The Index Theory of the German Energy Regulation Agency

What is the price of an energy transmission network?

or

How to construct a PPI by combining price indices of inputs?

Peter von der Lippe

1. Introduction

The problem dealt with in this paper is to compile a price index of an asset like a large (nationwide) "grid" for energy transmission. It arose from my work (winter 2007) as a consultant of a great private owner of a gas and electricity transmission network in Germany and its main result is part of the plaintiff's statement in a (still ongoing) lawsuit against our national regulation agency, the German Federal Network Agency (FNA for short).[1]

The FNA developed a price index in order to measure the actual market value (or "net present value")[2] of an energy network. This index basically aims at reflecting the production costs one would have to pay today for such grids, which, however, actually had been erected many years ago already and which are still in use. As our German official statistics presently does not (and actually never did) provide a suitable price index for precisely this sort of assets the FNA faced the task to define such an index on her own by using official price and wage indices as building blocs for such an "aggregated" (or combined) index. The solution of the agency was an asset price index P as a weighted average of an index of wages (λ_t, representing the factor "labour")[3] and "materials"[4] (μ_t as part of the German PPI) that is

(1) $P_t = w_L\lambda_t + w_M\mu_t$ (where $w_L + w_M = 1$).

For my theoretical discussion of the FNA approach I will assume that the index λ_t is a ratio of absolute wage levels, $\lambda_t = L_t/L_0$ (and correspondingly also $\mu_t = M_t/M_0$), just like a single price relative, although this is strictly speaking not the case

The FNA claimed that they had found empirically weights $w_L = 0.4$ and $w_M = 0.6$ for the year t = 2006, however, they did not give details about how they managed to find these figures and why they had been – as they said – unable to find such weights for years other than 2006.

There was much debate on the part of the grid owners as plaintiffs about whether costs of the network producing industry adequately reflect the true economic value of energy grids, and whether the selected sub-indices of labour λ_t and materials μ_t respectively really correctly cover the kind of work or goods in question. It was argued that it is far from clear to which sector the specific λ and μ index for a certain asset (facilities and equipments of energy nets) should refer. Should wages and prices be chosen that refer for example to the sector construction (section F in the NACE classification) or rather to the more comprehensive sector of the production industries.[5] Also contentious was why capital cost was excluded from this index P.

[1] or "Bundesnetzagentur (BNetzA)" in German.

[2] "Tagesneuwert".

[3] In what follows we use the term "wage" to denote both, wages as well as salaries.

[4] The intermediate consumption meant here should usually comprise raw materials and supplies as well as energy; however, it seems to me that the FNA only took goods serving as raw material into account.

[5] The "goods producing industries" comprise in addition to F (construction) also the sections B (mining and quarrying), C (manufacturing) and D (electricity, gas, steam and air conditioning supply). The position of the Federal Statistical Office (FSO) seems to be that if doubt be the broader delimitation should be preferred. I disagree, as there are two errors, not only one, to consider, that is the error to include irrelevant activities in the broader concept on the one hand and the error of excluding relevant activities when using the narrower concept. It is not clear that in any case the first error is less severe (as the FSO seems to think).

This is not my point in this paper although much (or most) of this kind of criticism clearly sounds reasonable and worth being discussed in detail.

However, what the paper is focused on is another topic, viz. the FNA's method to take into account the undeniable (and probably also labour saving) technical progress in producing energy transmission networks. The FNA did so by simply "down weighting" one of the components (that is the weight w_L of wages λ_t) while keeping the weight w_M of the other component (μ_t) constant. So the FNA "invented" the following producer price index P_t of energy grids

(2) $$P_t = \frac{w_L}{\pi_t} \cdot \lambda_t + w_M \cdot \mu_t = \omega_L \lambda_t + w_M \mu_t$$

where $\omega_L = w_L/\pi_t$ and data were used of the official German Statistics for the indices λ_t, μ_t, and π_t. Hence the FNA simply divided w_L by an index π_t of labour productivity, in order to establish an "adjusted" weight $\omega_L = w_L/\pi_t$ (such that $\omega_L + w_M \neq w_L + w_M = 1$) for λ_t.

Notice that – unlike a paper of Lawrence and Diewert to which I will refer at the end of this paper – no attempt is made to account for *total factor* productivity instead of *labour* productivity only. Also no attempt is made to re-estimate *both* weights in certain intervals. Another important difference is that the focus in the of the Lawrence/Diewert paper (and the New Zealand regulation to which it refers) is on productivity and profitability of the "line business" which uses such assets as inputs, while the FNA formula deals with costs and productivity producers of such assets are facing.

The decisions discussed and criticized in this paper with the formula above date back to August (electricity) and September (gas) 2007 and they were taken under the regime of a rather crude cost regulation or total cost benchmarking (not yet a more sophisticated "incentive regulation"[6]).

It is interesting to note that the agency only a year later (autumn 2008) realized that her approach (of eq. 2) is equivalent to using "unit labour costs" $\kappa_t = \lambda_t/\pi_t$ instead of the official wage index[7] λ_t, because

(2a) $$P_t = w_L\kappa_t + w_M\mu_t.$$

Obviously the index P_t is a weighted mean of unit labour costs and prices of materials while it is no longer a mean of the official wage index λ_t and the PPI-price index μ_t (because $\omega_L \neq w_L$) so that it may violate the mean value property $\mu_t < P_t < \lambda_t$ or (less likely) $\mu_t > P_t > \lambda_t$.[8]

The index according to (2) or (2a) is admittedly the agency's own invention (I did not find something remotely similar anywhere else in German official statistic). In what follows I therefore show

- how the FNA itself tried to justify its approach using arguments concerning the nature of the wage index λ_t as opposed to a price index like μ_t (section 2),
- in section 3, how I think, the formula (2) could be derived and justified formally (the FNA made no attempt of this kind),
- what is tacitly implied (in terms of an underlying production function) by "correcting" the weight of one factor taken in isolation (section 4), and finally

---

[6] The paper of Lawrence and Diewert refers to such advanced methods of regulation in New Zealand.
[7] Practical aspects of the compilation of this index in Germany will be discussed in more detail later
[8] In my view violation of this mean value property seems to be a significant shortcoming of the FNA's formula. However, I admit that it might be argued that the inequations are irrelevant because what matters is κ_t and not λ_t.

- in section 5 I try to summarize my ideas and present no less than five "open questions" for which I think it should be interesting to find an answer (and where I would be very glad if I could benefit somehow from the experts of the Ottawa Group).

Given that the energy supply assets are usable over a long period in time it is clear that we have an index problem with rather long time series involved, and that both, λ_t and μ_t are represented by *a number of* indices with different base years, and which therefore have to be *linked* together. It is of course also clear and generally agreed upon that the weights w_L and w_M will vary over the period of fifty and more years which is under consideration here. I do not argue in favour of constant weights w_L and $w_M = 1 - w_L$ over such a long time. My point only is that an isolated change of one weight ($w_L \rightarrow \omega_L$) "ceteris paribus" in a composite of two indices appears objectionable as it implies a rather odd and awkward underlying production function.[9]

Although I had plenty of time to think over my critique of the FNA-formula (which I developed in winter 2007/8 on behalf of the transmission network owners)[10] and I still tend to view the formula with suspicion I have to admit that I am not quite sure whether I got it right with my critique. I have no answer to quite a few problems, not only regarding the FNA formula but also its relation to a Laspeyres or Paasche approach to the problem under consideration. That's why I am seriously interested in a discussion of my opinion about the FNA formula (2) among the index experts of the Ottawa Group. Any comments are very welcomed indeed.

2. The mandate of the regulatory authority and how the FNA justified its formula

The FNA describes its mandate as follows: "to establish fair and effective competition in the supply of electricity and gas by ensuring non-discriminatory third-party access to networks and policing the use-of-system charges levied by market players." It is said that owners of such networks enjoy a "natural monopoly" - because it is as a rule not possible to build and operate competing networks - and they may therefore be tempted to misuse market power so that regulatory agencies also serve some valuable purposes concerning the general public.

Therefore the FNA has been given the power to take binding decisions which may possibly profoundly affect price formation mechanisms and thereby competition and of course also long term investment decisions in the energy sector. Even decisions *directly* affecting prices and profitability, like the decisions discussed in this paper are within the scope of the FNA's legislation (though contestable of course).[11]

The FNA offered in principle three arguments in favour of its formula (and the different treatment of L and M in particular). The first reveals a deplorable incompetence in index theory and is not worth being discussed in detail.[12] Thus only very briefly stated here it goes as follows: wages are already measured in Euro per hour (€/h), while prices of materials are reported in Euro per unit (number of items) €/n, and so labour productivity defined in terms of

---

[9] The correct procedure would be an empirical revision of *all* weights, w_L as well as w_M in certain intervals (of say five years or annually with chainlinking) which would also result in weights for λ_t and μ_t that add up to unity for all time periods t = 0, 1, …,T. It is also common to use expenditure shares for w_L and w_M respectively.

[10] To date there is still no decision of the court and therefore I continue to be interested quite a bit in this issue.

[11] It is of course clear that regulation can well do more harm than good, if for example the regulation policy fails to constitute the (long term) correct incentives. There has been much debate about possible mis-regulations, and regulators therefore developed a number of different (and perhaps increasingly more sophisticated) approaches. However, this is not our concern here. The point here is only, how to compile a price index in a situation in which no information is given concerning the producer prices actually obtained in the market place and where it may be sensible to combine somehow official price indices measuring various cost components in order to reflect the rising level of costs the network producers are facing.

[12] I was enormously astonished realising so much ignorance about index numbers in such a big office. The FNA is quite a sizeable administrative body which I used to think should not suffer from a shortage of economists.

units per hour (n/h) – which by the way is not correct – is needed to make a wage index commensurable to μ_t and P_t (measured in €/n). The FNA believed the equation in the respective dimensions $\frac{€/h}{n/h} = \frac{€}{n}$ exists as counterpart of the division of λ_t by π_t. It obviously was not known that an index has no dimension and is not a figure expressed in absolute monetary terms (€ or $). Some doubts should have occurred to the FNA if they have had looked at the kind and variety of goods and services combined in a CPI for example, and why to date nobody ever have had the idea to search for a common quantity unit to which prices for goods like bus ride, hair cut, potatoes, beer, monthly rent of a flat or a driving license may refer.[13]

The second argument sounds a bit more sophisticated. It was argued that the labour productivity reflects a substitution process. When (more) x substitutes, or replaces (now less) y it is clear what is meant by "substitution". The FNA, however, nowhere made clear what is x and what is y in this case, that is what substitutes labour. The agency simply maintained that we now use less labour only, but she did not reflect how and at what costs this came about. It was particularly ruled out that more capital was needed for a "substitution" of labour because capital costs were – in dissent with the industry – deliberately excluded from the formula (2) for P_t. Furthermore the FNA also did not consider the fact that rising wages are already to a certain extent reflective of an increased labour productivity and finally no attempt was made to study empirically a substitution process of whichever sort in the production of energy networks.

The third and final argument was built on the idea that there is a fundamental difference or asymmetry between price indices (for goods) and wage indices (for the production factor "labour"). The FNA repeatedly argued that technical progress (materialized in a rising labour productivity) is already accounted for in official *price* indices whereas official *wage* indices are not "adjusted" accordingly so that it is left to the user to make the necessary "corrections".

This argument obviously was brought into play by Hans Wolfgang Brachinger[14] as an FNA consultant. I nowhere found anything in publications of our Federal Statistical Office (FSO) which might be viewed as supporting this argument. For me therefore the difference, though evidently most important for the FNA's reasoning seems to be a misunderstanding and I think there simply is no such difference in the index methodology.[15]

On the contrary it is generally stated *that both types of indices, price and wage indices alike should comply with the same "principle of pure price comparison"* according to which a price index should reflect only the changing prices for *the same goods* or *the same type and amount of labour* respectively. The intention is not to measure directly (or to reflect indirectly) quantities of goods/labour effectively consumed (or what is theoretically deemed necessary) but rather to isolate the *price* component of such costs.

Formula (2) or (2a) as opposed to (1), however, raises the question which of the two indicators, λ_t (wages) or κ_t (unit labour costs[16]) is the "right" measure to reflect the price of labour

---

[13] I admit that I did not expect so much ignorance concerning index numbers in such a big office like the Federal Network Agency (FNA) which I think should not suffer from shortage of economists.

[14] Brachinger was consultant of the FNA in this case as I was consultant of the other party, the energy industry.

[15] Or more precisely, I never heard of a sort of asymmetry in terms of performing quality adjustments by accounting for increased labour productivity (in the case of μ_t), and abstaining from adjustments in the case of λ_t. If price indices and wage indices were in fact fundamentally different as regards the method (for example making or not making quality adjustments) what type of methodology then would apply to an index, like the one of the FNA, which combines both index types, λ_t as well as μ_t?

[16] The difference in this context is not about wages on the one hand and a broader more inclusive aggregate total compensation of labour (cost from the point of view of *employers*), but only whether or not wages are divided by labour productivity π.

as a component of the price P_t of one unit "output" (that is a unit of energy transmission network produced in t). It has been maintained that λ_t by contrast to κ_t tends to overstate the labour component (of costs) so that λ_t is "biased" upwards, whereas κ_t is "unbiased" and "quality adjusted". Some obvious questions inspired by this assertion are: Why does official statistics publish in the case of labour two indices, a biased (and unadjusted) index (λ_t) and an adjusted one (κ_t) whereas in the case prices for goods, we only have one type of index, viz. the quality adjusted one? If λ_t is biased why do we need λ_t in addition to the unbiased κ_t? I can't see any reason for using λ_t any more if this were the case. Furthermore if λ_t, is known to be biased why don't we have any official estimates of the amount of bias and why don't we see any attempts to make the necessary corrections of λ_t on the part of official statistics?

I admit that these are questions which are more or less confined to some formal aspects of the respective indices. Another sort of considerations may emerge when the economic interpretation is concerned: is κ_t perhaps from the economic point of view the more suitable indicator in general or at least in the context of "explaining" the price P_t? This, however raises the question: for which sort of economic problem, if any, λ_t should in turn be preferred over κ_t?[17] The co-existence of two indicators, λ_t and κ_t respectively, normally suggests that each of them has its specific merits and demerits. I know for example of κ_t as an indictor of competitiveness of a country, but I have not seen yet κ_t as a component of a price index for a certain type of goods. I frankly admit, however, that I am a bit irresolute and insecure about the answer to all these questions. This certainly is one of the points (not the only one) where I would very much appreciate some helpful comments.[18]

Digression

A short remark concerning the practicalities of λ_t in Germany might be in order. The index λ_t is according to the description given by the FSO[19] an "index of agreed wages and salaries"[20] (based on about 550 selected collective agreements[21] between unions and employers associations in Germany), and it "measures - for large areas of trade and industry and for central, regional and local authorities - the average change of hourly wages and monthly salaries that are fixed by collective agreements."[22] Interestingly the FSO also states that λ_t "is an important indicator of the general development of agreed remuneration and is used, among other things, to estimate staff expenditure and costs *in long-term contractual relationships* (price clauses)."[23] The Laspeyres type index[24] is weighted "according to the share that the employees in a specific economic branch have in the total number of employees in all economic branches covered."

---

[17] As is well known κ_t is used to measure competitiveness and it may also serve as an indicator or inflationary pressure but what is the use of λ_t, and more precisely: is λ_t or κ_t the correct variable to explain P_t?

[18] I will come back to these problems at the end with my first (out of five) questions.

[19] The source of the following quotes is the FSO's "Qualitätsbericht" (quality report) of the index in question which can be seen in the Internet.

[20] It is also called "contractual wages and salaries".

[21] The index therefore is not based on special surveys of enterprises but on the registration of contracts. A "representative" selection of the relevant contracts is made such that at least 75% of the employees in the respective economic branch will be represented.

[22] It is also - unlike the unit labour costs - part of the indicators of the dissemination standard of the International Monetary Fund (IMF).

[23] Emphasis added by v.d.L. This seems to mean that in order to mirror the *long term* development of the wage-level in Germany it suffices to simply link indices of different base periods together (as is assumed for λ_t in the formula), and that there is no need for an additional "correction" by π_t or some other index.

[24] It should be noted that the FSO wrote "The index … is computed as a Laspeyres *price* index with fixed base year, i.e. the index numbers refer to the employee structures of the base year applicable" (emphasis is again mine), which seems to indicate that there is no fundamental difference between such a wage index and a price index.

It should be noted that we also have a (quarterly) "labour cost index" (which only implies another definition of the remuneration aggregate and not a division by the index of labour productivity) in Germany. This is, however, an innovation only recently introduced and thus irrelevant for the long term price index problem under consideration here.[25]

It is most unlikely that the FNA's standpoint that wage indices and price indices should be viewed as fundamentally different is correct. I also found useful statements of other Statistical Institutes in the Internet that support my view. So the Australian Office for example states that "labour price indexes measure changes over time in the price of labour unaffected by changes in the quality or quantity of work performed." It has to be ensured "that only pure price changes are reflected" and "Price-determining characteristics of the jobs are detailed in fixed pricing specifications and any changes in labour payments due to changes in the pricing specifications do not contribute to index movements.

The following are examples of changes in price-determining characteristics which are not reflected in index movements:

- changes in the nature of work performed (e.g. different tasks or responsibilities)
- changes in the quantity of work performed (e.g. the number of hours worked)
- changes in the characteristics of the job occupant (e.g. age, apprenticeship year, successful completion of training or a qualification, grade or level, experience, length of service, etc.)."

It is also interesting to see that the (Australian) office states that the non-wage part of the index which includes among other things bonuses is not fully in line with the idea of a "pure price index because bonuses tend to reflect, at least partly, changes in the quality of work performed."

3. How a formula for the index P_t could be derived?

We may derive the formula of the FNA as follows. Assume two periods 0 (base period) and t (current period) respectively, and the following prices and quantities in 0 (and t) as *absolute* figures (prices in € for example or labour in terms of hours (h) worked so that wage is €/h)

	labour		materials*		final product	
	0	t	0	t	0	t
quantity	B_0	B_t	V_0	V_t	X_0	X_t
price	L_0	L_t	M_0	M_t	p_0	p_t
price index	$\lambda_t = L_t/L_0$		$\mu_t = M_t/M_0$		$p_t = P_t/P_0$	

* intermediate consumption
** "quantity" is here the number of "network units" which may sound a bit odd (the exact meaning of "units" here may be left open)

Then equality of sales-value (revenues) and total costs in period 0 means

(3) $X_0p_0 = B_0L_0 + V_0M_0$ and correspondingly in t we have

(4) $X_tp_t = B_tL_t + V_tM_t$.

From this it follows that

(5) $$p_t = \frac{1}{X_t/B_t}L_t + \frac{1}{X_t/V_t}M_t$$

and using labour productivity $X_t/B_t = \Pi_t$ and productivity of materials $N_t = X_t/V_t$ we get

[25] I found, however, in the Internet a description of the "OECD System of Unit Labour Cost Indicators" where Unit labour costs (ULCs) are defined in way consistent with our κ_t here. These indicators expressly intend to represent "a link between productivity and the cost of labour in producing output." Among the difference between ULCs and λ_t the OECD mentioned that adjustment for the self employed persons, but no hints were given as to the difference regarding uses and interpretation of λ_t and κ_t respectively.

(5a) $$p_t = \frac{1}{\Pi_t}\lambda_t L_0 + \frac{1}{N_t}\mu_t M_0 = \frac{1}{\Pi_t}\cdot L_t + \frac{1}{N_t}\cdot M_t$$

Division of (5a) by p_0 gives $P_t = \frac{p_t}{p_0} = \frac{1}{\Pi_t}\cdot\frac{L_0}{p_0}\cdot\lambda_t + \frac{1}{N_t}\cdot\frac{M_0}{p_0}\cdot\mu_t$

We now also introduce indices of the *change* of productivity viz. $\pi_t = \Pi_t/\Pi_0$ and $\nu_t = N_t/N_0$ and upon defining $w_L = \frac{L_0}{\Pi_0 p_0} = \frac{B_0 L_0}{X_0 p_0}$ and $w_M = 1 - w_L = \frac{M_0}{N_0 p_0} = \frac{V_0 M_0}{X_0 p_0}$ we get

(6) $$P_t = \frac{w_L}{\pi_t}\cdot\lambda_t + \frac{w_M}{\nu_t}\cdot\mu_t = \omega_L\lambda_t + \omega_M\mu_t.$$

Note that the weights $\omega_L = w_L/\pi_t = B_0L_t/P_0X_t$ and $\omega_M = w_M/\nu_t = M_0V_t/P_0X_t$ do not add up to unity. With unit-labour costs (ULC) $\kappa_t = \lambda_t/\pi_t$ and the (somewhat unfamiliar) unit costs of materials (UMC) $\theta_t = \mu_t/\nu_t$ we have, however, a weighted average

(6a) $P_t = w_L\kappa_t + w_M\theta_t$.

The difference between this formula and the FNA-index (2) is that the FNA implicitly assumed $\nu_t = 1$ or $N_t = N_0$ (or equivalently $\theta_t = \mu_t$).

If and only if $\nu_t = 1$ equations 6 and 6a reduce to (2) and (2a) respectively.

We now see that a choice can be made between a number of aggregated (or "mixed") indices combining price indices for inputs, λ_t and μ_t respectively or (equivalently) UCL and UCM indices (that is κ_t and θ_t). Table 1 summarizes the possibilities. It can be seen that the FNA solution seems to be a sort of "hybrid" approach, applicable only in a special case where the assumption $\nu_t = 1$ is justified.[26]

Table 1

type of approach	weights	variables
1a. Laspeyres	constant weights w_L, $w_M = 1 - w_L$	λ_t and μ_t
1b. Paasche	$w_{L^*} = L_0B_1/(L_0B_1 + M_0V_1)$ for λ_t and $w_{M^*} = M_0V_1/(L_0B_1 + M_0V_1)$ for μ_t	λ_t and μ_t
1c. updated weights	expenditure shares of period 1 (see below)*	λ_t and μ_t
2. eq. 6	variable weights $\omega_L = w_L/\pi_t$ and $\omega_M = w_M/\nu_t$ where $\omega_L + \omega_M \neq 1$	λ_t and μ_t
2. eq. 6a	constant weights w_L, $w_M = 1 - w_L$	unit costs κ_t and θ_t
3. FNA (hybrid weights)	ω_L (for λ_t, like 2) and w_M (for μ_t, like in 1) or	λ_t and μ_t
3. FNA (hybr. variables)	equivalently w_L and w_M	κ_t and μ_t

* if the example in the following section were carried out a bit further than just for t = 1 only it might be interesting to study the chain index approach.

We may also introduce updated weights $w_L^t = \frac{L_t}{\Pi_t p_t} = \frac{B_t L_t}{X_t p_t}$ and $w_M^t = \frac{M_t}{N_t p_t} = \frac{V_t M_t}{X_t p_t}$ so that $w_L = w_L^0$ and $w_M = w_M^0$ and calculate indices (perhaps a chain index) using constantly updated weights. It can easily be seen that the weights w_L^t and w_L^0 (and correspondingly w_M^t and w_M^0) are related as follows

[26] In the following section our task will be to unfold the implications and consequences of this very assumption.

(7) $$w_L^t = w_L^0 : \left(\pi_t \cdot \frac{P_t}{\lambda_t}\right) = w_L^0 \cdot \frac{\kappa_t}{P_t}\text{, and}$$

(7a) $$w_M^t = w_M^0 : \left(\nu_t \cdot \frac{P_t}{\mu_t}\right) = w_M^0 \cdot \frac{\theta_t}{P_t}.$$ [27]

This is not the kind of "updating" carried out in (6) where w_L is simply divided by π_t and w_M by ν_t in order to get $\omega_L = w_L/\pi_t$ and $\omega_M = w_M/\nu_t$.[28] While the Laspeyres approach (index P_L) consists in keeping the expenditure weights constant a Paasche index (P_P) would use weights

$$w_L^P = \frac{L_0 B_t}{L_0 B_t + M_0 V_t} \text{ and } w_M^P = \frac{M_0 V_t}{L_0 B_t + M_0 V_t} \text{ resulting in}$$

(8) $$P_P = \lambda_t w_L^P + \mu_t w_M^P = \frac{L_t B_t + M_t V_t}{L_0 B_t + M_0 V_t} = \frac{X_t p_t}{L_0 B_t + M_0 V_t}.$$

This shows that P_P will differ from the "correct" index (provided that our considerations we started with (3) and (4) are indeed correct) P_t (of eq. 6) to the extent to which the denominator $L_0B_t+M_0V_t$ differs from

(8a) $$X_t p_0 = L_0 \frac{\lambda_t}{P_t} B_t + M_0 \frac{\mu_t}{P_t} V_t = L_0^* B_t + M_0^* V_t \neq L_0 B_t + M_0 V_t\text{, recall } P_t = \frac{p_t}{p_0}.$$

In a similar vein, we see that the Laspeyres index $P_L = \lambda_t w_L + \mu_t w_M$ differs from P_t in (6) to the extent that the numerator of P_L that is $L_tB_0 + M_tV_0$ differs from X_0p_t, because

(9) $$P_L = \frac{L_t B_0 + M_t V_0}{X_0 p_0} \text{ and}$$

(9a) $$X_0 p_t = L_t \frac{P_t}{\lambda_t} B_0 + M_t \frac{P_t}{\mu_t} V_0 = L_t^\# B_0 + M_t^\# V_0 \neq L_t B_0 + M_t V_0.$$

We let this point unresolved here, that is leave open what is the correct index, P_t according to (6) or the familiar indices P_L and P_P.[29]

4. The implicit production function of the FNA formula

At this point I used to ask which type of production function might be tacitly assumed in deriving the FNA - formula (2) where a rising labour productivity $\pi_t > 1$ and at the same time a constant productivity of materials $\nu_t = 1$ was assumed. The FNA definitely was not aware of these implications and I suppose, the assumptions made were more or less unsubstantiated assertions only. I only later realised that already some simple transformation of definitions are able to demonstrate clearly enough the implications of the assumption $N_t = N_0$ which implies

(10) $$\frac{X_t}{X_0} = \frac{V_t}{V_0},$$

and this equation simply means, that output growth is determined solely by more or less input of materials, while labour input B_t as well as labour productivity π_t are completely irrelevant.

[27] It follows from (6a) that the sum of these weights adds up to unity.

[28] We will come back to this point at the end of the paper (section 5 discussing Question 4).

[29] If P_t were in fact correct, how could we justify the traditional indices P_L and P_P or perhaps even more so Fisher's index $P_F = (P_LP_P)^{1/2}$? These are the indices we are used to regard as the "correct" indices.

Moreover as V_0 and X_0 are constants $N_0 = X_0/V_0 = c_1$ is a constant as well, then what we found is that $\nu_t = 1$ simply implies the following rather odd production function

(11) $X_t = N_0 V_t = c_1 V_t$.

Thus both, B_t and π_t are irrelevant for the output X_t these variables will, however, together with input prices L_t and M_t influence the price p_t of the output. Using (11) we get with

(5b) $$p_t = \frac{1}{\Pi_t} \cdot L_t + \frac{1}{N_0} \cdot M_t$$

instead of (5a), an equation which basically serves the same purpose as (2).

To make the implications of the rather restrictive production function (11) clearer it may be useful to consider now an illustrative numerical example. The point in this example is that here the FNA formula in fact perfectly predicts the true price change, while both, the Laspeyres as well as the Paasche price index seem to be inadequate (because biased upwards).

Example 1

	X	p	X p	B	L	V	M	X/B = Π	X/V = N
0	100	30	3000	60	20	60	30	100/60 = 1.67	100/60 = 1.67
t = 1	150	40	6000	50	40	90	44.44*	150/50 = 3	150/90 = 1.67

* or more precisely 400/9

Evidently output increased by 50% (as $X_0 = 100$ rises to $X_t = 150$) just like the intermediate consumption V did (because $V_t/V_0 = 90/60 = 1.5$). Note that $w_L = 0.4$ and $w_M = 0.6$ and the prices changed as follows $p_t = 40/30 = 1.333$, $\lambda_t = 40/20 = 2$ and $\mu_t = 44.44/30 = (400/9)/30 = 400/270 = 1.48148$. Labour productivity rose by 80% ($\pi_t = 3/1.667 = 1.8$). It can easily be seen that under such conditions (as in fact $\nu_t = 1$) the FNA-formula is correct because

$$P_t = \frac{0.4}{1.8} \cdot \lambda_t + 0.6 \cdot \mu_t = \frac{4}{9} + \frac{8}{9} = \frac{12}{9} = 1.333$$

However, the mean value condition is violated because $1.333 < \mu_t = 1.48 < \lambda_t = 2$. By contrast the Laspeyres approach would yield $P_t^L = 0.4\lambda_t + 0.6\mu_t = 12/9 = 1.6889$ (P_t then should be 50.67 instead of 40).[30] Using updated weights $w_L^1 = 1/3$ and $w_M^1 = 2/3$ for the price relatives λ_t and μ_t would result in 1.6542 (is again not correct but in between $\mu_t = 1.4815$ and $\lambda_t = 2$).[31]

Summary of the results of example 1

type of approach	results	comments
1a. Laspeyres	760/450 = 1.68889	both indices > 1.333,
1b. Paasche	60/37 = 1.62162	Fisher's index: 1.6549 >
1c. updated weights	$w_L^1 = 1/3 \rightarrow$ index: 1.65432	and μ_t
2. eq. 6	$\omega_L = 2/9$ and $\omega_M = w_M = 0.6$ $\rightarrow$ index: 4/3 = 1.333	eq. 6 and 6a reduce to eq. 2 and 2a so the results are the same in the FNA approach
2. eq. 6a	$\kappa_t = 10/9 = 1.111$ (instead of $\lambda_t = 2$) and $\theta_t = \mu_t \rightarrow$ index: 4/3 = 1.333	

The underlying production function of the type (11) is in this first example

[30] The Laspeyres approach gives $(B_0L_t+V_0M_t)/(B_0L_0+V_0M_0) = 1.689$, and the Paasche formula in this case is given by $(B_tL_t+V_tM_t)/(B_tL_0+V_tM_0) = 1.6217$

[31] The index using updated weights also lies within the interval spanned by P_t^P and P_t^L and is only a bit smaller than the Fisher index $P_t^F = 1.655$.

(11a) $X_t = f_1(B_t, V_t) = (5/3)V_t = 1.667V_t = c_1V_t$.

so that X_t simply is proportional to V_t. It follows from above that for any other production function than just $X_t = c_1V_t$ the FNA formula will *not* hold true.

We may start with a variant of example 1 in which the underlying function now is

(12) $$X_t = \frac{5}{3}V_t + \frac{7}{3}B_t = c_1V_t + c_2B_t$$

instead of (11a). It is only with respect to c_2B_t that (12) differs from (11a). We leave the shaded parts in the table for example 1 unchanged and therefore also the expenditures

(3a) $X_0p_0 = B_0L_0 + V_0M_0 = 1200 + 1800 = 3000$ and

(4a) $X_tp_t = B_tL_t + V_tM_t = 6000$,

so that also the (base period) weights $w_L = 0.4$ and $w_M, = 0.6$ remain unchanged. Hence the modified assumptions are:

Example 2

	X	p = Xp/X	X p	X/B = Π	X/V = N
0	240	12.5	3000	240/60 = 4	240/60 = 4
t = 1	880/3*	20.45**	6000	293.33/50 = 5.867	293.22/90 = 3.259

* = 293.33
** rounded

Note that due to the different production function it is primarily X_0 and X_t (and therefore also the productivities) which differ from the first example. We get $X_0 = 60c_1 + 60c_2 = 240$ and $X_t = 50c_1 + 90c_2 = 880/3 = 293.33$ instead of $X_0 = 100$ and $X_t = 150$ respectively. The price change amounts to $P_t = p_t/p_0 = 20.45/12.5 = 1.63636$ and we get exactly this result for P_t using (6) or (6a).

Productivity changed as follows: $\pi_t = 5.867/4 = 1.4667$ and $\nu_t = 3.259/4 = 0.8148$ which explains the difference between weights (ω_L, ω_M) and weights (w_L, w_M) on the one hand and unit costs (κ_t, θ_t) and factor price indices (λ_t, μ_t) on the other.

Now consider the FNA-formula. The part $w_M\mu_t = 8/9 = 0.888$ remains unchanged (compared to example 1) and therefore we get according to (2) and (2a)

$$P_t^{FNA} = (0.4/1.4667)\cdot 2 + 0.8888 = 1.434343 < 1.63636.$$

The formula understates the price movement because it does not take into account that $\theta_t = \mu_t/\nu_t = 1.4815/0.8148 = 1.8181 > \mu_t = 1.4667$ because $\nu_t = 0.8148 < 1$ (productivity of materials decreased as productivity of labour increased).

The indices of Laspeyres and Paasche are functions only of quantities (B and V) and prices (L and M) of the two production factors. None of these figures has been changed. Thus there is no difference between the two examples in this respect. The same applies to the updating of weights. However, the results now come much closer to the correct figure 1.63636 than in the first example where a somewhat awkward production function (11a) was assumed.

Summary of the results of example 2

type of approach	results
1a. Laspeyres	same results as in example 1; however, they all now have more resemblance with the correct figure 1.63636
1b. Paasche	
1c. updated weights*	
2. eq. 6	$\omega_L = 0.272727$ and $\omega_M = 0.736363 \rightarrow$ index: 1.63636
2. eq. 6a	$\kappa_t = 12/8.8 = 1.3636$ ($\lambda_t = 2$) and $\theta_t = 2$ ($\mu_t = 1.4667$) $\rightarrow$ index: 1.63636
3. FNA	1.434343 < 1.63636

* The updated weights amount to 1/3 and 2/3 instead of 0.2727 and 0.7363 respectively in eq. 6, so that $\lambda_t = 2$ gets a slightly higher and $\mu_t = 1.4815$ a slightly lower weight in (6) than in the updated index. This explains that 1.65432 > 1.63636.

We now come to the nature of the underlying production functions (11a) and (12) respectively. It is clear that they both are linear homogeneous production functions (with constant returns to scale) which means that (assuming a sufficiently small interval between the points in time 0 and t)

(13) $$\begin{bmatrix} B_0 & V_0 \\ B_t & V_t \end{bmatrix} \cdot \begin{bmatrix} \partial X/\partial B \\ \partial X/\partial V \end{bmatrix} = \begin{bmatrix} X_0 \\ X_t \end{bmatrix}$$

or equivalently

(13a) $$dX = \frac{\partial X}{\partial B} dB + \frac{\partial X}{\partial V} dV .$$

With the figures of example 1 we get $\begin{bmatrix} 60 & 60 \\ 50 & 90 \end{bmatrix} \cdot \begin{bmatrix} \partial X/\partial B \\ \partial X/\partial V \end{bmatrix} = \begin{bmatrix} 100 \\ 150 \end{bmatrix}$ such that the marginal productivity of labour turns out to be zero $\partial X/\partial B = \frac{\begin{vmatrix} 100 & 60 \\ 150 & 90 \end{vmatrix}}{\begin{vmatrix} 60 & 60 \\ 50 & 90 \end{vmatrix}} = \frac{0}{2400} = 0$ as opposed to the positive $\partial X/\partial V = \frac{4000}{2400} = \frac{5}{3}$. The result of the second example sounds much more reasonable. The difference is due to the vector $\begin{bmatrix} X_0 \\ X_t \end{bmatrix} = \begin{bmatrix} 240 \\ 266{,}67 \end{bmatrix}$ instead of $\begin{bmatrix} 100 \\ 150 \end{bmatrix}$.

Thus we get in this case $\partial X/\partial B = \frac{5600}{2400} = \frac{7}{3} = c_1$ and $\partial X/\partial V = \frac{4000}{2400} = \frac{5}{3} = c_2$ which of course already follows from (12). Given the production function (12) the formula (6) and (6a) respectively seem to be correct as they correctly account for the rising labour productivity ($\pi_t = 1.4667$) and declining productivity of materials ($\nu_t = 0.8148$). Not only the FNA formula (2) but also the Laspeyres and Paasche formulas seem to be wrong. Moreover they seem to be unable to correctly reflect the difference between the two examples, 1 and 2, although the change of the output price was quite different (63.6% in example 2 compared to only one third in example 1).

In order to consider a more general production function I also assumed the Cobb Douglas function with constant returns to scale (linear homogeneous) $X_t = cB_t^{0.7} V_t^{0.3}$ where c = 5/3 and alternatively a function with disembodied technical progress at a constant rate of 0.4%

such that $X_t^* = cB_t^{0.7}V_t^{0.3}e^{0.004t}$. I chose figures for the inputs in such a way that the output remains approximately constant (≈ 100).

Example 3: Assumptions

t	B_t	L_t	V_t	M_t	total costs	Output X_t (X_t*)	price p (for X*)
0	60	20	60	30	3000	100 (100)	30 (30)
10	55	25	70	32	3490	98,54 (102.566)	35,42 (34.027)
20	52	30	85	35	4535	100,43 (108.798)	45,15 (41.683)
30	50	35	90	38	5170	99,40 (112.077)	52,01 (46.129)

Of course output is higher in the case X* of technical progress. This also makes productivities Π_t and N_t higher and prices p_t lower than in the case of no progress (i.e. the case of X). Furthermore N_t is constantly decreasing (so $\nu_t \neq 1$) whereas Π_t is increasing.[32] However, the most interesting result is that the FNA formula consistently understates the price movement which can be seen from comparing columns 3 and 2 or 6 and 5 in the following table. This understating applies also to the Laspeyres approach using constant weights w_L = 1200/3000 = 0.4 and w_M = 0.6 throughout the interval under consideration.[33] The resulting price movements are summarised in the following table:

Example 3: Results (concerning price movement)

	without technical progress (X)			with technical progress (X*)		
t	$P_t = p_t/p_0$	FNA (2)	P_L *	$P_t = p_t/p_0$	FNA (2)	P_L *
(1)0	(2)	(3)	(4)	(5)	(6)	(7)
10	1.180	1.105	1.14	1.134	1.087	1.14
20	1.505	1.218	1.30	1.389	1.178	1.30
30	1.734	1.347	1.46	1.538	1.280	1.46

* $P_L = 0.4\lambda_t + 0.6\mu_t$ Note that we again have the same result for P_L although the product (X* compared to X) and the productivities are quite different

More generally, for a Cobb-Douglas function like $X_t = cB_t^{0.7}V_t^{0.3}$ the assumptions $\pi_t > 1$ and at the same time $\nu_t = 1$ turn out to be contradictory. Assuming $\nu_t = 1$ is equivalent to require $\left(\frac{B_t}{V_t}\right)^{0.7} = \left(\frac{B_0}{V_0}\right)^{0.7}$ or simply $\frac{V_t}{B_t} = \frac{V_0}{B_0}$. On the other hand, under such conditions $\Pi_t = c\left(\frac{V_t}{B_t}\right)^{0.3}$ cannot differ from $\Pi_0 = c\left(\frac{V_0}{B_0}\right)^{0.3}$ because this would require $\frac{V_t}{B_t} > \frac{V_0}{B_0}$. Rather $\pi_t > 1$ and therefore $\frac{V_t}{B_t} > \frac{V_0}{B_0}$ implies $\frac{B_t}{V_t} > \frac{B_0}{V_0}$ that is $\nu_t < 1$. It can easily be verified that in example 3 we have $\pi_{10} = \left(\frac{V_{10}}{B_{10}} : \frac{V_0}{B_0}\right)^{0.3} = \left(\frac{70}{60} : \frac{55}{60}\right)^{0.3}$ = $(1.2727)^{0.3}$ = 1.075 > 1, and ν_{10} = $(1/0.2727)^{0.7} = 0.7857^{0.7}$ = 0.8447 < 1. So in general we cannot make assumptions about ν_t irrespective of what should hold for π_t. The two productivities are not unrelated as the FNA seems to believe.

[32] In fact ν_{10} = 1.41, ν_{20} = 1.181, and ν_{30} = 1.10 while π_{10} = 79, π_{20} = 1.93 and π_{30} = 1.99 (in the case of X).
[33] w_L decreased subsequently from 0.358 in t= 10 to 0.344, and 0.338 in t = 20 and t = 30.

Finally I would like to add the following consideration I owe to two colleagues of the University in Kassel (Germany).[34] Assuming $X_t = V_t^{\alpha}(A_t B_t)^{1-\alpha}$ and the turnover $Y_t = X_t p_t$, where $A_t = A_0(1+g)^t$ denotes labour saving technical progress.[35] Furthermore compensation of the inputs is given by $L_t = (\partial X_t / \partial B_t) p_t$ and $M_t = (\partial X_t / \partial V_t) p_t$ then factor shares are constant α and 1-α respectively (so that the progress is "neutral"). With constant output $X_0 = X_t = X$ and $\nu_t = \nu_0$ (for all t),[36], we get decreasing labour input and increasing labour productivity and wages as follows

$\pi_t = \pi_{t-1}(1+g)$ as well as $L_t = L_{t-1}(1+g)$ and $B_t = B_{t-1}/(1+g)$ for $t \geq 1$.

Another consequence is $B_t L_t = B_0 L_0$ and $V_t M_t = V_0 M_0$ and also $X_t p_t = X_0 p_0 = B_0 L_0 + V_0 M_0$ so that the price is constant as well (which means $P_t = 1$).

To sum up, we arrive at $\pi_t > \pi_{t-1} > \ldots > \pi_0$ and $\nu_t = \nu_{t-1} = \ldots = \nu_0$, however, the price is high because X, V, M, B and L all remain constant and the FNA-formula (2) applies but it specialises to the unrealistic and rather uninteresting identity $P_t = 1$ (as also $\mu_t = 1$ and $\lambda_t = \pi_t$).

So, after all I am inclined to conclude that the FNA concept of an asset price index cannot be justified. It seems to be consistently biased downward because it refrains from increasing the weight for the prices of materials, which, however ought to be done, when on the other hand the weight of labour is constantly reduced.

5. Conclusions and questions

To sum up, what seems to be the most interesting and perhaps also the most critical feature of the FNA-formula for an index combining two cost-components is arguably the isolated and quasi automatic (using the index of labour productivity π_t for this purpose) "correction" of one of the two weights only.[37] Much of what the FNA said in order to justify its formula was bluntly wrong (for example the alleged need to make prices of labour and of materials commensurable regarding the dimensions [units of measurement]). Of course the formula as a result of an inconclusive and perhaps unconvincing reasoning can still well be acceptable. However, I think this does not apply in this case.

I tried to demonstrate this by showing what the formula might mean in terms of an implicit production function and the core of my critique of the FNA approach was that this function is odd and implausible in that it implies a zero marginal productivity of labour and an output independent of the amount of labour input.

The FNA formula (2) amounts to taking unit labour costs κ_t instead of an index of wages λ_t in combination with fixed[38] weights w_L and w_M for possibly quite a long interval in time. I am irresolute and somewhat bewildered as to the comparative uses, advantages and interpretations of the two sorts of indices, λ_t and κ_t so I am interested in the answer to:

Question 1*: For which purposes is the index κ_t of unit labour costs (ULC) preferable to an index of wages λ_t? This may include some sub-questions: Does λ_t in general overstate the cost-push exerted by the factor labour, so that κ_t is the better gauge for the purpose of*

[34] Prof. Dr. F. Eckey and Prof. Dr. R. Kosfeld.

[35] Intermediate consumption here plays the part of capital.

[36] As X and ν are constant the amount of input V is constant as well ($V_t = V_0$) and so is its price $M_t = M_0$ while labour input B_t is decreasing at the constant rate 1/(1+g) which justifies the term "labour saving". Thus the "intensity" (factor proportion) $i_t = V_0/B_t$ and labour productivity π_t is rising. More precisely, we get $\pi_t = (1+g)\,\pi_t$ for all t = 0, 1, …, T.

[37] Another point mentioned above was the possibly fallacious substantiation the FNA gave for its formula (see section 2), however, is in itself not reason enough to reject it.

[38] I know that to use fixed weights in such a situation may in itself already be objectionable.

compiling a "cost index" as proxy for a PPI? And if so, what use can be made of λ_t after all? In particular: is there a fundamental methodological difference between price indices on the one hand and wage indices (λ_t) on the other?

In our attempt to find a justification for the FNA formula (the FNA's economists themselves, however, never tried to "derive" their formula from some sort of theoretical model) we started with equations 3 and 4. The idea was to set the sales-value $X_t p_t$ in all periods t equal to the total costs $B_t L_t + V_t M_t$ and to proceed from this starting point to a formula for $P_t = p_t/p_0$.

***Question 2**: Is the model of equations (3) and (4) I have chosen in order to justify the FNA-formula (2) and its underlying more general formula (6) a correct approach or should we choose a fundamentally different way to attain a price index from cost components (for example to set the price p_t equal to the marginal costs - assuming perfect competition - rather than $X_t p_t$ to the total costs)*[39]

Given that (3) and (4) may indeed establish a legitimate concept of price formation the price index formula (6) seems to be a correct implication. The interesting (and amazing) point then is for me that the result differs from the traditional price-index approach (Laspeyres, Paasche, Fisher) of weighted averages of price relatives (or sub-indices).

***Question 3**: Given that (6) is indeed a correct implication of a "model" combining input prices and output prices*[40] *how can we explain that we get quite different results with indices of Laspeyres, Paasche and Fisher (P_L, P_P, P_F), and perhaps also a chain index? Compared to the "model" leading to (6), what then it the apparently different idea (or reasoning) behind a P_L - P_P - or chain-index type cost-index $P(\lambda_t, \mu_t)$? Moreover: how can we explain that such indices don't yield different results in evidently quite different (as regards the production function and thus also the cost function) situations? If there in fact are some different choices possible in this case: What is the correct model explaining output prices p_t in relation to input prices λ, and μ_t?*

A closely related problem and phenomenon then is the rationale behind different weights. Indices like P_L and P_P can be viewed as using different sets of expenditure weights for the input-price-indices, λ_t and μ_t respectively. We also considered an index using updated weights which subsequently may be chained. What seems to be intriguing is to study the difference between these weights. Using (7) and (7a) and the definitions of the variables we get

(14) $$w_L^t = w_L^0 \cdot \frac{\kappa_t}{P_t} = \omega_L \cdot \frac{\lambda_t}{P_t} = \omega_L \cdot \frac{1}{P_0} \cdot \frac{\lambda_t}{p_t}, \text{ and}$$

(14a) $$w_M^t = w_M^0 \cdot \frac{\theta_t}{P_t} = \omega_M \cdot \frac{\mu_t}{P_t} = \omega_M \cdot \frac{1}{P_0} \cdot \frac{\mu_t}{p_t}.$$ [41]

For the difference between ω-weights and w^t-weights some sort of index of "real" input prices λ_t/p_t and μ_t/p_t seem to be responsible. How can we explain this? How can we explain in particular a Laspeyres chain (chL) index approach with constantly changing expenditure weights of the previous period

(15) $$p_T^{chL} = \left(w_L^0 \frac{L_1}{L_0} + w_M^0 \frac{M_1}{M_0} \right)\left(w_L^0 \frac{L_1}{L_0} + w_M^0 \frac{M_1}{M_0} \right) \ldots \left(w_L^{T-1} \frac{L_T}{L_{T-1}} + w_M^{T-1} \frac{M_T}{M_{T-1}} \right)$$

[39] In this case, however, I can't yet see clearly how "productivities" can be introduced which on the other hand is highly desirable as just these productivities play an important part in the considerations of the FNA regarding the necessary updating of weights. It is of course clear, that a cost function is related to a production function and relates costs to input quantities, B_t and V_t.

[40] I mean the consideration I started with equations (3) and (4) and which lead to (6).

[41] It follows from (6a) that the sum of these weights adds up to unity.

$$= \left(\frac{w_L}{\pi_0}\lambda_1 + \frac{w_M}{\nu_0}\mu_1\right)\cdot\frac{1}{P_1}\left(\frac{w_L}{\pi_1}\lambda_2 + \frac{w_M}{\nu_1}\mu_2\right)\cdot\frac{1}{P_2}\left(\frac{w_L}{\pi_2}\lambda_3 + \frac{w_M}{\nu_2}\mu_3\right)\ldots\cdot\frac{1}{P_{T-1}}\left(\frac{w_L}{\pi_{T-1}}\lambda_T + \frac{w_M}{\nu_{T-1}}\mu_T\right)$$

($w_L = w_L^0$ and $w_M = w_M^0$) as opposed to the direct Laspeyres index $P_L = w_L\lambda_T + w_M\mu_T$ or the direct index according to (2) $\frac{w_L}{\pi_T}\lambda_T + \frac{w_M}{\nu_T}\mu_T$.

If the chL-index is regarded as the only sensible index, then one could also criticise the FNA formula of course for other reasons than only the assumption $\nu_1 = \nu_2 = \ldots = \nu_T = 1$.

So in addition to the possibly unacceptable FNA-formula one might think of quite a few reasonable price index-formulas somehow combining input-prices This raises the

> ***Question 4****: which of the index formulas (perhaps one which may be quite different from formulas introduced here in this paper) can be recommended instead of the FNA-formula.*

As aforesaid the crucial and critical point in the FNA's reasoning, however, seems to be the assumptions made concerning the "productivity" of the inputs. As output X is a function of both inputs, B and V such that X = f(B, V) assumptions made for labour productivity $\pi = X/B$ and those for the productivity of materials (or in a broader sense, intermediate consumption including energy) $\nu = X/V$ cannot be independent of one another since $\frac{\pi_t}{\nu_t} = \frac{V_t/V_0}{B_t/B_0}$. I tried to work out the implications of this relationship but I am not sure that I saw all of them.

> ***Question 5****: The FNA formula refers to the special case of constant productivity of materials $\nu_t = 1$. I think that this situation amounts to assuming a production function with a vanishing marginal productivity of labour $\partial X/\partial B$ which seems to be a rather odd function, suggesting that the FNA formula seems to be seriously misconceived. I tried to demonstrate this with a linear homogenous production function and some numerical examples. Is this conclusion correct and can we conclude generally (using an arbitrary production function) that the assumptions $\pi_t > 1$ and $\nu_t = 1$ are not reconcilable? My considerations certainly are not general enough. At the end of section 4 I quoted a rather unrealistic theoretical model which gives rise to $\pi_t > 1$ and at the same time $\nu_t = 1$ therefore my question is: Are there other (perhaps more realistic) situations we might think of where also $\pi_t > 1$ and $\nu_t = 1$ may occur? Is it correct to say that as output X is a function of both inputs, B and V such that X = f(B, V) we are not free to make any assumptions concerning productivities π_t and ν_t as is tacitly made in the FNA approach? And are such arguments "general" enough and sufficient to declare the FNA-formula inappropriate or even invalid?*

I hope the general problem to define a PPI on the basis of price indices reflecting changing prices of inputs is sufficiently interesting so that it appears worthwhile considering such questions.

Appendix

Erwin Diewert gave me one his papers on price index problems within the framework of price regulation in the field of energy transmission:

D. Lawrence and W. E. Diewert (2006), "Regulating Electricity Networks: The ABC of Setting X in New Zealand", in Chapter 8 *Performance Measurement and Regulation of Network Utilities*, T. Coelli and D. Lawrence (eds.), Cheltenham: Edward Elgar Publishing, pp. 207-241.

I add here with his permission a synoptic table (see next page) in which I contrasted his method with the one of the FNA which I criticized in the present paper.

Difference between the approach of the German Federal Network Agency (FNA) and Lawrence/Diewert (LD)

There is obviously an enormous difference between these two approaches to index problems within the framework of regulation, which may be summarized in a table as follows:

Subject	Lawrence/Diewert	German FNA
Principal objective of the task	Incentive regulation (CPI-X approach) of prices in New Zealand electricity lines business, i.e. methods to set a maximum change of output prices while also providing incentives to cut costs and to catch-up for the below average firms	Updating of asset valuations in electricity and gas network firms using a price index which is meant to reflect production costs of the respective assets; no incentive regulation, no benchmarking and inter-industry comparisons intended
The sector on which the focus lies	Production (costs, productivity) of services provided by the transmission and distribution businesses; comparisons between firm and economy wide growths and levels of performance parameters	Prices in producing new assets (lines, constructions, equipments etc.); focus on the supply side only, not on the demand on the part of transmission network owners (or on their performance parameters)
End product of the task	Definition of factors (B, C1, C2) to be applied in the framework of price setting for electricity suppliers, based on average (economy wide) and relative (firm specific) level and growth of productivity and also (for C2) profitability	Definition of a price index (reflecting dynamics in producer prices) for producing assets in order to inflate or deflate the economic value of transmission facilities. No data of line businesses considered, only official price and wage indices.
Output	Three kinds of output (throughput, capacity, number of connections), aggregated using weights gained from a cost function	No indicators of real output of asset producers, much less of the service output of the owners of transmission nets
Input	Quantities and prices of five types of inputs: operating expenses (OpEx, including labour and materials), and various assets (stocks), e.g. overhead and underground network etc. "Direct physical assets measures"[1)] were preferred to simply updating given valuations of assets	Index combines prices of labour (wages) and materials only; not clear how weights for these respective sub-indices were derived. No OpEx or other costs incurred in the line businesses (in particular no estimates of the "amount" of assets). Index serves to update valuations of assets.
Productivity	Total factor productivity (TFP) defined as ratio of chained Fisher quantity indexes of output and (total) input (also partial TFPs for the five types of inputs)	Only labour productivity (of the commodities producing sectors and construction sectors) to (solely) "adjust" the weight of wages in the index
Econometric estimates	Estimation of input demand equations (input quantities depending on output and time trend) within the framework of cost functions (separately for each firm)[2)]	No estimation of asset or other input requirements in order to satisfy demand for transmission services. No econometric work at all on the part of the FNA known
Problems in methodology	In order to make transitive comparisons of productivity levels: CCD-indices[3)]	All in all method was econometrically anything but sophisticated

1) Estimation of physical quantities of the principal assets
2) The cost function also provided weights for aggregating the three output components
3) Caves-Christensen-Diewert; transformed Törnqvist indices to make transitive multilateral (here across firms) comparisons

For me the Lawrence/Diewert paper again demonstrates that the formula of our German Federal Network agency (FNA), is a primitive and theoretically not well reasoned one. This is in particular so as it only accounts for the labour productivity and not for the TFP. Moreover no considerations can be found as to the implicit assumptions (in terms of production and cost functions) made when the FNA established its formula.