bias relative to the theoretic Konus cost of living index, I^t_{Konus} ,$I^t_{CD} > I^t_{Konus}$. But it is clear from equation (9) that the difference between the Cobb-Douglas index, I^t_{CD} , and the Geometric Lowe, I^t_{GLo} , stems from the correlation from price changes between one period and a subsequent period: not a correlation between price and quantity changes and it is the latter that defines a substitution bias. Equation (9) which is an exact representation of the difference between the formulas makes no mention of quantity changes, and this has an intuition given their difference is in price-updating. If such effects lead to $I^t_{GLow} > I^t_{CD} > I^t_{Konus}$, then they work against I^t_{GLo}; if $I^t_{GLow} < I^t_{CD}$ then they work in favor of I^t_{GLo}.

We also examine how sensitive changes in the difference between the formulas are to changes in relative prices. For this we employ a different framework in the next section.

V. SENSITIVITY OF THE RELATIONSHIP BETWEEN COBB-DOUGLAS AND GEOMETRIC LOWE INDEXES

Let's examine the relationship between the Cobb-Douglas and Geometric Lowe indexes by taking the ratio of the two. Dividing equation (5) by equation (4) one obtains:

$$I^t_{GLo} \Big/ I^t_{CD} = \prod_{i=1}^{n} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^{b0}} \Big/ \prod_{i=1}^{n} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^{b}} \tag{10}$$

Note the following relationship between the price-updated weights and reference weights.

$$s_i^{b0} = \frac{p_i^b q_i^b \left(\dfrac{p_i^0}{p_i^b} \right)}{\sum_{i=1}^{n} p_i^b q_i^b \left(\dfrac{p_i^0}{p_i^b} \right)} = \frac{p_i^b q_i^b \left(r_i^{bo} \right)}{\sum_{i=1}^{n} p_i^b q_i^b \left(r_i^{bo} \right)} = \frac{s_i^b r_i^{b0}}{\overline{r}^{bo}}, \tag{11}$$

Where $r_i^{b0} = \dfrac{p_i^0}{p_i^b}$ and

$\overline{r}_i^{bo} = \dfrac{\sum_{i=1}^{n} p_i^b q_i^b \left(r_i^{bo} \right)}{\sum_{i=1}^{n} p_i^b q_i^b}$, the weighted arithmetic average price relative between *b* and 0.

Substituting equation (11) into equation (10) one derives the following:

$$= \prod_{i=1}^{n} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^b r_i^b / \overline{r}^{b0}} \Big/ \prod_{i=1}^{n} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^b} \tag{12}$$

Noting that $\frac{p_i^t}{p_i^0} = r_i^{0t}$ we obtain:

$$= \prod_{i=1}^{n} \left(r_i^{0t} \right)^{s_i^b r_i^b / \overline{r}^{b0}} \Big/ \prod_{i=1}^{n} \left(r_i^{0t} \right)^{s_i^b} \tag{13}$$

$$= \prod_{i=1}^{n} \left(r_i^{0t} \right)^{s_i^b r_i^b / \overline{r}^{b0} - s_i^b}$$

$$= \prod_{i=1}^{n} \left(r_i^{0t} \right)^{s_i^b \left[r_i^{b0} / \overline{r}_i^{b0} - 1 \right]}$$

It is more helpful to examine this expression in its logarithmic form:

$$= \exp \left[\sum_{i=1}^{n} s_i^b \left(r_i^{b0} / \overline{r}^{b0} - 1 \right) \ln r_i^{0t} \right] \tag{14}$$

From equation (14), we can assess the direction of difference between the two indexes. If there is no price change between *b* and 0, then there would be no difference. However, if there are price changes between *b* and 0, then differences will occur. Items that have greater than average price changes, $\left(r_i^{b0} / \overline{r}^{b0} > 1 \right)$, will have greater importance in the Geometric Lowe and items with less than average price changes, $\left(r_i^{b0} / \overline{r}^{b0} < 1 \right)$, will have less importance in the Geometric Lowe. The combined effect would be to make the $I_{GLo}^t > I_{CD}^t$ as long as there is no offsetting price drop between 0 and *t* (which would make $\ln r_i^{0t}$ negative). If there is a decline in prices between 0 and *t,* the degree of difference between the two will be determined by the strength of the price decreases in the period 0 to *t*.

If, on average, prices move in the same direction in both periods, then $I_{GLo}^t > I_{CD}^t$. If, on average, prices move in opposite directions in both periods, then $I_{CD}^t > I_{GLo}^t$ Table 2 presents a summary of the difference between the Geometric Lowe and Cobb-Douglas price indexes.

Table 2 Direction of Change and Level Differences between the Cobb-Douglas and Geometric Lowe Indexes

Relative Price Change	$r_i^{b0} \big/ \bar{r}^{b0} > 1$ and $r_i^{0t} \geq 1$	$r_i^{b0} \big/ \bar{r}^{b0} < 1$ and $r_i^{0t} < 1$	$r_i^{b0} \big/ \bar{r}^{b0} > 1$ and $r_i^{0t} < 1$	$r_i^{b0} \big/ \bar{r}^{b0} < 1$ and $r_i^{0t} > 1$
Geometric Lowe vs. Cobb Douglas	$I_{GLo}^t > I_{CD}^t$	$I_{GLo}^t > I_{CD}^t$	$I_{GLo}^t < I_{CD}^t$	$I_{GLo}^t < I_{CD}^t$

What can we deduce from this table? In normal circumstances such as with the Consumer Price Index, prices will usually rise between the weight reference period *b* and when the new weights are introduced in period 0. Then, after introduction, the CPI will normally rise over the time from period 0 to *t*. These two circumstances lead us to the shaded portion in the lower second column of Table 3 and indicates that after the new weights are introduced we will observe that $I_{GLo}^t > I_{CD}^t$.

VI. Comparison of Cobb-Douglas with the Törnqvist Price Index

While it is very difficult to construct a true cost of living index (COLI), there are several ideal target indexes that approximate the COLI. One of these, as noted previously, is the Törnqvist price index. The Törnqvist index formula is:

$$I_T^t = \prod_{i=1}^{n} \left(\frac{p_i^t}{p_i^0} \right)^{\left(s_i^0 + s_i^t\right)/2}, \text{where } s_i^0 = \frac{p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} \text{ and } s_i^t = \frac{p_i^t q_i^t}{\sum_{i=1}^{n} p_i^t q_i^t} \qquad (15)$$

We can compare the Cobb Douglas price index to the Törnqvist by taking the ratio of equation (5) to equation (15):

$$I_{CD}^t \Big/ I_T^t = \prod_{i=1}^{n} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^b} \Big/ \prod_{i=1}^{n} \left(\frac{p_i^t}{p_i^0} \right)^{\left(s_i^0 + s_i^t\right)/2}$$

$$= \prod_{i=1}^{n} \left(r_i^{0t}\right)^{s_i^b} \Big/ \prod_{i=1}^{n} \left(r_i^{0t}\right)^{\left(s_i^0 + s_i^t\right)/2} \qquad (16)$$

This expression provides a measure of the difference in levels between the I_{CD}^t and the I_T^t. If, on average, there is no change in prices from 0 to *t* ($r_i^{0t} = 1, i = 1, \ldots, n$) then there will be no difference between the two. Similarly, there is no difference if the shares in period *b* are equal to the average of the shares in periods 0 and *t* (i.e., $s_i^b = (s_i^0 + s_i^t) /2$ i=1, . . ., *n*. This is not as restrictive as requiring $s_i^b = s_i^0 = s_i^t$, i.e., price elasticity of substitution of unity across the whole time from *b* to 0 to *t*,). Under normal circumstances of consumer behavior, households shift their purchases away from items with greater price change to ones with less change. In both of these formulas quantities purchased can vary. In the Cobb Douglas they vary in an assumed predetermined manner with the quantities changing inversely with the price change to keep the shares constant. In the Törnqvist the shares change over time reflecting the actual change in shares between periods 0 and *t*. We need to know the elasticity of substitution over time between *b*, 0 and *t* to evaluate difference.

The evidence from the U.S. presented earlier indicates, on the one hand, that U.S. households have an elasticity of substitution less than unity so that the commodities with relatively higher price trends will receive less importance in I^t_{CD} than in the I^t_T. In such a circumstance, the $I^t_{CD} < I^t_T$, and the direction of the bias will be downward. If, on the one hand, some countries' households have an elasticity of substitution equal to or greater than unity, then the commodities with relatively higher price trends will receive more importance in I^t_{CD} than in the I^t_T. In such a circumstance, the $I^t_{CD} > I^t_T$ and the direction of the bias is upward.

VII. EMPIRICAL RESULTS

The empirical results and their outline here are based on preliminary work and will be developed in further versions.

The data used are the elementary aggregate indexes for the U.S. CPI and their weights over the period January 1998 to December 2009 from the U.S. Bureau of Labor Statistics. The elementary aggregate indexes are for about 211 item strata (product groups) for the large part derived using geometric means.[10] We stress that the compilation of the U.S. urban CPI is based on 211 item strata *for 38 area strata*, that is, 8018 cells. Our analysis is a counterfactual analysis of the effect of using different formula to measure the US CPI if only item weights were available, as is the case with many countries.

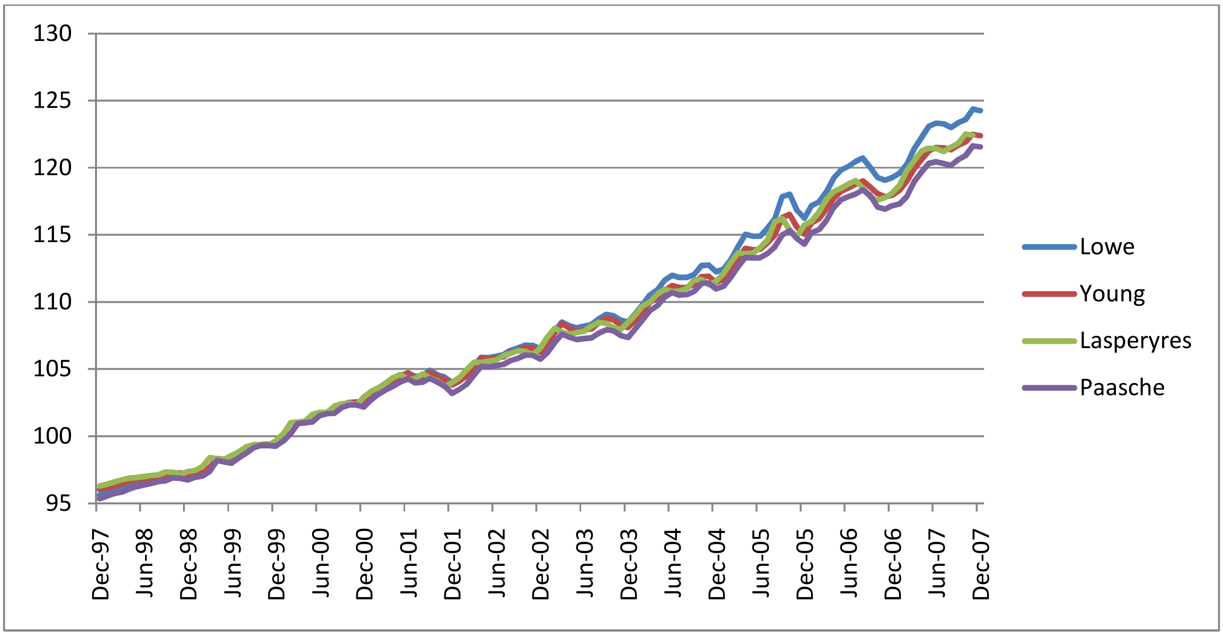
The weights used over this period are given in the table below. Following BLS procedures for their arithmetic aggregation at the higher level, they were price updated from the mean of the mid-two months of the expenditure period to the December prior to their use in the index. Note that the mean annual 1993-95 urban US expenditures for the 211 CPI item strata were the basis of the CPI weights for the four years from January 1998 through December 2001. Unlike the subsequent expenditure weights, these expenditures are (i) from a 3-year period (not a 2-year period), (ii) were used in the CPI for a 4-year period (not a 2-year period), and (iii) were price updated to December 1997 from about 2½ years (not 2 years) earlier—from the midpoint, June-July 1994.

[10] For product groups using arithmetic means see BLS, January 2008 *CPI Detailed Report*, Table 3, ff. 6 at: http://www.bls.gov/cpi/#tables.

Mean-annual expenditures	Basis of weights for:
1993-1995	Jan98-Dec01
1999-2000	Jan02-Dec03
2001-2002	Jan04-Dec05
2003-2004	Jan06-Dec07
2005-2006	Jan08-Dec09
2007-2008	Jan10-Dec11

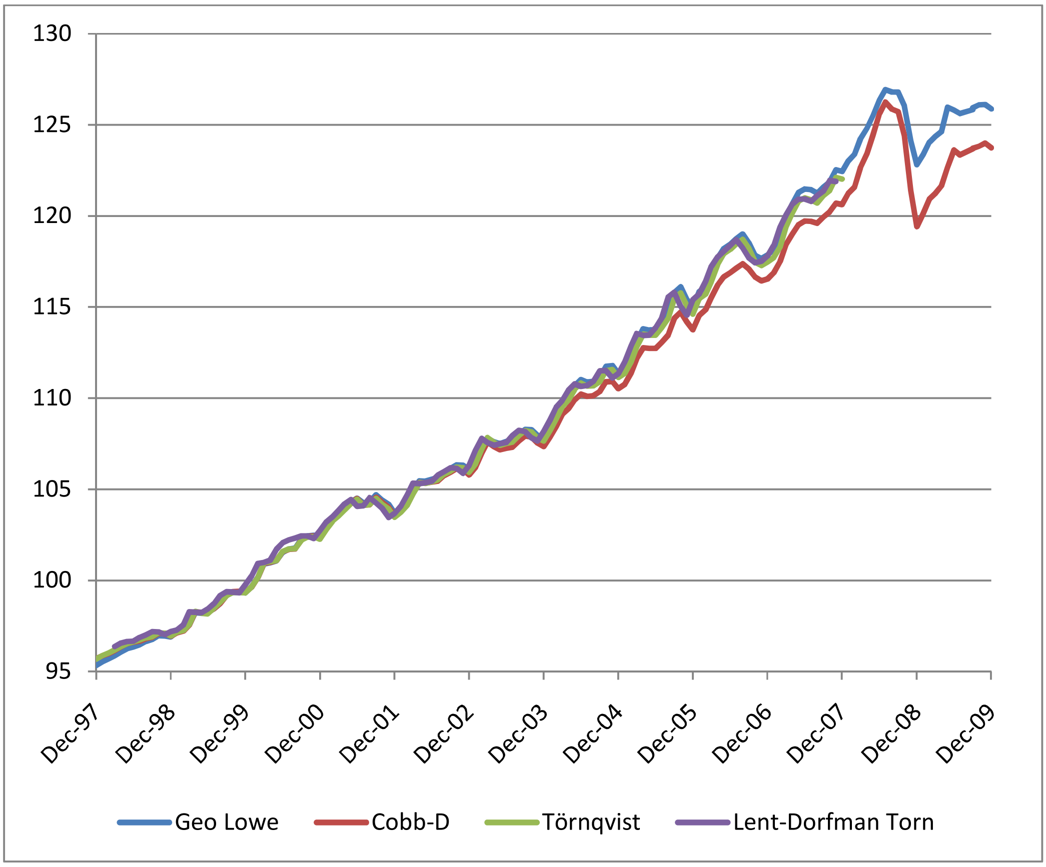
We provide some initial results her to be developed in later versions of the paper.

We start with the standard arithmetic indices:



Lowe is above Laspeyres, as expected, and Young much closer to Laspeyres and, thus, the desirable Laspeyres-Paasche interval.

We next move to our geometric counterparts and also include a Lent-Dorfman (2009) approximation to the Törnqvist index. The approximation is outlined in section III above but in this formulation is an approximation to a Törnqvist where the weight is estimated as a (4-point) moving average, lagged one period, as the ratio of (Lowe *minus* Törnqvist) to (Lowe *minus* CD)—see Lent and Dorfman (2009, page 143).

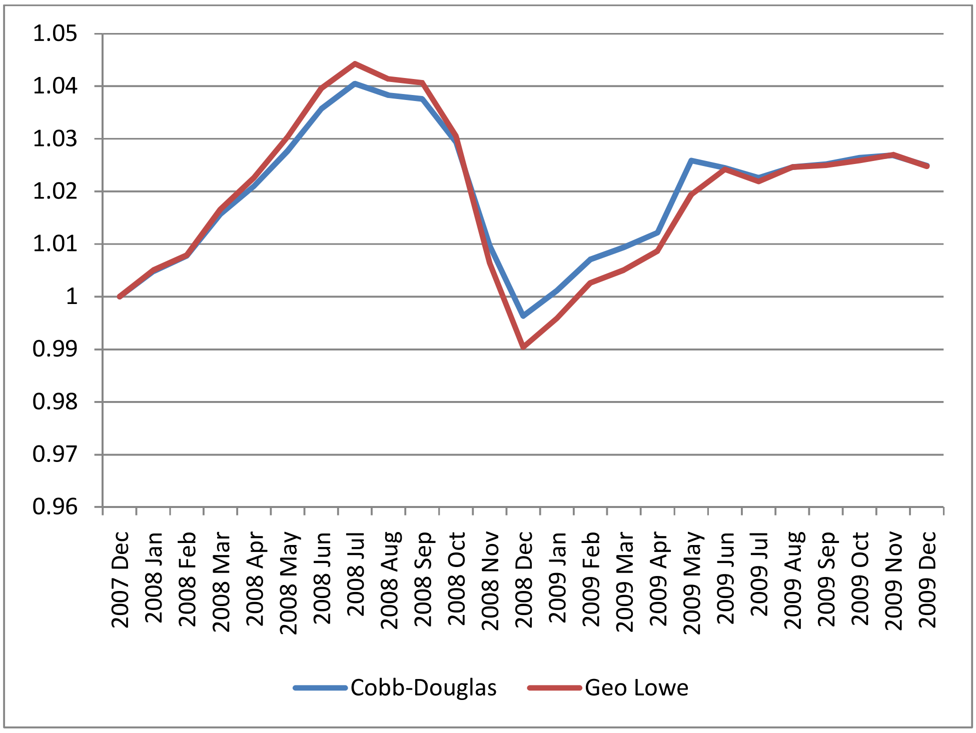


Here at the beginning of the series, all indices seem to track each other quite closely. After December 2003, the Geometric Young and Cobb-Douglas drift apart. The geometric Lowe is closer to the Törnqvist from December 2003 to December 2007, providing evidence, at least for this data set, in its favor. Lent-Dorfman appears to track Törnqvist even better. After December 2007, due to the lack of current weights, the Törnqvist is unavailable. towards the end of December the period, providing evidence, at least for this data set, in its favor. From December 2007 the difference between the Geometric Lowe and cob-Douglas increases. The decomposition in equation (9) is of interest.   This is considered in the table below. The indexes have been re-referenced to December 2007=1.0000 for convenience.

	Cobb-Douglas	Geometric Lowe	Geometric Lowe/Cobb Douglas	$\rho_{x,y}^{w^b}$	$\sigma_y^{w^b}$	$cv_x^{w^b}$	$\exp\left(\rho_{x,y}^{w^b} cv_x^{w^b} \sigma_y^{w^b}\right)$
2007 Dec	1.0000	1.0000	1.0000				
2008 Jan	1.0048	1.0050	1.0002	0.1695	0.0131	0.0821	1.0002
2008 Feb	1.0078	1.0079	1.0001	0.0687	0.0180		1.0001

2008 Mar	1.0157	1.0166	1.0010	0.3879	0.0303	1.0010
2008 Apr	1.0210	1.0226	1.0016	0.4846	0.0393	1.0016
2008 May	1.0277	1.0304	1.0027	0.5535	0.0583	1.0027
2008 Jun	1.0357	1.0396	1.0037	0.5659	0.0790	1.0037
2008 Jul	1.0405	1.0443	1.0037	0.5156	0.0875	1.0037
2008 Aug	1.0383	1.0414	1.0030	0.5011	0.0728	1.0030
2008 Sep	1.0376	1.0407	1.0029	0.5552	0.0644	1.0029
2008 Oct	1.0294	1.0305	1.0011	0.2764	0.0466	1.0011
2008 Nov	1.0097	1.0064	0.9967	-0.4754	0.0836	0.9967
2008 Dec	0.9963	0.9904	0.9941	-0.5572	0.1294	0.9941
2009 Jan	1.0012	0.9959	0.9948	-0.5264	0.1211	0.9948
2009 Feb	1.0071	1.0026	0.9956	-0.4937	0.1091	0.9956
2009 Mar	1.0094	1.0051	0.9957	-0.4709	0.1104	0.9957
2009 Apr	1.0122	1.0087	0.9965	-0.4100	0.1032	0.9965
2009 May	1.0259	1.0194	0.9937	-0.3996	0.1940	0.9937
2009 Jun	1.0245	1.0242	0.9997	-0.0544	0.0751	0.9997
2009 Jul	1.0226	1.0219	0.9994	-0.0963	0.0809	0.9994
2009 Aug	1.0246	1.0246	1.0000	-0.0054	0.0788	1.0000
2009 Sep	1.0252	1.0250	0.9998	-0.0304	0.0820	0.9998
2009 Oct	1.0264	1.0259	0.9996	-0.0609	0.0813	0.9996
2009 Nov	1.0269	1.0270	1.0001	0.0091	0.0765	1.0001
2009 Dec	1.0249	1.0248	1.0000	-0.0051	0.0787	1.0000

The correlation coefficient dictates which of the two formulas exceeds the other. Since they start at different values in the above chart, we re-reference the chart to December 2007 below. Note both indexes fall in December 2008, the Cobb-Douglas more so. Note the relatively high dispersion in relative prices at this time given by the high standard deviation combined with the high (negative) correlation. But in the latter period, the two indexes are fairly close and very low values of dispersion and correlation both contribute to this. The two parameters generally move together, high correlation with high variance, though the exceptional $\sigma_y^{w^b} = 0.19$ for May 2009 is not reflected in a very high difference due to a lower correlation coefficient.



VIII. CONCLUDING REMARKS

The two geometric formulations most readily available for compilers are the Cobb-Douglas and the Geometric Lowe price indexes. The former maintains the weight reference period expenditure shares while the latter maintains a fixed quantity basket when compiling the CPI. A formal exact decomposition of the difference between the two was derived and identified a formula bias on the part of the geometric Lowe that has the potential to lead to excess drift and volatility. Analysis of the index formulas indicates that for the normal type of price behavior, i.e., prices increasing between the weight reference period and the price reference period and prices further increasing between the price reference period and the current period, the Cobb-Douglas price index is preferred to the Geometric Lowe.

Neither of the indexes provides the ideal solution. The ultimate goal for NSOs should be to compile the target indexes, even if they can only be prepared on a retrospective basis. In such scenario, a Törnqvist index might be prepared on a lagged basis with a Cobb-Douglas index produced in the most recent periods. However, NSOs should establish a regular program for a household expenditure survey on a 3 – 5 year cycle so that the CPI weights can be updated more frequently. Such a program would permit calculation of a target index such as a Törnqvist, unfortunately only with a lag of two or more years.

The empirical results of the differences in price index number formulas for the U.S. CPI indicates that the series using a Cobb-Douglas formula is close to the preferred series using a Törnqvist formula. There is a definite upward drift in the series using the Lowe and Young formulas. This provides empirical support for the use of the Cobb-Douglas formula for current index compilation as suggested in the *CPI Manual*.

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