A recursive Jevons formulation

DRAFT VERSION

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Abstract

The unweighted geometric mean value, denoted Jevons index for elementary aggregates, is commonly applied to either a fixed base or rolling base price index with backwards chaining. It is here suggested a Jevons formulated as a recursive combination of the two alternatives in order to accommodate a changing product universe as well as honoring a fixed base, simultaneously.

Keywords: Jevons, recursion, fixed base, rolling base

1 Introduction

Over the past two decades, there has been an accentuated desire of using census-like data in CPI baskets. Come transaction data, standard index methodology has been either adapted or obsolete, opening for elaboration through innovative methods. Although obvious benefits from censuses, which in effect is a new feature, no clarity exists on how to preferentially deal with all parameters: price, product churn and current quantity weights. This ideal new data reality appears not too ideal to handle in practice. In this note, a suggestion is made for applying the most standard elementary aggregation formula, the Jevons, recursively to exploit the full product universe. Jevons is here formulated as partly fixed and partly chained, on a recursively incorporated fixed base.

2 A recursive Jevons formulation

The principle of the fixed base formulation is *latest* occurrence comparison with *first* occurrence, usually a fixed month like December the preceding year. In contrast, the monthly chaining formulation is of *latest* occurrence with the *last* occurrence, running throughout the year and fixated to December the preceding year. In a non-changing product universe(*), the two formulations coincide as the chaining collapses into the first and last occurrences. Emphasis

can be on either of the two methods regarding which intersecting universe of products to use in standard applications of Jevons.
The following suggestion adheres principally to a fixed basket regime with the amendment of multiple fixed baskets with merely one chained link, each due to provenance.

2.1 The product universe evolvement over thirteen months

Denote a product group *c* consisting of items *g*: $g \in c$. This is the product universe U_c. By adding a time dimension, at base period *0*, say December year *y-1*, the current items are g^0. The following month, January year *y*, the universe is g^1. The intersection universe is $g^{1,0}$ and the first amended universe is $g^{1,\bar{0}}$, i.e. items added to the universe with elapse of time. The new universe is $U_c^1 = g^{1,0}+g^{1,\bar{0}}$. Items that existed in g^0 and no longer exist in g^1 are in the null subset $g^{\emptyset}$ and these need no further treatment.

In the third month, February, the intersection universe is $g^{2,0}$, the second amended universe is $g^{2,\bar{0}}$ from the *current* month and $g^{2|1,\bar{0}}$ from the previous month for items still available that originated in January. Items in the null subset are again not considered. The new universe is $U_c^2 = g^{2,0}+g^{2,\bar{0}}+ g^{2|1,\bar{0}}$. To emphasize, the amendment $g^{2,\bar{0}}$ (or any $g^{t,\bar{0}}$) comprises items that first occurred in the current month *t* and $g^{2|1,\bar{0}}$ comprises current items that occurred in the conditionally given month (January in this case) but were not in the base, hence $\bar{0}$. Without loss of generality, $g^{2|1,\bar{0}}$ can be expressed as $g^{2|1}$, implicitly stating that genesis is in period 1.

Also, note that items in $g^{2,0}$, the intersection of February and the base, may or may not have been in $g^{1,0}$ – implicit conditioning is on existence in the base g^0. The pattern is formalized in Table 1.

Table 1 Subsets of a changing product universe

Period	Intersection	Amendment	Total Universe
0	g^0	-	$U_c^0 = g^0$
1	$g^{1,0}$	$g^{1,\bar{0}}$	$U_c^1 = g^{1,0}+g^{1,\bar{0}}$
2	$g^{2,0}$	$g^{2,\bar{0}}$, $g^{2\|1,\bar{0}}$	$U_c^2 = g^{2,0}+g^{2,\bar{0}}+ g^{2\|1,\bar{0}}$
3	$g^{3,0}$	$g^{3,\bar{0}}$, $g^{3\|2,\bar{0}}$, $g^{3\|1,\bar{0}}$	$U_c^3 = g^{2,0}+g^{3,\bar{0}}+ g^{3\|2,\bar{0}}+g^{3\|1,\bar{0}}$
4	$g^{4,0}$	$g^{4,\bar{0}}$, $g^{4\|3,\bar{0}}$, $g^{4\|2,\bar{0}}$, $g^{4\|1,\bar{0}}$	$U_c^4 = g^{4,0}+g^{4,\bar{0}}+ g^{4\|3,\bar{0}}+g^{4\|2,\bar{0}}+g^{4\|1,\bar{0}}$
⋮	⋮	⋮	⋮
t>1	$g^{t,0}$	$g^{t,\bar{0}}$, $\sum_{k=1}^{K=t-1} g^{t\|t-k,\bar{0}}$	$U_c^t = g^{t,\bar{0}}+ \sum_{k=1}^{K=t-1} g^{t\|t-k,\bar{0}}$

The evolvement of the product universe over time is seen outlined in Table 1, with a growing number of possible amendment sets $g^{t|k,0}$ for time points *k* between the base and current period t. Note that this does not necessarily imply a growing set of *actual* items when summarizing the amendment sets. It can be realized that amendment sets are all conditional on respective *first* occurrence after the initial base period, e.g. $g^{3|1,\bar{0}}$ reflects items in March that first occurred in January, whereas $g^{3|2,\bar{0}}$ reflects items in Match that first occurred in February. This is not equivalent to $g^{3|2,1,\bar{0}}$, which is the intersection universe of items that existed both in January and February as well as March, while not in the base.

2.2 A recursive Jevons formulation

The standard Jevons for the fixed basket formulates after period 1 with a recursive index amendment R_t as

$$IX_t = \left[\frac{P_t}{P_0}\right]^q \times [R_t]^{(1-q)},\ t>1 \qquad (1)$$

following standard notation for prices in the current period t: p_i^t and the base period p_i^0 are multiplied $\prod_{i=1}^{i=n(g^{t,0})}\left(\frac{p_i^t}{p_i^0}\right)$ to obtain $\frac{P_t}{P_0}$. No item subscripting is needed henceforth and the number of items from the intersecting universe is denoted $n(g^{t,0})$. It is seen that (1) becomes a recursive statement for *t*>1.

The relative importance between the fixed basket Jevons part and the recursive Jevons amendment R_t is regulated through *q*, *q*≤*1*. The choice of *q* requires elaboration as it replaces the application of expenditure shares, hence the first part of the right hand side in (1) is a simultaneous proxy for a geometric Laspeyres (§16.75,§22.32, ILO 2004) and a geometric Paasche (§16.76, §16.80, *ibid.*) given the fixed base. A dynamic *q* based on past as well as most recent observations of expenditures, in say t, may result in an uncontrolled drifting index due to the decadence rate in the recursive formulation. The choice should be perhaps policy based or deterministic throughout the index year.

In period t=1, R_t is by definition null as outlined in the previous subsection. As of period t=2, the recursive amendment formulates as

$$R_t = \prod_{k=0}^{K=t-2} \sqrt[K]{\left(\frac{P_{t|1+k}}{P_{1+k|1+k}} \times IX_{1+k}\right)},\ t>1. \qquad (2)$$

The recursive part R_t in (2) applies to the pattern outlined in Table 1 accordingly as the following example for the first two periods after January, t>1; t=2 and t=3:

in t=2; $R_2 = \frac{P_{2|1,\bar{0}}}{P_{1|1,\bar{0}}} \times \left(\frac{P_1}{P_0} = IX_1\right)$ for $g^{2|1,\bar{0}}$, and

in t=3; $R_3 = \sqrt[2]{\left(\left(\frac{P_{3|2,\bar{0}}}{P_{2|2,\bar{0}}} \times IX_2\right) \times \left(\frac{P_{3|1,\bar{0}}}{P_{1|1,\bar{0}}} \times IX_1\right)\right)}$ for $\left(g^{3|2,\bar{0}} \cup g^{3|1,\bar{0}}\right)$.

It is again seen that R_t will not benefit immediately from the entire amendment – the current period amendment $g^{t,\bar{0}}$ is not used until the immediately following time period when it has its first price development observable, $g^{t|1+k}$ and consecutively from *k*=(0,1..,*K*=t-2). This is then multiplied with its corresponding recursive formulation IX_{1+k} relating it to the base.

In one sense, this is a group mean imputation method (depending on the choice of *q*) for periods between the fixed base and the first occurrence. The main difference from a forward-carried base price imputation is the temporary disregard of $g^{t,\bar{0}}$: upcoming items are assigned their own price development in the second month after first occurrence, analogous to a monthly chaining and resampling strategy in which items hibernate one month when appearing for the first time ever.

3 A numerical example

1% every month, q varies

2% to the 6th month, q varies

2% every month, q varies

2% to the 12th month, q varies, but 1 month dips to -4%.

Compare with Jevons Fixed & chained: basket attrition – different development.

If relaunches have identical dev. – no problem.

If not, estimate proximity to chaining.

Realized that everything has the same base, no drift possible!

References

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Appendix

Extracts from the ILO (2004) Manual for CPI:

9.36 The Jevons index does not imply, or assume, that expenditure shares remain constant. Obviously, the Jevons can be calculated whatever changes do, or do not occur in the expenditure shares in practice. What the economic approach shows is that if the expenditure shares remain constant (or roughly constant), then the Jevons index can be expected to provide a good estimate of the underlying cost of living index.

9.34 On the basis of the economic approach, the choice between the sample Jevons and the sample Carli rests on which is likely to approximate the more closely to the underlying cost of living index: in other words, on whether the (unknown) cross-elasticities are likely to be closer to unity or zero, on average. In practice, the crosselasticities could take on any value ranging up to plus infinity for an elementary aggregate consisting of a set of strictly homogeneous items, i.e., perfect substitutes. It should be noted that in the limit when the products really are homogeneous, there is no index number problem, and the price ‘‘index’’ is given by the ratio of the unit values in the two periods, as explained later. It may be conjectured that the average cross-elasticity is likely to be closer to unity than zero for most elementary aggregates so that, in general, the Jevons index is likely to provide a closer approximation to the cost of living index than the Carli. In this case, the Carli index must be viewed as having an upward bias.

9.35 The insight provided by the economic approach is that the Jevons index is likely to provide a closer approximation to the cost of living index for the elementary aggregate than the Carli because, in most cases, a significant amount of substitution is more likely than no substitution, especially as elementary aggregates should be deliberately constructed in such a way as to group together similar items that are close substitutes for each other.

9.36 The Jevons index does not imply, or assume, that expenditure shares remain constant. Obviously, the Jevons can be calculated whatever changes do, or do not occur in the expenditure shares in practice. What the economic approach shows is that if the expenditure shares remain constant (or roughly constant), then the Jevons index can be expected to provide a good estimate of the underlying cost of living index. Similarly, if the relative quantities remain constant, then the Carli index can be expected to provide a good estimate, but the Carli does not actually imply that quantities remain fixed.

9.37 It may be concluded that, on the basis of the economic approach as well as the axiomatic approach, the Jevons emerges as the preferred index in general, although there may be cases in which little or no substitution takes place within the elementary aggregate and the Carli might be preferred. The index compiler must make a judgement on the basis of the nature of the products actually included in the elementary aggregate.