A proposal for a new system of aggregation in the Swedish Consumer Price Index

(revised and extended version of paper presented at the
fifth meeting of the International Working Group on Price Indicies
under the title **Some issues in index construction**)

by

Jörgen Dalén
Jörgen Dalén Statistical Consulting
Victor Balcks väg 158
S - 122 40 Enskede
E-mail: jorgen.dalen@stockholm.mail.telia.com

November 1999

Abstract: This paper was prepared for the KPI Commission in Sweden, which has reviewed the basic principles of the the Swedish CPI. It proposes a solution to the problem of estimating a cost of living index within the practical and time constraints that exists in a national statistical agency. The solution is essentially a chain index with whole years as base periods in each link and with the superlative Walsh price index for estimating the year to year price changes. The last link in the chain comparing the current month with the full year two years back is a Laspeyres index, but at the turn of a year the last link is replaced by a Walsh index. In this way the overestimating Laspeyres bias does not influence the movements of the index series as such.

In the absence of proper weights at lower levels of the index, some empirical evidence is cited to justify the proposition that, as a main rule, unitary price elasticity should be assumed. This rule is not without exceptions, however. Approximations of the Walsh formula under various assumptions of price elasticity and access to weights are given. Some practical, computational issues in lower level aggregation are further discussed.

Key words: Cost of living index, superlative index, elementary aggregate index, Walsh index.

1. Introduction

This report deals with the basic index construction problems for a Consumer Price Index such as the Swedish *KPI* (Konsumentprisindex). Basic index construction has essentially two distinct parts:

The chain/link choice. In any price index one must decide on a weight reference and a price reference period within an *index link*. One must also determine the frequency in time between changes of reference periods. In practice, there are two main approaches to this problem. The most common approach internationally is the *fixed base index* in which reference periods are kept constant for several years – 3-10 or more. The other approach is the *chain index* with annual links, used in Sweden and some other countries, where the price reference period is changed annually. An additional issue is the choice of *price reference period*, which could be either a month or a year. In chain indices, the usual practice so far has been to use a month (December or January) whereas for fixed base indices the practice is mixed.

Aggregation formulae. Within each link, functional forms for combining prices and weights to measures of price change must be decided. In practice, we have both differences in consumer behaviour in different markets and differences in access to weight data in different parts of the index. For this reason, there is a need to look at the functional form problem separately at different aggregation levels.

This report was originally written for the KPI Commission in a slightly different version. This Commission was appointed in 1997 and presented its final report – KPI Commission (1999) - in November 1999[1]. Its purpose was to establish the basic principles of the index, which were not changed since the Commission of 1952. It covers many of the fundamental problems of a CPI including: i) Basic index construction (the subject of this paper), ii) Owner-occupied

[1] The report is in Swedish but some Annexes are in English and it includes an English summary.

housing, iii) Estimation of KPI bias, iv) Improving index reliability and v) Measures of underlying inflation. No decision as to the actual application of the Commission Report to the KPI is yet taken, however. Its proposals will be open to public debate and referred to various institutions and Government bodies for consideration. After this process a final Government decision will be taken.

This report first discusses, in Section 2, criteria for what we regard as a good index construction. In Section 3 the present KPI system and its history are presented and its strengths and weaknesses are discussed. Section 4 compares several possible strategies for setting up a CPI system and analyses the choice of system at the highest aggregation level. Section 5 discusses the approximations that will be necessary at lower levels, where the access to weight data is less than perfect. Section 6 quotes the Commission summary of its proposals with regard to index construction.

2. Criteria for index construction

In line with a long Swedish tradition, KPI construction is proposed to be guided by microeconomic theory and index theory in general and by the theory of the cost-of-living index in particular, as far as possible. It means that the index ought to monitor the cost over time of obtaining a constant standard of living with regard to the area of household consumption considered in scope for the index. At the same time, a number of practical considerations must inevitably be addressed.

The following criteria are therefore considered:

1. *Minimal long-term bias in relation to cost-of-living index/superlative index*. This requirement presupposes that the index is constructed so that there is a clear connection to established index theory. The requirement means that there should be a minimal aggregate bias over a period of many years so that under- or overcompensation in, e.g., social benefits does not occur.
2. *Minimal bias and disturbances in short-term changes*. This especially applies to 12-month changes, which is an important statistic for the purpose of inflation monitoring. It means that a short-term change, measured as the ratio between index numbers in the series, should not be disturbed by other effects than the aggregate price changes over the time horizon concerned.
3. *Axiomatic considerations*. It is desirable that an index link is constructed to meet important index axioms such as those given in Balk (1995).
4. *Correct handling of seasonal products*. All products should be given a proper and balanced treatment in the long-term development of the index regardless of their seasonal price and consumption pattern. This is not an easy criterion to formulate exactly but nonetheless important.
5. *Timeliness.* It must be possible to perform index calculations so that monthly index estimates can be published no later than today.
6. *Easy to interpret and analyse*. Index producers and users are used to today's structure of the index which could be viewed as a fixed basket of products or a *pure price index* in the terminology of Diewert (1999). The index is then possible to express as a product sum of weights and subindices. At present the KPI has this property down to a level of around 300 products. This structure facilitates various analyses such as construction of satellite

measures, “what-happens-if ” analyses etc. It is an advantage if this structure could be retained or else that another structure is not more difficult to handle.

7. *Non-revisability*. An index number for a month, once published cannot be revised. This is desired because of the legal implications, when using the index in private or public contractual agreements.
8. *Co-ordination with the HICP*. This is a very practical kind of consideration. It is desirable that duplication of work is not needed for the KPI and the HIKP. For example, low level indices should preferably be possible to compute from the same set of data.
9. *Resource aspects*. The index should not be excessively expensive to produce.

Since the primary use of the KPI is thought to be for compensation purposes, criterion 1 on unbiasedness stands out as the most important one.

3. The present KPI construction

First a note on notation: Below, P_k^t (Q_k^t) stands for an aggregate price (quantity) of product k in period t and $V_k^t = P_k^t\, Q_k^t$ for a value of the same product. We will also denote the value share in period t by $W_k^t = V_k^t \Big/ \sum_k V_k^t$. When there is no risk for confusion, the subscript k will sometimes be omitted.

3.1. Historical background

The present structure of the KPI dates back to SOU (1943), a Government Commission that was appointed for reviewing consumer price measurement during wartime conditions when many goods were rationed and the consumption structure therefore changed rapidly. At this time the Cost of Living Index (*Levnadskostnadsindex*, LKI, as it was then called) used a commodity basket based on a Household Budget Survey from 1933 and a need was felt for a more frequent updating of the basket.

The 1943 commission went into some depth with its task and derived a unique expression for a Consumer Price Index. Starting with two points in time - 0 and t – and a set of products (goods and services) of fixed quality, it defined prices and quantities for these products. It further set up the task of separating the change in total consumption value of these products into two parts: one due to price change (PC_{0t}) and one to quantity change (QC_{0t}). The value identity was the starting point:

$$PC_{0t} QC_{0t} = \frac{\sum P_t Q_t}{\sum P_0 Q_0} \qquad (1)$$

According to an idea developed by Divisia (1925), who considered the change to be divided into infinitesimal parts, the following integral expression of value change was obtained:

$$\log PC_{0t} + \log QC_{0t} = \sum \int_0^t V_\tau \frac{dP_\tau}{P_\tau} + \sum \int_0^t V_\tau \frac{dQ_\tau}{Q_\tau}\,, \qquad (2a)$$

where $V_\tau = P_\tau Q_\tau$ is the value at time τ. It now seems natural to define price change as

$$\log PC_{0t} = \sum \int_0^t V_\tau \frac{dP_\tau}{P_\tau}\,, \qquad (2b)$$

and to define quantity change correspondingly.

The Commission now considered a division of the period of study (0 to t) into smaller parts – 0, 1, 2, …, t. Under the assumption that values $\boldsymbol{V}$ were constant within each such small period, say 0 to 1, it derived the following expression for the price index:

$$\boldsymbol{I}_{01} = \left[\prod \left(\frac{\boldsymbol{P}_1}{\boldsymbol{P}_0} \right)^{\boldsymbol{V}} \right]^{\frac{1}{\sum \boldsymbol{V}}} \qquad (3)$$

Assuming instead that quantities were constant, the following expression was obtained:

$$\boldsymbol{I}_{01} = \frac{\sum \boldsymbol{P}_1 \boldsymbol{Q}}{\sum \boldsymbol{P}_0 \boldsymbol{Q}} \qquad (4)$$

The Commission then argued that by choosing V or Q so that they reflected the whole period under consideration (from 0 to 1 in this case) a better approximation to the integral expression in (2a-b) was obtained than by choosing them according to conditions prevailing at either the beginning or the end of the period.

Choosing between (3) and (4), it further pointed out that (3) corresponded to the assumption of normal (unit) price elasticity whereas (4) corresponded to an assumption of zero elasticity. Under normal circumstances, it argued, the geometric formula would correspond better to the statistical data, since unit elasticity was a more reasonable assumption than zero elasticity. However, under the wartime conditions then prevailing, this was less clear. The Commission therefore concluded that (3) and (4) were equally satisfactory from a theoretical point of view.

Moving then to practical considerations, the Commission noted two advantages with the arithmetic mean compared with the geometric mean: i) It is possible to give (4) but not (3) a fixed basket interpretation and ii) (4) is somewhat simpler to apply computationally. These considerations led it to propose formula (4) for use in the LKI.

However, it rounded up the argument by suggesting that the formula question ought to be reconsidered when returning to more normal consumption conditions after the war. If then the index link was to be extended to several years and (4) was transformed into a Laspeyres' index, then a geometric index should instead be considered.

The derivation that is summarised above could clearly be subject to objections. How could quantities and values be defined for infinitesimal points of time? Also, there is no reference to the cost of an unchanged standard of living in this derivation.

The Divisia index is defined at points of time rather than for periods. A practical consequence of this particular index definition is that the target dates of the KPI have been the 15th of a month rather than (the average price of) the whole month.

An interesting historical fact is that, during the years 1937-1949, the Central Bank of Sweden (as well as the Central Bank of Finland during a similar period) calculated a Consumption Price Index according to the geometric formula (3), although with lagging weights.

The chain index definition of the LKI was put into practice immediately in 1943. Up to June 1954 the LKI was computed quarterly and new weights according to (4), referring to the current year, were introduced as they became available, which in practice was in the December index. In July 1954 the new monthly *Konsumentprisindex* series - as proposed by SOU (1953), a Government Commission appointed in 1952 - was initiated and the distinction between a short-term and a long-term index to be presented below was formulated. The practical implementation of the proposals in the two Government Commissions is carefully described in Socialstyrelsen (1961).

3.2 Present practice

The present index construction thus follows the principles of formula (4) above. The KPI is a chain index with annual links going from December one year to December next year which are multiplied when calculating the long index series which at present has 1980 as its base year. For each new link, weights are recalculated based on new information.

We make a distinction between a long-term link (L), which uses quantity weights Q_y from year y and a short-term link (S), which uses quantity weights Q_{y-1} from year y-1. The definitions of the links are

$$L_{y-1,12}^{y,12} = \frac{\sum_k P_k^{y,12} Q_k^y}{\sum_k P_k^{y-1,12} Q_k^y} \qquad (5)$$

and

$$S_{y-1,12}^{y,m} = \frac{\sum_k P_k^{y,m} Q_k^{y-1}}{\sum_k P_k^{y-1,12} Q_k^{y-1}}, \qquad (6)$$

where summation is over N products with subscript k (which will later be dropped).

The chained index from December of the index base year 0 to a month m in year Y (a factor measuring price changes from the average of year 0 to December year 0 is added in practice) will now be:

$$KPI_{0,12}^{Y,m} = S_{Y-1,12}^{Y,m} \prod_{y=1}^{Y-1} L_{y-1,12}^{y,12} \qquad (7)$$

This means that in the long run the KPI series only depends on the long-term links; the short-term links are successively replaced by their long-term counterparts for December each year.

In practice, of course, no quantities could be directly observed at the higher KPI aggregation levels. The National Accounts (NA) are instead the first choice for weights. Up-to-date NA consumption values today exist for about 90 categories. During the annual weight revision, which takes place in January and early February, new NA values are brought into the index. For example, in early 98 values for 97 – denoted V^{97} - were used both for replacing the short-term link of December 97 with a long-term link *and* for the new short-term link of 98. The values are price updated (or “backdated”) to the price reference period of the link in question. Continuing with the 97 example, we obtain:

Long-term weight for 97 = $\boldsymbol{W^{97,L}} = \left(\boldsymbol{V^{97}} \Big/ \dfrac{\boldsymbol{P^{97}}}{\boldsymbol{P^{96,12}}} \right) \Big/ \sum \boldsymbol{V^{97}} \Big/ \dfrac{\boldsymbol{P^{97}}}{\boldsymbol{P^{96,12}}}$ (8a)

to be plugged into $\boldsymbol{L}^{97,12}_{96,12} = \sum \boldsymbol{W}^{97,\boldsymbol{L}} \dfrac{\boldsymbol{P}^{97,12}}{\boldsymbol{P}^{96,12}}$ and (8b)

short-term weight for 98 = $\boldsymbol{W^{98,S}} = \dfrac{\boldsymbol{V^{97}} \dfrac{\boldsymbol{P^{97,12}}}{\boldsymbol{P^{96,12}}} \Big/ \dfrac{\boldsymbol{P^{97}}}{\boldsymbol{P^{96,12}}}}{\sum \boldsymbol{V^{97}} \dfrac{\boldsymbol{P^{97,12}}}{\boldsymbol{P^{96,12}}} \Big/ \dfrac{\boldsymbol{P^{97}}}{\boldsymbol{P^{96,12}}}}$ to be plugged into (9a)

$\boldsymbol{S^{98,m}_{97,12}} = \sum \boldsymbol{W^{98,S}} \dfrac{\boldsymbol{P^{98,m}}}{\boldsymbol{P^{97,12}}}$. (9b)

The indices $\dfrac{\boldsymbol{P}^{97}}{\boldsymbol{P}^{96,12}}$ are computed as averages of 12 monthly short-term indices from January 97 to December 97.

Below the NA weight level, procedures are not entirely consistent at present. Various information is used for weights, much of which is not from the right year and a special procedure is used within the category of imputed rent. Still price "redating" is done according to (8)-(9) which leads to some problems discussed more below.

3.3 Strengths and shortcomings of the present index construction

The first, major advantage of the KPI index construction outlined above is that it is largely able to avoid the so called upper level substitution bias (the Laspeyres overestimation of a true cost of living index). This advantage is due to the use of weights that are between the two periods for which prices are compared. A simple measure of the magnitude of this substitution bias is the difference between the short-term and the long-term index, although this difference is to some extent disturbed by other kinds of new information that is brought into the long-term index. Table 0 gives these differences between 1979 and 1998. We note that the difference is positive in 18 years out of 20. The order of size of the mean difference, 0.1-0.2, is similar to estimates of upper level substitution bias in the U.S. CPI, see for example Aizcorbe and Jackman (1993) who estimated this bias for the years 1982-91. Below, we will return to other measures of substitution bias for the KPI.

Table 0: Differences between short- and long-term indices, Dec y-1 to Dec y.

1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
0.14	**0.52**	**0.29**	**0.07**	**0.03**	**0.07**	**-0.06**	**0.03**	**0.25**	**0.11**	**0.03**
1990	1991	1992	1993	1994	1995	1996	1997	1998	**Mean**	
0.21	**0.15**	**0.02**	**0.15**	**0.32**	**0.28**	**0.26**	**0.50**	**-0.14**	**0.16**	

The second advantage of the present index construction is that it is possible to take in new products and up-to-date weights every year. This advantage is, however, shared with other countries that apply chain indices and is not due to the specific long-term index definition.

However, there are also some problems. The first one is that there is a slight asymmetry in the long-term index formula. Prices are generally measured in the week in which the 15th occurs.

This means that the period between the two points in time that the 1943 Commission referred to in formulae (3) and (4) above is Dec 16, y-1 to Dec 15, y rather than Jan 1, y to Dec 31, y as in today's long-term index. Another way to look at this problem is to consider the following decomposition of the long-term index.

$$L_{y-1,12}^{y,12} = \frac{\sum_k P_k^{y,12} Q_k^y}{\sum_k P_k^{y-1,12} Q_k^y} = \frac{\sum_k P_k^y Q_k^y}{\sum_k P_k^{y-1,12} Q_k^y} * \frac{\sum_k P_k^{y,12} Q_k^y}{\sum_k P_k^y Q_k^y} \qquad (10)$$

Here we have divided L into two factors. The first one is a Paasche index from December y-1 to the year y, a period of 6½ months on average. The second one is a Laspeyres index from the year y to December y, a period of 5½ months on average. This means that the long-term index is slightly biased in the Paasche direction, which may lead to an underestimating substitution bias! This may seem as a minor point but an additional factor may tend to aggravate this bias. This is to the extent that price changes tend to be concentrated to the turn of the year, in January, or else to the first part of the year so that quantity weights would largely reflect the consumption pattern *after* the price increase. This problem will be looked at empirically in the next section.

A second problem with the way that the long-term index is estimated is that price updating is applied below the level where there is new information on consumption values. Haglund (1992) points out that this procedure amounts to an assumption of unitary price elasticity and that an underestimating bias occurs if the price elasticity is smaller than one. Norberg (1996) shows that the type of ratio estimator used for estimating (8b) gives rise to an underestimating sampling bias.

The third problem is due to seasonal products, especially those for which there is no consumption and thus no price in December. It is difficult to cover them in a consistent way in an index construction that has a single month as its price reference period.

The fourth problem is of a more practical nature and concerns the easy-to-analyse criterion. When the 12-month rate of inflation is now published, the effect of the long-term index is removed. The official motive for this practice is that the annual revision of the figures may refer to changes (substitutions), which are not within the 12 months to which the inflation measure refers. However, it gives rise to some confusion among the index users to have a measure of 12-month inflation, which is not the simple ratio of the index numbers.

4. Basic (upper level) index construction

4.1 Superlative aggregation formulae

The approach to index construction put forward by the 1943 Commission could be seen as an early attempt to estimate a true cost of living index. Today, we know that as far as the functional form is concerned, the best approach to this estimation is through the concept of a *superlative index* as defined by Diewert (1976).

Three important examples of superlative index number formulae, based on four vectors of prices and quantities (P^1,P^2,Q^1,Q^2) and referring to two periods in time, 1 and 2, are

$$\textit{Fisher's ideal index: } I^2_{1;F} = \sqrt{\frac{\sum_k P^2_k Q^1_k}{\sum_k P^1_k Q^1_k}\,\frac{\sum_k P^2_k Q^2_k}{\sum_k P^1_k Q^2_k}} = \sqrt{\frac{\sum_k V^1_k \dfrac{P^2_k}{P^1_k}}{\sum_k V^2_k \dfrac{P^1_k}{P^2_k}}}, \tag{11}$$

$$\textit{Törnqvist's index: } I^2_{1;T} = \prod_k \left(\frac{P^2_k}{P^1_k}\right)^{(V^1_k+V^2_k)/2} \text{ with } V^t_k = \frac{P^t_k Q^t_k}{\sum_k P^t_k Q^t_k} \text{ for t = 1 or 2 and} \tag{12}$$

$$\textit{Walsh's index: } I^2_{1;W} = \frac{\sum_k P^2_k \sqrt{Q^1_k Q^2_k}}{\sum_k P^1_k \sqrt{Q^1_k Q^2_k}} = \sum_k W_{k;W}\,\frac{P^2_k}{P^1_k}, \tag{13}$$

$$\text{where } W_{k;W} = \sqrt{\frac{V^1_k V^2_k}{P^2_k / P^1_k}} \Bigg/ \sum_k \sqrt{\frac{V^1_k V^2_k}{P^2_k / P^1_k}} \tag{13b}$$

An index formula, which numerically comes very close to being a superlative index and will be considered later, is:

$$\textit{Edgeworth's index: } I^2_{1;E} = \frac{\sum_k P^2_k\left(Q^1_k + Q^2_k\right)}{\sum_k P^1_k\left(Q^1_k + Q^2_k\right)} = \sum_k \frac{P^1_k\left(Q^1_k + Q^2_k\right)}{\sum_k P^1_k\left(Q^1_k + Q^2_k\right)}\,\frac{P^2_k}{P^1_k} = \sum_k W_{k;E}\,\frac{P^2_k}{P^1_k}, \tag{13½}$$

$$\text{with } W_{k;E} = \frac{V^1_k + \dfrac{V^2_k}{P^2_k / P^1_k}}{\sum_k V^1_k + \sum_k \dfrac{V^2_k}{P^2_k / P^1_k}} \tag{13½b}$$

The fundamental difficulty of all superlative indices is that they require weights from both ends of the comparison. The demand for timeliness normally prevents the use of weights for the last period in a short-term indicator like the CPI.

They also run into difficulties in a chain index where the price reference period is only a month, since the weights would then also refer to only a month. Besides the lack of monthly consumption statistics, it cannot be reasonable even in principle to choose a month's consumption as the basis for weights, since there is a seasonal variation that would make such a choice less representative of a household's total consumption. If, on the other hand, we choose a year's consumption as our quantity weight in an index with a month as a price reference period we introduce an asymmetry into the index which results in a loss of its true superlativity property. Below, we will look at these effects empirically.

4.2 Index formulae using only period 1 weights

At least for the last link in a chain index it will be necessary to use a base-weighted index, i.e. an index formula where only period 1 quantities are included. Some such formulae that we will consider are (remember that $W_k^1 = V_k^1 \Big/ \sum_k V_k^1$):

Laspeyres' index: $I_{1;L}^{2} = \dfrac{\sum_k P_k^2 Q_k^1}{\sum_k P_k^1 Q_k^1} = \sum_k W_k^1 \dfrac{P_k^2}{P_k^1}$ (14)

The *geometric index:* $I_{1;G}^{2} = \prod_k \left(\dfrac{P_k^2}{P_k^1} \right)^{W_k^1}$ (15)

The *Constant Elasticity of Substitution (CES) index:* $I_{1;C}^{2} = \left[\sum_k W_k^1 \left(\dfrac{P_k^2}{P_k^1} \right)^{1-\sigma} \right]^{1/(1-\sigma)}$ (16)

The Laspeyres index is only consistent with zero price elasticity and tends to overestimate a cost of living index, where elasticity is larger than that. The geometric index is only consistent with unitary price elasticity and over-/underestimates a cost of living index where elasticity is larger/smaller than one[2]. The CES index is a generalisation of these two indices and is consistent with a price elasticity of substitution, which is equal to σ between all pairs of products and assumed to be known in advance. (In practice, one would use data from before the present index link to estimate it). The CES index is a relatively novel theoretical invention. It was first proposed by Lloyd (1975) and later rediscovered by Moulton (1996).

At the highest aggregation level, empirical results suggest a price elasticity between zero and one. Based on U.S. CPI data for 1988-1995 Shapiro and Wilcox gave a value of $\sigma=0.7$ as the best estimate. Moulton and Stewart (1997) present series of both fixed weight and chained Laspeyres and geometric indices and compare them to Fisher and Törnqvist indices based on U.S. CPI data for 1987-1995. Their results likewise show that the superlative indices fall below the Laspeyres but above the geometric index – and closer to the geometric than to the Laspeyres. In both these studies, a disaggregation of the U.S CPI data into 207 item groups and 44 geographic strata was used.

4.3 The chain/link choice

Four approaches to the chain/link choice will be investigated.

Fixed base indices. In this alternative, we would choose one of (14)-(16) as our aggregation formula. It would be natural to have a full year as the reference period for both prices and quantities. The link could last 3-10 years, below we will look at 5-year links. The long run development of the chained index would then be like

[2] We adopt the convention of using positive values for elasticity.

$$\boldsymbol{I_0^{Y,m} = I_0^5 I_5^{10} ... I_{5n}^{Y,m}} \tag{17}$$

Annually chained indices, linking by month. This is the present chaining method used in the KPI with December as the link month. The long run development of the index follows the following chain formula[3]:

$$\boldsymbol{I_0^{Y,m} = I_{Y-1,12}^{Y,m} \prod_{y=1}^{Y-1} I_{y-1,12}^{y,12} \ x \ I_0^{0,12}} \tag{18}$$

Annually chained indices, linking by year. For reasons discussed below we want to try a different form of the chain index, not used earlier, in which we use a full year as our reference period for both prices and quantities. With this option the chained index will be:

$$\boldsymbol{I_0^{Y,m} = I_{Y-1}^{Y,m} \prod_{y=1}^{Y-1} I_{y-1}^{y}} \tag{19}$$

Delayed chain index, linking by year. There are some practical problems with implementing (19), which will be discussed more below. Another option would be to have two-year Laspeyres link in the end of the chain, allowing more time for weight data for earlier years to arrive. This gives the following expression for the index:

$$\boldsymbol{I_0^{Y,m} = I_{Y-2}^{Y,m} \prod_{y=1}^{Y-2} I_{y-1}^{y}} \tag{20}$$

We will now take a closer look at the various set-ups that are possible within these four frameworks and compare them numerically based on historic data.

4.4 Simulations

A number of numerical experiments were carried out for comparing various index alternatives in order to estimate the sizes of biases. For this purpose, a database of KPI subindices for 72 National Accounts (NA) purposes was used, covering the period from 1980 to 1998. Imputed rent for owner-occupied housing was excluded from the simulations and so were a few other, very small categories for which comparable data for the whole period were not available.

The simulations were thus designed to measure upper level substitution bias, i.e. the bias resulting from failure to use a superlative index in the final aggregation step where subindices for these 72 purposes are being put together into an All Item Index. Substitution biases at lower levels are not covered by this simulation.

The annual values thus obtained were price updated to December – the link base month. Algebraically, the indices and weights in the data base are defined in the following way (the superscript *y,m* means month *m* in year *y* and only *y* means the whole year *y*):

---

[3] The extra link from 0 to 0,12 is needed for having a full year as the index reference period. Its exact definition is: $I_0^{0,12} = I_{-1,12}^{0,12} \Big/ \frac{1}{12} \sum_m I_{-1,12}^{0,m}$

The short-term index (***STIX***): $$S_{y-1,12}^{y,m} = \frac{\sum_k P_k^{y,m} Q_k^{y-1}}{\sum_k P_k^{y-1,12} Q_k^{y-1}} = \sum_k W_k^{STIX} \frac{P_k^{y,m}}{P_k^{y-1,12}}, \tag{21}$$

where the weights are $$W_k^{STIX} = \frac{V_k^{y-1} \dfrac{P_k^{y-1,12}}{P_k^{y-1}}}{\sum_k V_k^{y-1} \dfrac{P_k^{y-1,12}}{P_k^{y-1}}} \tag{21b}$$

The long-term index (***LTIX***): $$L_{y-1,12}^{y,12} = \frac{\sum_k P_k^{y,12} Q_k^{y}}{\sum_k P_k^{y-1,12} Q_k^{y}} = \sum_k W_k^{LTIX} \frac{P_k^{y,12}}{P_k^{y-1,12}}, \tag{22}$$

where the weights are $$W_k^{LTIX} = \frac{V_k^{y} \dfrac{P_k^{y-1,12}}{P_k^{y}}}{\sum_k V_k^{y} \dfrac{P_k^{y-1,12}}{P_k^{y}}} \tag{22b}$$

The data base consists of the indices and weights defined above. However, in the simulations we need the actual values, which are not in the database. The value shares were instead obtained by "backdating" the weights defined above[4].

Throughout the analysis, we will use the two present index types defined in (5) and (6) as standards with which we compare our alternatives. Further, when comparing two whole years, we will use the following computational rules:

$$S_{y-1}^{y} = S_{y-2,12}^{y-1,12} S_{y-1,12}^{y} / S_{y-2,12}^{y-1} \text{ and} \tag{23}$$
$$L_{y-1}^{y} = L_{y-2,12}^{y-1,12} S_{y-1,12}^{y} / S_{y-2,12}^{y-1}, \text{ where} \tag{24}$$
$S_{y-1,12}^{y}$ and $S_{y-2,12}^{y-1}$ will be geometric or arithmetic averages of the monthly indices.

4.4.1Fixed base indices

We will use 5-year links in our simulations. Our base-weighted formulae will now be defined as follows:

Laspeyres - FL: $$I_y^{y+n,m} = \frac{\sum_k P_k^{y+n,m} Q_k^{y}}{\sum_k P_k^{y} Q_k^{y}}, \text{ where } 0<=n<=5. \tag{25}$$

The form of the Laspeyres' index to be used in the simulations will be the following:

$$I_{y,12}^{y+5,12} = \sum_k W_k^{y} \frac{P_k^{y+5,12}}{P_k^{y,12}} \text{ and } W_k^{y} = \frac{P_k^{y,12} Q_k^{y}}{\sum_k P_k^{y,12} Q_k^{y}}, \tag{25b}$$

[4] The different levels of values and quantities over the years get lost in this exercise. This problem is not essential. However, it is a slight problem when estimating the Edgeworth index.

which are identical to the short-term weights of year ***y+1***. Note that (25b) is identical to the ratio of ***y+5,12*** to ***y,12*** indices in a run of strict Laspeyres' indices with year y as the reference period.

The second option is a CES index: based on the same weights and subindices as in (25):

$$\textbf{CES - FC}(\sigma)\textbf{:}\ I_{y,12}^{y+5,12} = \left[\sum_k W_k^y \left(\frac{P_k^{y+5,12}}{P_k^{y,12}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \qquad (26)$$

We will compute (26) for σ=0.1, 0.2, … 0.9. For σ=0 it coincides with ***FL*** and for σ=1 it turns into the geometric index as the limiting case:

$$\textbf{Geometric - } \textbf{\textit{FG}}\text{: } I_{y,12}^{y+5,12} = \prod_k \left(\frac{P_k^{y+5,12}}{P_k^{y,12}}\right)^{W_k^y} \qquad (27)$$

In Table 1 we look at simulated indices for rolling 5-year links, i.e. 79-84, 80-85, 81-86 etc. We present one-year averages of 5-year changes. We notice that the fixed base Laspeyres is on average 0.27 points above the fixed base geometric index. These two variants are the extremes and the CES indices effectively interpolate the whole interval in between for σ=0-1. Comparing with present index construction we see that the long-term index (taken as our preliminary yardstick) corresponds to a value of about σ=0.7 which happens to be the same value as found by Shapiro and Wilcox (1997). We also see that the difference FC(.7) – LTIX over the whole period varies from +0.25 to –0.13. This difference can be interpreted as the annual bias, if an FC index were used. Also, the chained short-term indices (STIX) which uses recent weights are on average smaller than the fixed base Laspeyres, indicating that bias tends to increase with the age of the weights.

4.4.2 Chain indices: linking by month

When linking by month, all index formulae have to be somewhat twisted, since quantity and price data will necessarily be from different periods. We choose only one "superlative" index in as our reference here.

$$\textbf{Törnqvist - } \textbf{\textit{MT}}\text{: } I_{y-1,12}^{y,12} = \prod_k \left(\frac{P_k^{y,12}}{P_k^{y-1,12}}\right)^{W_{k;T}} \qquad (28)$$

with $W_{k;T} = \frac{1}{2}\left(W_k^{y-1} + W_k^y\right)$. (28b)

We immediately note the asymmetry in the formula with weights reflecting years and prices being for months. This problem is difficult to avoid when linking by month since consumption values with good quality exist only by year and since we want to represent the whole year in our index. This index would still have to be a long-term index, since year y values are available only early in year y+1.

$$\textbf{Edgeworth - } \textbf{\textit{ME}}\text{: } I_{y-1,12}^{y,12} = \frac{\sum_k P_k^{y,12}\left(Q_k^{y-1} + Q_k^y\right)}{\sum_k P_k^{y-1,12}\left(Q_k^{y-1} + Q_k^y\right)} = \sum_k W_k \frac{P_k^{y,12}}{P_k^{y-1,12}}, \qquad (29)$$

where $$W_k = \frac{V_k^{y-1}\dfrac{P_k^{y-1,12}}{P_k^{y-1}} + V_k^{y}\dfrac{P_k^{y-1,12}}{P_k^{y}}}{\sum_k V_k^{y-1}\dfrac{P_k^{y-1,12}}{P_k^{y-1}} + \sum_k V_k^{y}\dfrac{P_k^{y-1,12}}{P_k^{y}}} \qquad (29b)$$

Two variants of geometric indices have been simulated. In both of those we change the weights in (28) so that for:

Geometric - ***MG1***: As (28) with weights $= W_k^{y-1}$

Geometric - ***MG2***: As (28) with weights= W_k^{STIX} in (21b) above.

The CES-index now takes on the following shape:

CES - ***MC(σ):*** $$I_{y-1,12}^{y,12} = \left[\sum_k W_k^{STIX}\left(\frac{P_k^{y,12}}{P_k^{y-1,12}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \qquad (30)$$

The numerical results of this comparison are shown in Table 2. For brevity, only one CES index is shown - the one with the “optimum” σ value. Some noteworthy features are:

1. The present long-term index on average well approximates a Törnqvist index. Over the years, LTIX-MT goes from +0.12 to –0.11.

2. The σ value in the MC index which best approximates a Törnqvist index is 0.6. Looking at single years shows, however that MC(.6) - MT varies from +0.20 to -0.25. This means that, if we have MC(.6) as a short-term index and a superlative long-term index, then the shift when going from a short- to a long-term index would be between these numbers. As a comparison, the present difference between STIX and LTIX varies between -0.08 and 0.27. That is, STIX is biased but with somewhat smaller variation than MC(.6).

4.4.3 Chain indices: linking by year

When linking by year it becomes necessary to establish a connection between the annual and the monthly index, since the $\boldsymbol{I}_{y-1}^{y}$ in (19) or (20) have to be built up from monthly indices. For each product group k, we need to aggregate month-on-month indices $\frac{P_k^{y,m}}{P_k^{y-1,m}}$ or price levels $P_k^{y,m}$ and $P_k^{y-1,m}$ to year-to-year indices $\frac{P_k^{y}}{P_k^{y-1}}$. For the simulations, we used the geometric time aggregation principle throughout, that is

$$\frac{P_k^{y}}{P_k^{y-1}} = \prod_m \left(\frac{P_k^{y,m}}{P_k^{y-1,m}} \right)^{\frac{1}{12}}, \tag{31}$$

(Below, in Section 5, we discuss the calculation method of the year-to-year index further.)

The following index types were explored in the simulations:

Törnqvist - *YT* : $I_{y-1}^{y} = \prod_k \left(\frac{P_k^{y}}{P_k^{y-1}} \right)^{W_{k;T}}, \qquad (32)$

where the weights $W_{k;T}$ are defined according to (28b)

Walsh – YW: $I_{y-1}^{y} = \sum_k W_{k;W} \frac{P_k^{y}}{P_k^{y-1}}, \qquad (33)$

$$\text{where } W_{k;W} = \sqrt{\frac{V_k^{y-1} V_k^{y}}{P_k^{y} / P_k^{y-1}}} \Bigg/ \sum_k \sqrt{\frac{V_k^{y-1} V_k^{y}}{P_k^{y} / P_k^{y-1}}} \tag{33b}$$

Edgeworth - *YE*: $I_{y-1}^{y} = \sum_k W_{k;E} \frac{P_k^{y}}{P_k^{y-1}}, \qquad (34)$

$$\text{where } W_{k;E} = \left(V_k^{y-1} + \frac{V_k^{y}}{P_k^{y} / P_k^{y-1}} \right) \Bigg/ \left(\sum_k V_k^{y-1} + \sum_k \frac{V_k^{y}}{P_k^{y} / P_k^{y-1}} \right). \tag{34b}$$

Note that the weights in (32), (33) and (34) are now symmetric with respect to the pricing periods. Indices according to (32) and (33) are thus, by definition, true superlative indices. This is a clear advantage of the year-to-year index, which no other index formulation can fully meet.

For the final year-to-month index, a base-weighted index is necessary and we again define the three formulae that are possible for this situation.

Laspeyres - *YL*: $I_{y-1}^{y,m} = \sum_k W_k^{y-1} \frac{P_k^{y,m}}{P_k^{y-1}} \qquad (35)$

The year-to-year index that will be used in the simulations below will then be:

$$I_{y-1}^{y} = \sum_k W_k^{y-1} \frac{P_k^{y}}{P_k^{y-1}} \tag{35b}$$

Geometric – YG: As (28) with weights $= W_k^{y-1}$.

CES - *YC(σ)*: $$I_{y-1}^{y,m} = \left[\sum_k W_k^{y-1} \left(\frac{P_k^{y,m}}{P_k^{y-1}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{36}$$

The year-to-year index that will be used in the simulations below will then be:

$$I_{y-1}^{y} = \left[\sum_k W_k^{y-1} \left(\frac{P_k^{y}}{P_k^{y-1}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{36b}$$

In Table 3, we present the simulation results on year-to-year indices. For reference, we have also included the monthly Törnqvist index. We now note that:

1. The true superlative indices YW, YT and YF are very close together as is also YE, an "almost superlative index". They differ by at most 0.01 and usually much less. On average, the difference is 0.001-0.002. For all practical purposes, they could thus be considered numerically equivalent[5].
2. The long-term index is below the superlative indices, on average by 0.04. It is larger, however, in 7 years out of 17 so the difference is not necessarily significant. It could still be interpreted as some evidence of the Paasche effect demonstrated in (10) above.
3. The Laspeyres' index is about 0.05 and the short-term index about 0.08 larger than the superlative indices. These differences are persistent over almost all years and clearly significant.
4. A CES index with σ slightly smaller than 0.4 approximates a superlative index on average over the whole period. Looking at single years shows that [YC(.4) – YT] varies from 0.06 to -0.17. In most years the difference is smaller than 0.1. This means that, if we would have YC(.4) as a short-term index, then we could expect the shift when going from a short- to a long-term index to be between these numbers.
5. The geometric index with old weights significantly underestimates a superlative index, on average by about 0.08.
6. The linking-by-month index (MT) is biased downwards by 0.02 on average. The reason for this is that the weights on average tend to be "biased backwards" in time, moving MT to some extent towards a base-weighted geometric index.

4.5 Data access and index construction in practice

The present production of an index series according to (5) – (7) involves an annual reweighting process based on various data sources, which takes place in January-February each year.

[5] This statement has to be qualified for the situation where one price is zero – YT then collapses to zero or infinity. This situation does not show up in simulations at this high aggregation level but happens at lower levels from time to time.

We will here discuss briefly the data and timing problems that the other chain/link choices will give rise to.

Weight data access. The new household expenditure surveys (HES) are planned to run continuously and calendar year results will become available about six months after the reference year. However, the sample size is small (2000 households) and large non-response rates are expected. National Accounts (NA) values are available for some 90 categories (purposes). A preliminary version, used at present, is available in early January, t+1 (immediately after the end of the year). A revised version is available in November, t+1 and a final version in November, t+2. An ambition, not yet fully reached, is also to have a division into finer categories, finished in a preliminary version by November, t+1 and revised in November, t+2.

Fixed base indices. In this case, it is no point in using the rather crude estimates in January, t+1. Thus a new link with base year t would start from January, t+2, (t+3 if one wants to wait for the revised values). In the case of a 5-year link continuing to December, t+6 (t+7), weights would on average, over the link, be 3½ (4½) years old.

Annually chained indices, linking by month. In this case, the presently used formulae – (5)-(7) - are one choice. Other combination of long and short-term links to be entered into (18) would also be possible such as (with the above acronyms) i) MT+MG1, ii) MT+MG2, iii) MT+MC, iv) ME + STIX, v) ME + MC and vi) only STIX. The list is not exhaustive.

Annually chained indices, linking by year. Here one idea would be to have a superlative index (YF, YT, YW or YE, although the latter is not strictly superlative) for the year-to-year links and either a Laspeyres, a geometric or a CES index for the final year-to-month link. Another option would be to have Laspeyres indices throughout.

4.6 An evaluation of construction alternatives

We will now look at how our various alternatives perform with regard to the criteria set out in Section 2.

4.6.1 Minimal long-term bias in relation to cost-of-living index/superlative index.
Table 4 gives average biases over 18 years according to our simulations, for the directly comparable formulae. Some alternatives are not possible to compare directly with YT. We then used LTIX as an intermediate standard and added the bias of LTIX (–0.04) to their estimated biases according to Tables 1 and 2. Results are in Table 5.

Table 4: Bias of index formulae, with YT as the standard, according to Table 3, 1981-98

Formula	Average bias	Range of bias variation
YW, YF[6]	0	.
YE	0.00	-0.02 to +0.01
YL	+0.05	-0.03 to +0.11
YG	-0.09	-0.38 to +0.01
YC(.4)	-0.00	-0.17 to +0.06
MT	-0.02	-0.15 to +0.07
ME	+0.02	-0.05 to +0.08
STIX	+0.08	-0.02 to +0.19
LTIX	-0.04	-0.27 to +0.13

Table 5: Average bias of various index formulae, with YT as the primary standard and LTIX as the intermediate standard, according to Table 1-2, 1981-97

Formula	FL	FG	FC(.7)	MC(.6)	MG1
Average bias	0.15	-0.11	0	0	-0.10

Bias variation over the years is only meaningful to estimate with the YT standard. For the alternatives in Table 5, it is likely to be at least as large as for those in Table 4, however.

We see that the fixed base options with FL and FG both have non-negligible biases, in different directions. The CES index (FC) appears to be unbiased on average, but this is partly illusory, since it requires perfect knowledge of σ, the elasticity of substitution, which we do not have. Since σ could be assumed to be rather stable FC would give small average biases, though.

Linking by month will not either give us unbiased estimates in the long run, although with the MT or ME formulae, biases seem to be quite small. Again, the MC index would be nearly unbiased. Since the σ would now be possible to reestimate each year we would be in a better position with MC than with FC. The LTIX or STIX will not provide unbiased indices – according to the simulations the present index construction adds additive biases of –0.04 each year on top of each other, whereas the alternative with only STIX leads to annual biases of +0.08.

Linking by year with a superlative or Edgeworth index for the year-to-year index does give us an unbiased index series. This is independent of the choice of year-to-month index, since its influence is cancelled out in the long run. YL or YG are both biased in opposite directions, by half a decimal point.

4.6.2 Minimal bias and disturbance in short-term changes

The 12-month change of the index is a key economic statistic, which is much analysed by central banks, and economists engaged in macro-economic analyses. It is therefore desirable that this statistic in itself is not biased and that a time series of such changes is not unduly influenced by other factors than pure price changes. It is therefore instructive to look at the definition of this statistic under different chain/link constructs.

---

[6] The small differences between the superlative indices are not indicative of any bias, since neither of them can be proved superior on theoretical grounds.

Fixed base index: $I_{Y-1,m}^{Y,m} = \dfrac{I_0^{Y,m}}{I_0^{Y-1,m}}$ (37)

12-month change: linking by month (December): $I_{Y-1,m}^{Y,m} = \dfrac{I_{Y-1,12}^{Y,m} I_{Y-2,12}^{Y-1,12}}{I_{Y-2,12}^{Y-1,m}}$ (38)

Linking by year according to (19): $I_{Y-1,m}^{Y,m} = \dfrac{I_{Y-1}^{Y,m} I_{Y-2}^{Y-1}}{I_{Y-2}^{Y-1,m}}$ (39)

Linking by year according to (20): $I_{Y-1,m}^{Y,m} = \dfrac{I_{Y-2}^{Y,m} I_{Y-3}^{Y-2}}{I_{Y-3}^{Y-1,m}}$ (40)

Now, in the fixed base index the 12-month change will usually be within an index link. The numerator and the denominator use the same weights and the statistic will only depend on the fixed and the actual price changes. The long-term bias is rather stable and thus a small problem, when looking at a series of such changes. This last property is an advantage for example for central banks that want to monitor inflation by looking at how series of 12-month changes develop.[7]

In a chain index, on the other hand, the 12-month change will usually be between two consecutive index links so that there is a weight change in between. This weight change is an unwanted disturbance and, at the turn of each year, there will be a break in the time series so that the change in the 12-month rate when going from December to January will be only partly due to real price changes and partly to weight changes and other technical adjustments to the index. (A chain index based on linking only short-term indices according to (6) would not have this problem.) On the other hand, but perhaps less importantly for inflation analysts, the estimates will be unbiased on average.

The sizes of these disturbances have been simulated. We do not present these simulations in detail here but on average the absolute sizes of these differences are around 0.1 for all the major chain index alternatives discussed here, except the one with only short-term indices. At the maximum, the “annual jumps” can be as large as 0.4. For the delayed chain index, the jumps are the largest but the differences are not great.

4.6.3 Axiomatic considerations
It is only meaningful to look at axiomatics within each index link. The basic index tests - monotonicity, identity, proportionality and dimensional invariance (see Balk, 1995 for their definition) - are satisfied by all formulae considered here.

The time reversal test is satisfied by all superlative formulae (and Edgeworth) but not the base period weighted formulae. We consider this test important, since it guarantees some kind of symmetry between the periods compared that indicates unbiasedness.

[7] On the other hand, in the end of every link it will eventually be possible to recalculate these changes based on the new weights. This creates a situation where inflation history will be rewritten which can also create problems.

The factor reversal test is only satisfied by Fisher's index. We do not consider this test crucial for a CPI, however, since a dual volume index is not directly asked for.

The geometric indices, including Törnqvist, break down for zero prices, which the other indices can manage. This property has some importance, since prices for some public services sometimes change from/to zero.

Consistency in aggregation is only satisfied by the base period weighted formulae, not by the superlative indices. However, we believe that this test is too strong to be of importance. A weaker additivity criterion is discussed below under the heading *easy to analyse*.

Diewert (1999) recently proposed a test, which distinguishes the Edgeworth index, which does not satisfy it, from the other indices, which do satisfy it. He calls this the *invariance to proportional changes in current quantities test* with the following definition[8]:

$$P(p^0,p^1,q^0,\lambda q^1) = P(p^0,p^1,q^0,q^1) \text{ for all } p^0,p^1,q^0,q^1 \text{ and all } \lambda>0 \qquad (41)$$

This test is more relevant in a spatial comparison between two countries of different sizes. In a temporal comparison, especially between two consecutive years, the quantity vectors are sufficiently close for this problem not to be significant.

In summary, we consider the Fisher, Walsh and Edgeworth indices to satisfy the important criteria, with a slight disadvantage for Edgeworth.

4.6.4 Correct handling of seasonal products

This criterion speaks strongly for a year base period and against the present system with month base periods. The criterion does not distinguish between formulae, since these could all, with some difficulty, be adapted to take care of seasonal products.

4.6.5 Timeliness

Chain indices have some problems with the first monthly index in a new link because of the introduction of new weights in that index which takes extra time and causes a delay. In a fixed base index, one simply continues with the old weights until the new are finalised and thus "solves" the problem. In the delayed chain index it should be possible to avoid the annual delays, since there will be more time for the weight preparations.

4.6.6 Easy to interpret and analyse

Neither the Fisher, nor the geometric or Törnqvist indices can easily be looked at as monitoring the cost for a fixed basket of products. They cannot either be written in the established algebraic form of a sum of weights times subindices, which the producers and analysts are used to handle. This is a considerable disadvantage for such a broadly used, popular index as the CPI.

---

[8] "There is a potential problem with the use of the Edgeworth Marshall price index that has been noticed in the context of using the formula to make international comparisons of prices. If the price levels of a very large country are compared to the price levels of a small country using formula then the quantity vector of the large country may totally overwhelm the influence of the quantity vector corresponding to the small country.[8] In technical terms, the Edgeworth Marshall formula is not homogeneous of degree 0 in the components of both q^0 and q^1. To prevent this problem from occurring … (*we ask that the price index)* satisfy the following *invariance to proportional changes in current quantities test*"

However, both the Walsh and the Edgeworth index (like the Laspeyres index) possess these important properties, since they are defined as the ratio of the costs for two baskets with the quantities being averages of those of the base and the comparison period. Thereby, they also possess a simple additivity property, in the sense that weights for subindices can simply be added to obtain weights for higher aggregates. This criterion can be seen as a weaker form of the consistency in aggregation test and we believe that it covers what is needed for practical CPI purposes.

4.6.7 Non-revisability

All our proposals in this report satisfy non-revisability by design. If this criterion were not crucial, various other possibilities would exist which could lead to greater reliability in both long and short term comparisons. Such solutions would probably also add to the complexity of the system, however.

4.6.8 Co-ordination with the HICP

The HICP is defined as a "Laspeyres-type index" with a year as the index reference period. At the same time, there is a requirement to update weights and introduce new products annually, if such measures are important for comparability. None of the solutions discussed in this report fully satisfies HICP requirements. However, the additional work for adapting a KPI system to the HICP requirements are not judged to introduce any new problems compared with today.

4.6.9 Resource aspects

Changing over to a new aggregation system of course leads to a one time extra cost. It is outside the scope of this report to estimate this cost. The current costs for running the systems are lowest for a fixed base system but it depends on the ambition, when it comes to introducing new products and outlets at the lower level of the index. The cost for having a year as a base period is probably somewhat higher, since the calculation systems at lower levels will involve more links and longer time spans (see below). The difference is not believed to be large, however.

4.6.10 Summarised evaluation

With the criterion *minimal long-term bias in relation to a cost-of-living index/superlative index* as our main criterion, the above evaluation clearly boils down to a recommendation to choose a **delayed chain index** according to equation (20). This choice immediately makes a superlative index aggregation over some 90 product groups possible and, with better National Accounts and other sources for weights, it opens for extending superlative aggregation to much finer levels of the index.

For the year-to-year links in this index, all superlative formulae (including Edgeworth) are virtually equivalent from a bias point of view. The criterion *easy to interpret and analyse* points at Walsh or Edgeworth as the clearly best choices. A small edge is given in favour of **Walsh**, due to its true superlative properties and meeting more tests.

For the year-to-month link, simplicity speaks in favour of the **Laspeyres** index. Its bias does not influence the long-term bias of the chain index as a whole.

5. Index aggregation at lower levels

The analysis above has led us to recommend the following basic KPI construction.

The index from reference year 0 to year Y, month m should be a chained index according to the following formula:

$$I_{0}^{Y,m} = I_{Y-2}^{Y,m} \prod_{y=1}^{Y-2} I_{y-1}^{y} \tag{42}$$

For the year-to-year index, the Walsh index should be used:

$$I_{y-1}^{y} = \frac{\sum_k P_k^{y} \sqrt{Q_k^{y} Q_k^{y-1}}}{\sum_k P_k^{y-1} \sqrt{Q_k^{y} Q_k^{y-1}}} = \sum_k W_{k;W} \frac{P_k^{y}}{P_k^{y-1}}, \tag{43}$$

where $W_{k;W} = \sqrt{\frac{V_k^{y} V_k^{y-1}}{P_k^{y} / P_k^{y-1}}} \Big/ \sum_k \sqrt{\frac{V_k^{y} V_k^{y-1}}{P_k^{y} / P_k^{y-1}}}$. (43b)

For the year-to-month index, the Laspeyres index should be used:

$$I_{Y-2}^{Y,m} = \sum_k W_k^{Y-2} \frac{P_k^{Y,m}}{P_k^{Y-2}}, \qquad \text{where } W_k^{Y-2} = V_k^{Y-2} \Big/ \sum_k V_k^{Y-2} \tag{44}$$

This computational scheme will be possible to use down to at least 90 product groups for which consumption values from the National Accounts will be available. Work is going on to extend this breakdown further down the product hierarchy.

In this section, we will deal with the problems that emerge below the level, where these value weights are available.

5.1 Aggregation steps

For the purposes of the present analysis, it is useful to speak of four aggregation steps. In actual index calculation, there may sometimes be more or (less often) fewer steps than that but they can then be traced back to one of these four steps. We will, somewhat unnaturally number them from top to bottom. We will give examples of the kind of situations that we have in mind as we go along.

The first step - the combination of some 90 NA purposes into the All Item Index
In this step, current NA value weights are guaranteed in our proposed system. The purposes are according to the international Coicop (Classification Of Individual Consumption by Purpose) system and are similar to those used in the HICP.

The second step - the combination of 3-400 product groups into 90 NA purposes
In this step NA values are also possible but not guaranteed and we have to prepare for some less than superlative aggregation method as the best estimate.

Aggregation is in this case over products within the same product group. In some cases, these are fairly close substitutes such as pork and beef within the meat category. In other cases, substitutability is much smaller such as between dishwashers and refrigerators in the household appliances category.

The third step – combining elementary aggregates into product groups
This step is quite heterogeneous in nature. In most cases value weights will not be available, although exceptions exist. In the third step product groups subindices are constructed, sometimes in several steps, from subgroup, outlet and/or regional elementary aggregates. For this step, there is a great variety of situations. A few examples are given here.

*Local price collection***.** For some 150 representative products (mainly goods), local price collectors monitor the prices directly in the outlets. For these products there is a stratification into industries (outlet types) according to SNI 92 (the Swedish version of the NACE system for industrial classification). (Elementary aggregation is done nationally, over all outlets in one industry.) Weights over industries are based on a survey planned to be 5-yearly, reflecting the distribution of sales of a certain product group between industries.

*Daily necessities***.** The daily necessities system introduces an additional stratification compared with the local price system. Here, e.g., the industry *Supermarkets* (SNI 52.112) is further divided into three strata according to the chain a certain outlet belongs to (*ICA/KF/DAGAB*).

*Electricity***.** In this subsystem aggregation is over the following dimensions: i) Network fees vs. energy fees, ii) type of housing, iii) fixed vs. variable (=according to consumed amount) fees and iv) type of tariff used (partly according to the consumer's choice).

*Municipal services for owner-occupiers***.** Here, aggregation is over i) type of service (water and sewage, refuse collection and chimney sweeping) and ii) municipalities.

*Telephone services***.** Aggregation is over i) main type of services (mobile services, residential services, and Internet services), ii) company providing the service and iii) detailed services according to the tariff of the provider.

*Health care***.** Aggregation is over i) type of service and ii) organisation providing the service, where the organisation is usually a county council (*landsting*) so that there is also a geographical division involved in this aggregation step.

We observe that the third level displays a very mixed picture with regard to substitutability. The subgroups sometimes represent entities between which substitutability is large (e.g. outlet chains) and sometimes very small (e.g. municipality of residence).

The fourth step – aggregating price quotes into elementary indices
At this level, single observed prices are aggregated to elementary indices. In practice, there are several different cases here. Fairly typical examples in the KPI are:

1. A representative item (e.g. a refrigerator, a TV set, a restaurant meal) is defined in the central office. Varieties of this item are selected by price collectors and are thus aggregated over a certain type of randomly (PPS[9]) selected outlets (5-digit NACE number) in all of Sweden. The available weight information is the size of the outlet in terms of its number of employees (with a 2-year lag).
2. For most item groups in daily necessities (e.g. bread, cereals, frozen fish, washing detergents etc.), typically purchased in supermarkets, randomly (PPS) selected items (e.g. cod fillet, Findus, 400g) are aggregated over randomly (PPS) selected outlets, divided into three major wholesaler/retailer chains, in all of Sweden. In addition to the size of the outlet, the weight of the item in terms of its national sales 1-2 years ago is known.
3. Water and sewage charges are aggregated over a number of randomly selected municipalities. The population size of the municipality is used as weight.
4. Complete information of the sales of all alcoholic beverages by the Government retail monopoly (Systembolaget) which uses national prices. The exact quantities/values sold of each bottle/can of beer, wine or spirit are known.
5. Electricity or telephone charges are aggregated over a number of providers. Usually, good information on quantities (kWh, call minutes etc.), although sometimes with a lag, is available.

As a rule, substitutability within the aggregates will be large[10]. Sometimes there are also weights involved at this level, sometimes not. In some cases such as tariff prices in item 5 of the list, there are specific problems, which will not be dealt with here.

5.2 Estimates of price elasticity

For determining the best aggregation method in the absence of current value weights, we will use the price elasticity criterion. We will assume that the relation between price elasticity and index formula is given by the CES price index in (16) above.

With this formula in mind, the empirical issue becomes one of determining the value of the price elasticity σ in different cases. Unfortunately, the available information is not abundant. Most estimates are done in the U.S., only one recent Swedish source is known. We will first look at these estimates at a level roughly corresponding to the second aggregation step. Then we will look at evidence relating to the third and fourth steps.

5.2.1 Second step elasticities

The only recent Scandinavian source for price elasticities is Edgerton et al. (1996). They analyse food demand in the Nordic countries based on NA data from 1963-89 for Sweden and similar time periods for the other Nordic countries. They use a three-level aggregation, where food consumption is divided into animalia, vegetabilia, beverages and miscellaneous and each of these categories is in turn divided into three subgroups, the lowest level considered in their study. Table 6 gives their estimated own-price elasticities for each of the 12 groups:

---

[9] Probability Proportional to Size

[10] Although an aggregate often covers the whole country, price changes do not usually follow a regional pattern. This means that the observed variation within an aggregate often represents variation between nearby outlets.

Table 6: Own-price elasticities (reversed sign) for the Nordic countries, estimated by Edgerton et al (1996)

	Denmark	*Finland*	*Norway*	*Sweden*
Animalia *Meat*	*0.5*	*0.2*	*0.7*	*0.4*
Fish	*0.9*	*0.3*	*0.8*	*0.3*
Milk, cheese and eggs	*0.4*	*-0.1*	*0.2*	*0.0*
Beverages *Soft drinks*	*1.1*	*1.2*	*0.7*	*0.6*
Hot drinks	*0.2*	*0.2*	*0.3*	*0.1*
Alcoholic drinks	*0.5*	*0.6*	*0.9*	*0.9*
Vegetabilia *Bread and cereals*	*0.4*	*0.1*	*0.4*	*0.7*
Fruit and vegetables	*0.5*	*0.3*	*0.6*	*0.6*
Potatoes	*0.5*	*0.5*	*0.5*	*-0.1*
Miscellaneous *Fats and oils*	*0.6*	*0.3*	*0.2*	*0.3*
Sugar	*0.7*	*0.3*	*0.2*	*-0.8*
Confectionery etc.	*0.8*	*0.4*	*0.1*	*0.4*

The groups in Table 6 are, however, groups, that belong to the upper level of KPI aggregation, where current consumption values are available, at least in principle[11].

McLelland (1999), at the U.S. Bureau of Labor Statistics (BLS), makes detailed comparisons between outcomes of different index formulae within the expenditure class level of the U.S. CPI (70 groups divided into 207 item strata). In the KPI, this level roughly corresponds to the division into 300 subindices mentioned above. He finds that the estimated elasticity of substitution is close to or greater than unity in the majority of cases. In comparing the Fisher, Laspeyres and Geometric indices he finds that Laspeyres is significantly different from the Fisher indices in 65 to 80 percent of the cases, whereas the Geometric index (with old weights) only differs significantly in about 15 percent of the cases. Thus the Geometric index is judged to be a better estimator of a cost-of-living index than the Laspeyres, when only old weights (from period 0) are available. McLelland's main objective is to establish the best methodology at the elementary index level. He argues that, due to increased substitution at the lower levels of the index, the elasticity generally increases as we move down in the index hierarchy. This causes the differential biases of the Laspeyres and the Geometric indices in relation to Fisher to move further in favour of the Geometric index.

A survey of published estimates of demand elasticity in the U.S. shows a somewhat mixed picture. Three important recent works in this area are Huang (1993), Berry, Levinsohn and Pakes (1995) and Hausman (1997). Huang is concerned with the food sector. He estimates the elasticity of food at home as a whole to be 0.19 but as the level of disaggregation increases the elasticity also generally increases, although with some exceptions. In table 7, we present a summary of his estimates.

[11] At present, the NA values for these food categories are not based on current information but are just extrapolated from earlier years in fixed value proportions. It will, however, be possible to improve these matters with some reasonable effort.

Table 7: Price elasticities (reversed sign) in some food categories for the U.S. according to Huang (1993)

Subgroups of meat and fish	0.12 to 1.87
Flour, rice, eggs, milk and cheese	0.04 to 0.25
Subgroups of fruit	0.19 to 1.18
Subgroups of fresh vegetables	0.08 to 0.62
Subgroups of processed fruit and vegetables	0.17 to 0.74
Subgroups of beverages	0.18 to 0.56
Subgroups of sugar and fats	0.01 to 0.24

The difficulties in this area are underlined by some very different estimates by other researchers; Nelson (1994) estimates elasticity for milk to 1.15 and Heien and Pompelli (1989) to 0.69. Unpublished Swedish experiences from the recent period when milk subsidies were abolished and prices thus suddenly increased by some 20% were that purchasing volumes were hardly influenced at all, a fact that supports the low elasticity estimates for milk. Hausman gives 0.90 as his estimate for breakfast cereals as a whole. For cars as a whole, three estimates are 0.87 (McCarthy, 1996) 0.85 (Levinsohn, 1988) and 1.43 (Trandel, 1991).

The great variability of the elasticity estimates, also for the same product groups, precludes strong conclusions for any particular group. The estimates cover the whole range from slightly above zero up to values above one and thus make a definite judgement on average price elasticity difficult.

We consider McLelland's results as the single strongest evidence relating to the formula choice at this level, since he compared index formulas with their superlative counterparts afterwards and with actual CPI data. His conclusion is that the estimated elasticity of substitution is close to or greater than unity in the majority of cases and thus that the geometric mean estimator, which is based on an assumption of price elasticity being equal to one, estimates a superlative index better than a Laspeyres index does.

With Table 7 in mind, it may be possible to make informed guesses as to categories where an assumption of unitary price elasticity is not reasonable and apply different aggregation principles in the two cases.

5.2.2 Low-level elasticities

Let us first ask: What is the economic interpretation of an elementary aggregate (EA) and of the kind of consumer behaviour that takes place within it? For food, daily necessities and some other products, the EA in the KPI consists of a set of varieties in a set of outlets defined either as a chain of retailers or as an outlet type (example hypermarkets, radio/TV-retailers etc.) defined by the 5-digit NACE level (SNI 92, the Swedish NACE version). In contrast to most other countries, no regional demarcations are applied. How, then, do consumers behave in response to price changes within such an EA?

Firstly, we note that much of the low-level substitution would take place also between two EAs, for example between a hypermarket and a local supermarket or between a Konsum and an ICA supermarket in the same area. This calls for a “substitution allowing” aggregation principle between different outlet groups for the same product. Within an EA, there will be substitution effects unless the price changes display a strong regional pattern. Regional effects are not strong, since much of the price change is known to be due to short-term fluctuations, "price bouncing" etc. Therefore, within an EA substitution effects are generally large.

What substitution effects are we talking about more exactly at this level? Dalton et al (1998) made an excellent account of this when they explained the BLS change to the geometric mean formula at the EA level. *Ice cream* was used as their exemplifying product:

> "Substitution can take several forms corresponding to the types of item- and outlet-specific prices used to construct the basic indexes:
>
> - Substitution among *brands of products*, for example, between brands of ice cream;
> - Substitution among *product sizes*, for example, between pint and quart packages of ice cream;
> - Substitution among *outlets*, for example, between a brand of ice cream sold at two different stores;
> - Substitution across *time*, for example, between purchasing ice cream during the first or second week of the month;
> - Substitution among *types of items* within the category, for example, between ice cream and frozen yoghurt;
> - Substitution among *specific items in different index categories*, for example, between ice cream and cupcakes.
>
> Thus, in response to an increase in the price charged by a store for a certain brand of ice cream, a consumer could respond by redistributing purchases along any of several dimensions represented by other priced items in the category: to another brand of ice cream whose prices had not risen, to a larger package of ice cream with a smaller price per ounce, to ice cream at a different store where ice cream is on sale, or to a brand of frozen yoghurt. The consumer also could respond by postponing the ice cream purchase until a later date. …
>
> Finally, the consumer could substitute from the ice cream brand to a specific alternative dessert item, such as cupcakes or apples, that is in another CPI category."

The distinctions between all the kinds of substitution elements described in this quotation are not well developed in economic research[12]. What can be said under these circumstances?

In general it is reasonable to expect elasticities to increase when we move down the product hierarchy to more detailed levels. This is because we find closer substitutes at the lower levels than at the higher (pork vs. beef instead of meat vs. fish, Brand A vs. Brand B coffee instead of coffee vs. tea, Coca Cola in outlet A vs. the nearby outlet B instead of Coca Cola vs. Fanta). However, not many studies of low level elasticities have been done so far.

Berry et al estimates elasticity for cars and give estimates between 3.08 (Lexus LS400) and 6.52 (Nissan Sentra) at the brand level as compared with the above-mentioned estimates for cars as a whole between 0.85 and 1.43.

---

[12] A research line that is in its infancy is that of consumer search theory. Reinsdorf (1994) discusses the implications for the cost of living index of searching consumers in markets with price dispersion for products of constant quality.

Hausman deals with breakfast cereals, also at the brand level. His estimates of elasticity for particular brands range from 1.93 (Cheerios) to 3.18 (Frosted Wheat Squares) as compared to 0.90 for the group as a whole.

Reinsdorf (1996) carried out a comparison between different index formulas based on A.C. Nielsen scanner data for outlets in Washington and Chicago for coffee, 1993-94. His data showed that the geometric mean index (with base period weights) is in most cases closer to the Fisher index than the Laspeyres index. Other unpublished BLS research on scanner data hints at even stronger results; the most typical pattern seems to be Laspeyres>Geomean>Fisher which is consistent with a price elasticity of more than one.

We carried out a small experiment based on Swedish scanner data for some item groups, where we estimated the following average within-outlet price elasticities for some item groups[13].

Table 8: Average price elasticities for EAN codes (within brands) in some item groups based on scanner data, 1994-1996.

Item group	**Weighted median price elasticity of demand**
Frozen fish	2.35
Breakfast cereals	2.00
Washing detergents	2.05
Fats	1.21

These estimates (where very crude estimation methods were used) turn out to be in the lower range of the brand-type elasticities seen in U.S. data but still well above unity.

5.2.3 Summary of information on price elasticities

Tentative conclusions based on the scarce information given above would be:

1. For the second aggregation step, own-price elasticities vary from slightly above zero up to well above one. There are some inelastic products (for example milk, potatoes, flour and rice) as well as more elastic one like new cars and breakfast cereals.
2. At lower levels, elasticities are generally much higher and often well above unity for brands in competitive product areas.

5.3 Approximations to the Walsh formula for the second aggregation step

Where current consumption values are not available, two special cases of the Walsh formula are of interest.

Firstly, assume zero elasticity. This is consistent with the assumption that the *quantity ratios* between products remain intact despite the change in relative prices. Say that in (43) we have $\boldsymbol{Q_k^y = \lambda Q_k^{y-1}}$ for all k . The Walsh index then reduces to the Laspeyres index

[13] The data covered 117 weeks from 9440 to 9652. All purchases in 26 outlets were included. The number of item descriptions averaged over was around 100. The first week in which the item appeared in a shop was taken as the base for shopwise calculations. The median of the estimated price elasticities was taken over shops and weeks and finally the medians were averaged over the item descriptions. Weights, $(p-p_0)(q-q_0)$, were applied.

$$I_{y-1}^{y}=\frac{\sum_{k}P_{k}^{y}Q_{k}^{y-1}}{\sum_{k}P_{k}^{y-1}Q_{k}^{y-1}}=\sum_{k}\frac{V_{k}^{y-1}}{\sum_{k}V_{k}^{y-1}}\frac{P_{k}^{y}}{P_{k}^{y-1}} \tag{45}$$

If the known quantities (or values) are from a period before y-1 (from period s, say), then the invariance of the quantity ratios implies that $Q_{k}^{y}=\lambda Q_{k}^{y-1}=\lambda\mu Q_{k}^{s}$ for all k . Instead of (45) we then obtain:

$$I_{y-1}^{y}=\frac{\sum_{k}P_{k}^{y}\sqrt{\lambda\mu Q_{k}^{s}\mu Q_{k}^{s}}}{\sum_{k}P_{k}^{y-1}\sqrt{\lambda\mu Q_{k}^{s}\mu Q_{k}^{s}}}=\sum_{k}\frac{V_{k}^{s}\left(P_{k}^{y-1}/P_{k}^{s}\right)}{\sum_{k}V_{k}^{s}\left(P_{k}^{y-1}/P_{k}^{s}\right)}\frac{P_{k}^{y}}{P_{k}^{y-1}} \tag{46}$$

Price updating the weights from period s to period y-1 is thus a logical choice in this case.

Secondly, assume unitary price elasticity. This is consistent with the assumption that the *value ratios* between products remain intact despite the change in relative prices. This implies that in (43) we have $V_{k}^{y}=\lambda V_{k}^{y-1}$ for all k. The Walsh index then reduces to the following index:

$$I_{y-1}^{y}=\frac{\sum_{k}V_{k}^{y-1}\sqrt{P_{k}^{y}/P_{k}^{y-1}}}{\sum_{k}V_{k}^{y-1}/\sqrt{P_{k}^{y}/P_{k}^{y-1}}}=\sum_{k}\frac{V_{k}^{y-1}/\sqrt{P_{k}^{y}/P_{k}^{y-1}}}{\sum_{k}V_{k}^{y-1}/\sqrt{P_{k}^{y}/P_{k}^{y-1}}}\frac{P_{k}^{y}}{P_{k}^{y-1}}, \tag{47}$$

If the known values are from a period before y-1 (from period s, say), then the invariance of the value ratios implies that $V_{k}^{y-1}=\mu V_{k}^{s}$ for all k . We can then put the period s values in the period y-1 values' place in (47) without any other changes. Especially, no price updating is called for in this case.

Therefore, if only old value weights are known, then (45)-(47) can be chosen given an assessment of the most likely value of the elasticity of substitution. The analysis above suggests that unitary price elasticity is more often the best approximation, which would make (47) the main option.

For the year-to-month there is no question of approximation since the Laspeyres formula applies also in the second step. If the weights are from a period before y-1 then price updating should be done only where zero price elasticity is assumed, i.e. where the Laspeyres index is used also in the year-to-year index.

5.4 The third aggregation step

The same logic essentially applies for the third step as for the second step. As noted above this aggregation can represent many different situations with regard to substitutability and elasticity. For the case of unitary price elasticities, the internationally recognised geometric mean formula provides an alternative that can be proved (see Annex 1) to give almost the same answer as (47)[14].

$$I_{y-1}^{y} = \prod_{k}\left(\frac{P_k^{y}}{P_k^{y-1}}\right)^{\frac{V_k}{\sum_k V_k}} \tag{48}$$

In Annex 1, it is demonstrated that (47) and (48) give the same results to almost the third order according to a Taylor approximation, which only requires that the coefficient of variation of the price relatives be bounded.

The choice between these two formulae is thus a minor one. In favour of (47) one can argue that it provides formula consistency at all levels. For (48) speaks the fact, that it is the internationally accepted formula.

In the third step, elasticities are often larger than in the second step. Also, there is no need for presenting subindices at this level to users. Therefore, here we could consider aggregation according to (47) or (48) also in the year-to-month index in order to reduce substitution bias. For example:

$$I_{Y-2}^{Y,m} = \sum_{k}\frac{V_k \Big/ \sqrt{P_k^{Y,m} \big/ P_k^{Y-2}}}{\sum_k V_k \Big/ \sqrt{P_k^{Y,m} \big/ P_k^{Y-2}}}\,\frac{P_k^{Y,m}}{P_k^{Y-2}}, \tag{49}$$

5.5 Elementary aggregation

At the lowest level single prices are to be aggregated into indices. Aggregation based on unitary price elasticity should be the rule here in both the year-to-year and the year-to-month links. Here we denote weights by w_j and assume that they sum to one. In principle, they are value weights although they are often crude size measures. For the year-to-year index, the basic form would be either:

$$I_{y-1}^{y} = \frac{\sum_j w_j \sqrt{p_j^{y} \big/ p_j^{y-1}}}{\sum_j w_j \Big/ \sqrt{p_j^{y} \big/ p_j^{y-1}}} \quad \text{or} \tag{50}$$

$$I_{y-1}^{y} = \prod_{j}\left(\frac{p_j^{y}}{p_j^{y-1}}\right)^{w_j} \tag{51}$$

In these formulae, the price in each year is ideally interpreted as a unit value over months, i.e.:

$$p_j^{y} = \sum_{m=1}^{12} p_j^{y,m} q_j^{y,m} \Bigg/ \sum_{m=1}^{12} q_j^{y,m}\ , \tag{52}$$

[14] Here, we drop the time superscript for V_k, since the value weights are often older than y-1.

where $q_j^{y,m}$ is the quantity sold in month y,m. Often, equal quantities in all months will need to be assumed.

If it is desired to bring in new products or outlets annually, *link months* (*lm*) need to be introduced, so that an overlap is established in *lm*. This would change (50) and (51) into:

$$I_{y-1}^{y}=\frac{\sum_{j\in U_1} w_{1j}\sqrt{p_j^{lm}/p_j^{y-1}}}{\sum_{j\in U_1} w_{1j}/\sqrt{p_j^{lm}/p_j^{y-1}}}\;\frac{\sum_{j\in U_2} w_{2j}\sqrt{p_j^{y}/p_j^{lm}}}{\sum_{j\in U_2} w_{2j}/\sqrt{p_j^{y}/p_j^{lm}}}\quad\text{or}\qquad(53)$$

$$I_{y-1}^{y}=\prod_{j\in U_1}\left(\frac{p_j^{lm}}{p_j^{y-1}}\right)^{w_{1j}}\prod_{j\in U_2}\left(\frac{p_j^{y}}{p_j^{lm}}\right)^{w_{2j}},\qquad(54)$$

where U_1 and U_2 denote the two universes in the first and second links.

For the year-to-month index there will usually be a need to divide it into three links according to the following scheme:

$$I_{Y-2}^{Y,m}=\frac{\sum_{j\in U_1} w_{1j}\sqrt{p_j^{lm1}/p_j^{Y-2}}}{\sum_{j\in U_1} w_{1j}/\sqrt{p_j^{lm1}/p_j^{Y-2}}}\;\frac{\sum_{j\in U_2} w_{2j}\sqrt{p_j^{lm2}/p_j^{lm1}}}{\sum_{j\in U_2} w_{2j}/\sqrt{p_j^{lm2}/p_j^{lm1}}}\;\frac{\sum_{j\in U_3} w_{3j}\sqrt{p_j^{Y,m}/p_j^{lm2}}}{\sum_{j\in U_3} w_{3j}/\sqrt{p_j^{Y,m}/p_j^{lm2}}}\quad\text{or}\qquad(55)$$

$$I_{Y-2}^{Y,m}=I_{Y-2}^{lm1}I_{lm1}^{lm2}I_{lm2}^{Y,m}=\prod_{j\in U_1}\left(\frac{p_j^{lm1}}{p_j^{Y-2}}\right)^{w_{1k}}\prod_{k\in U_2}\left(\frac{p_k^{lm2}}{p_k^{lm1}}\right)^{w_{2k}}\prod_{l\in U_3}\left(\frac{p_l^{Y,m}}{p_l^{lm2}}\right)^{w_{3k}}\qquad(56)$$

Note that it is not necessary to have the same link months in all aggregates. Especially, where a product is seasonally unavailable, a month where it is available should be chosen as the link month. In most cases, it will be desirable to choose December or January, since the computation of an annual average price will then be facilitated.

Dividing the index into “sublinks” will sometimes be necessary also in the second and third aggregation steps. It is straightforward to apply the formulae for elementary aggregates for those cases also.

In Annex 2, we provide a table with the aggregation scheme from top to bottom.

The above description covers most practical situations. In some cases, however, the situation at hand calls for other solutions.

5.5.1 Superlative elementary weights

For some products, above all alcohol, there is detailed and current data available for weighting. They should then be used in a manner that combines the Walsh index with unit values over time for homogeneous products.

5.5.2 Inelastic aggregation and zero prices

If elementary aggregation is over observations between which price elasticity is small, a Laspeyres-type index should be used. By this, we mean the following index (written in its general form):

$$I_{01} = \frac{\sum q_b p_1}{\sum q_b p_0}, \tag{57}$$

where period b, usually before 0, is the most recent period for which quantity data is available.

This is also the preferred approach, where a price is zero in one end of the comparison. The existence of a zero price means that price elasticity in that point of the demand curve is zero[15], which in turn means that the index solution sought in this situation must be consistent with zero elasticity.

6. Summary

For a summary of this report we choose to quote the following excerpts from the official English summary of the KPI Commission (1999), which is entirely in line with this report.

> *"It is proposed that the CPI be a chain index consisting of year-to-year links that measure the relation between the average price level during two consecutive years. The chain index for a specific month is the product of these year-to-year links and a final link, which measures the relation between the price level during that month and the average price level during the calendar year two years earlier.*
>
> *On the relatively aggregated level, where up-to-date weight data are available, the year-to-year links should be based on the average composition of consumption during the two compared years (a Walsh index). For calculation of the year-to-month link, information on the composition of consumption in the base year should be used (a Laspeyres link).*
>
> *On a lower level, where relevant and up-to-date weight data are not available, calculations should be based, in most cases, on the assumption that shares in value terms are unchanged (unitary elasticity of substitution). Exceptions from this rule could be made in cases where it is not reasonable to assume that relative volumes are affected by changes in relative prices. When aggregating to higher levels, however, where alternative groupings of the index are made for analytic purposes, weights in the year-to-month link should be of the Laspeyres type, i.e. based on the consumption volumes in the base year."*

---

[15] Price elasticity is in general defined as $e = \frac{dq}{dp}\frac{p}{q}$, which means that it is identical to zero for p=0 if the demand function is differentiable.

7. References

Aizcorbe, A.M. and Jackman, P.C. (1993): *The Commodity Substitution Effect in CPI Data, 1982-91*. Monthly Labor Review, 116 (December), 25-33.

Balk B.M. (1995): *Axiomatic Price Index Theory: A Survey*. International Statistical Review, 63, 1, 69-93.

Berry, S., Levinsohn, J. and Pakes, A. (1995): *Automobile Prices in Market Equilibrium*. Econometrica, 63:4, 841-890.

Dalén, J. (1992): *Computing Elementary Aggregates in the Swedish Consumer Price Index*. Journal of Official Statistics, Vol. 8, No.2, 129-147.

Dalton, K.V., Greenlees, J.S. and Stewart, K.J. (1998): *Incorporating a Geometric Mean Formula into the CPI.* Monthly Labor Review, October, 3-7.

Diewert, W.E. (1976): *Exact and Superlative Index Numbers*. Journal of Econometrics, 115-145.

Diewert, W.E. (1978): *Superlative Index Numbers and Consistency in Aggregation*. Econometrica, 883-900.

Diewert, W.E. (1999): *The consumer price index and index number purpose.* Paper presented at the Fifth Meeting of the International Working Group on Price Indices (The Ottawa Group), Reykjavik, Iceland, August 25-27, 1999; Revised: September, 1999.

Divisia, F. (1925): *L'indice monetaire et la théorie de la monnaie*. Revue d'Economie Politique, 39, 980-1008.

Edgerton, D.L., Assarsson, B., Hummelmose, A., Laurila, I. P., Rickertsen, K. and Vale, P.H. (1996): *The Econometrics of Demand Systems,* Kluwer Academic Publishers, Dordrecht.

Haglund, M. (1992): *Vägningstal och priselasticiteter*. Internal Memo presented to the Swedish Index Board. Statistics Sweden, March 12.

Hausman, J. A. (1997): *Valuation of New Goods under Perfect and Imperfect Competition*." In Bresnahan, T., and R.J. Gordon, eds., *The Economics of New Goods*, 209-247. NBER Studies in Income and Wealth, vol. 58. Chicago: University of Chicago Press.

Heien, D. and Pompelli, G. (1989): *The Demand for Alcoholic Beverages: Economic and Demographic Effects*. Southern Economic Journal, Vol 55, no. 3, 759-770.

Huang, Kuo (1993): *A Complete System of U.S. Demand for Food*. Technical Bulletin Number 1821, Economic Research Service, United States Department of Agriculture.

KPI Commission (1999): *Konsumentprisindex, betänkande från utredningen om översyn av konsumentprisindex.* Justitiedepartementet, Stockholm. SOU 1999:124.

Levinsohn, J. (1988): *Empirics of Taxes on Differentiated Products: The Case of Tariffs in the U.S. Automobile Industry*. In Robert Baldwin, ed. *Trade Policy Issues and Emprical Analysis.* Chicago: University of Chicago Press, 11-40.

Lloyd, P.J. (1975): *Substitution Effects and Biases in Nontrue Price Indices*. American Economic Review 65, 301-313.

McCarthy, P.S. (1996); *Market Price and Income Elasticities of New Vehicle Demands. Review of Economics and Statistics,* 78:3, 543-547.

McLelland, R. (1999): *On the Use of the Geometric Mean and Laspeyres Formulas in the CPI*. Unpublished Report, Bureau of Labor Statistics, Washongton D.C.

Moulton, B.R. (1996): *Constant Elasticity Cost-of-Living Index in Share Relative Form*. Bureau of Labor Statistics, Washington D.C., December.

Moulton, B.R. and Stewart, K.J. (1997): *An Overview of Experimental U.S. Consumer Price Indices*. In Proceedings of the Third Meeting of the International Working Group on Price Indices. Centraal Bureau voor de Statistiek, Research Paper no. 9806, Voorburg.

Nelson, J. (1994): *Estimation of Food Demand Elasticities Using Hicksian Composite Commodity Assumptions.* Quarterly Journal of Business and Economics, Vol. 33, no. 3, 51-68.

Norberg (1996): *Systematisk effekt på KPIs långtidsindex av rent slumpmässiga fel i prisindex för representantvaror*. Internal Memo presented to the Swedish Index Board. Statistics Sweden, September 25.

Reinsdorf, M. (1994): *The effect of Price Dispersion on Cost of Living Indexes*. International Economic Review, Vol. 35, No. 1.

Reinsdorf, M. (1996): *Constructing Basic Component Indexes for the U.S. CPI from Scanner data: A test Using Data on Coffee*. Working Paper 277, U.S. Bureau of Labor Statistics.

Shapiro, M.D. and Wilcox, D.W. (1997): *Alternative Strategies for Aggregating Prices in the CPI. Review*. Federal Reserve Bank of St. Louis, Volume 79, 3, 113-126.

Socialstyrelsen (1961). *Konsumentpriser och indexberäkningar åren 1931-1959*. Sveriges Officiella Statistik, Stockholm.

SOU (1943): *Betänkande angående levnadskostnadsindex*, SOU 1943:8, Finansdepartementet, Stockholm.

SOU (1953): *Konsumentprisindex, betänkande angående omläggning av levnadskostnadsindex*, SOU 1953:23, Civildepartementet, Stockholm.

Trandel, G. A. (1991): *The Bias Due to Omitting Quality when Estimating Automobile Demand.* Review of Economics and Statistics, 73:3, 522-525.

Annex 1

Taylor series approximations of two index formulae

Here we present, without proofs, Taylor series approximations of the two most important formulae discussed above. Approximations are done for those formulas, which are functions of price relatives only and are based on series expansions around the unit vector. We use the notation $r_k = p_k^1 / p_k^0$ for price relatives and w_k for weights, summing to one over k. In this notation the formulae are:

$$W = \frac{\sum_k w_k \sqrt{r_k}}{\sum_k w_k \,/\, \sqrt{r_k}} \text{ and} \tag{A.1}$$

$$G = \prod_k r_k^{\,w_k} \tag{A.2}$$

Approximations are expressed as functions of the first, second and third weighted central moments of the distribution of the r_k as follows:

$$\mu = \sum_k w_k r_k \,; \qquad \sigma^2 = \sum_k w_k (r_k - \mu)^2 \,; \qquad \gamma = \sum_k w_k (r_k - \mu)^3 \tag{A.3}$$

Now, the third order Taylor approximations of these are:

$$W \approx \mu - \sigma^2/2 + 3\gamma/8 + \sigma^2(\mu-1)/2 \text{ and} \tag{A.4}$$

$$G \approx \mu - \sigma^2/2 + \gamma/3 + \sigma^2(\mu-1)/2 \tag{A.5}$$

We thus have

$$W - G \approx \gamma/24 \tag{A.6}$$

which is not very much!

The result has been numerically tested. The approximation is quite reliable for those kinds of distributions of price relatives that are common in practice, where σ is below 0.2. It collapses where σ becomes very large (above 0.5, say).

Annex 2

Simplified aggregation scheme in four steps

Aggregation level	**Year-to-month index**	**Year-to-year index**
All Item level, aggregated from NA groups, de-noted h	$I^{y,m}_{y-2} = \sum_{h} W_h I^{y,m}_{y-2;h}$, where (A2.1) $W_h = \dfrac{V_h^{y-2}}{\sum_{h} V_h^{y-2}}$ (A2.1b)	$I^{y}_{y-1} = \sum_{h} W_h I^{y}_{y-1;h}$ where (A2.5) $W_h = \sqrt{\dfrac{V_h^{y} V_h^{y-1}}{I^{y}_{y-1;h}}} \Big/ \sum_{h} \sqrt{\dfrac{V_h^{y} V_h^{y-1}}{I^{y}_{y-1;h}}}$ (A2.5b)
NA group level, aggregated from product subgroups, de-noted g.	$I^{y,m}_{y-2;h} = \sum_{g \in h} W_g I^{y,m}_{y-2;g}$, where (A2.2) $W_g = \dfrac{V_g^{y-2}}{\sum_{g \in h} V_g^{y-2}}$ (A2.2b)	$I^{y}_{y-1;h} = \sum_{g \in h} W_g I^{y}_{y-1;g}$, where (A2.6) $W_g = \dfrac{V_g \Big/ \sqrt{I^{y}_{y-1;g}}}{\sum_{k} V_g \Big/ \sqrt{I^{y}_{y-1;g}}}$ (A2.6b)
Subgroup level, aggregated from elemen-tary aggre-gates, denoted d.	$I^{y,m}_{y-2;g} = \sum_{d \in g} W_d I^{y,m}_{y-2;d}$ where (A2.3) $W_d = \dfrac{V_d \Big/ \sqrt{I^{y,m}_{y-2;d}}}{\sum_{d \in g} V_d \Big/ \sqrt{I^{y,m}_{y-2;d}}}$ (A2.3b)	$I^{y}_{y-1;g} = \sum_{d \in g} W_d I^{y}_{y-1;d}$, where (A2.7) $W_d = \dfrac{V_d \Big/ \sqrt{I^{y}_{y-1;d}}}{\sum_{d \in g} V_d \Big/ \sqrt{I^{y}_{y-1;d}}}$ (A2.7b)
Elementary level, linking is done in link months (*lm*, *lm1*, *lm2*) once a year, usually in December or January	$I^{y,m}_{y-2;d} = I^{lm1}_{y-2;d}\, x I^{lm2}_{lm1;d}\, x I^{y,m}_{lm2;d} =$ $\dfrac{\sum_{j \in d_1} w_{1j} \sqrt{p_j^{lm1} / p_j^{y-2}}}{\sum_{j \in d_1} w_{1j} / \sqrt{p_j^{lm1} / p_j^{y-2}}}$ x $\dfrac{\sum_{j \in d_2} w_{2j} \sqrt{p_j^{lm2} / p_j^{lm1}}}{\sum_{j \in d_2} w_{2j} / \sqrt{p_j^{lm2} / p_j^{lm1}}}$ x (A2.4) $\dfrac{\sum_{j \in d_3} w_{3j} \sqrt{p_j^{y,m} / p_j^{lm2}}}{\sum_{j \in d_3} w_{3j} / \sqrt{p_j^{y,m} / p_j^{lm2}}}$	$I^{y}_{y-1;d} = I^{lm}_{y-1;d}\, x I^{y}_{lm;d} =$ $\dfrac{\sum_{j \in d_1} w_{1j} \sqrt{p_j^{lm} / p_j^{y-1}}}{\sum_{j \in d_1} w_{1j} / \sqrt{p_j^{lm} / p_j^{y-1}}}$ x (A2.8) $\dfrac{\sum_{j \in d_2} w_{2j} \sqrt{p_j^{y} / p_j^{lm}}}{\sum_{j \in d_2} w_{2j} / \sqrt{p_j^{y} / p_j^{lm}}}$

TABLE 1: Fixed base indices compared to chained short and long term indices. December-December. One-year averages.

YEAR	STIX	LTIX	FC(.7)	FG	FL
	← *chained* →				
79-84	110.801	110.613	110.753	110.684	110.913
80-85	108.977	108.880	108.935	108.884	109.049
81-86	107.829	107.726	107.617	107.540	107.782
82-87	106.746	106.619	106.574	106.500	106.729
83-88	105.822	105.681	105.770	105.721	105.881
84-89	105.510	105.373	105.481	105.426	105.603
85-90	106.552	106.370	106.617	106.553	106.757
86-91	107.279	107.139	107.218	107.103	107.477
87-92	106.437	106.351	106.250	106.116	106.562
88-93	106.213	106.134	106.014	105.891	106.307
89-94	105.451	105.368	105.256	105.134	105.549
90-95	103.891	103.836	103.710	103.612	103.945
91-96	102.539	102.475	102.422	102.350	102.587
92-97	102.821	102.710	102.752	102.686	102.910
93-98	101.843	101.766	101.798	101.752	101.904
MEAN	105.914	105.803	105.811	105.730	105.997

TABLE 1 (cont.)

YEAR	FC(.9)	FC(.8)	FC(.7)	FC(.6)	FC(.5)	FC(.4)	FC(.3)	FC(.2)	FC(.1)
79-84	110.707	110.730	110.753	110.777	110.799	110.822	110.845	110.868	110.891
80-85	108.901	108.918	108.935	108.951	108.968	108.984	109.000	109.017	109.033
81-86	107.566	107.592	107.617	107.642	107.666	107.690	107.713	107.737	107.759
82-87	106.525	106.550	106.574	106.597	106.620	106.643	106.665	106.687	106.708
83-88	105.738	105.754	105.770	105.787	105.803	105.819	105.834	105.850	105.865
84-89	105.445	105.463	105.481	105.499	105.517	105.534	105.552	105.569	105.586
85-90	106.575	106.596	106.617	106.638	106.658	106.679	106.699	106.718	106.738
86-91	107.142	107.180	107.218	107.256	107.293	107.330	107.367	107.404	107.441
87-92	106.161	106.205	106.250	106.295	106.339	106.384	106.428	106.473	106.517
88-93	105.932	105.973	106.014	106.055	106.096	106.138	106.180	106.222	106.264
89-94	105.174	105.215	105.256	105.297	105.339	105.381	105.422	105.464	105.507
90-95	103.645	103.677	103.710	103.743	103.776	103.809	103.843	103.876	103.910
91-96	102.374	102.398	102.422	102.445	102.469	102.493	102.517	102.540	102.564
92-97	102.708	102.730	102.752	102.774	102.797	102.819	102.842	102.864	102.887
93-98	101.767	101.783	101.798	101.813	101.829	101.844	101.859	101.874	101.889
MEAN	105.757	105.784	105.811	105.838	105.865	105.891	105.918	105.944	105.971

TABLE 2: Chained indices, linking by month.Change December-December.

YEAR	STIX	LTIX	MC(.6)	MT	ME	MG1	MG2
80-81	109.639	109.444	109.518	109.458	109.541	109.405	109.439
81-82	111.073	110.917	111.008	110.933	110.995	110.911	110.965
82-83	110.722	110.675	110.655	110.685	110.699	110.659	110.611
83-84	108.235	108.157	108.199	108.165	108.196	108.190	108.175
84-85	105.320	105.303	105.294	105.312	105.311	105.312	105.276
85-86	103.980	103.767	103.741	103.719	103.873	103.462	103.562
86-87	105.607	105.335	105.543	105.421	105.471	105.433	105.497
87-88	106.015	105.888	105.968	105.952	105.952	105.983	105.936
88-89	106.645	106.591	106.590	106.605	106.618	106.589	106.553
89-90	110.627	110.381	110.391	110.494	110.504	110.273	110.240
90-91	107.572	107.571	107.317	107.545	107.572	107.166	107.153
91-92	101.531	101.518	101.411	101.401	101.525	101.281	101.332
92-93	104.901	104.816	104.826	104.873	104.859	104.823	104.776
93-94	102.875	102.797	102.790	102.752	102.836	102.710	102.739
94-95	102.684	102.586	102.635	102.610	102.635	102.609	102.603
95-96	100.754	100.704	100.665	100.698	100.729	100.620	100.605
96-97	102.934	102.686	102.796	102.715	102.810	102.682	102.710
97-98	100.004	100.087	99.943	100.020	100.046	99.955	99.901
MEAN	105.618	105.512	105.516	105.520	105.565	105.448	105.448

TABLE 3: Chained indices, linking by year. Change from whole year Y-1 to Y.

YEAR	YW	YT	YF	YE	YC(.4)	YL	YG	MT	KTIX	LTIX
82	109.953	109.953	109.954	109.957	109.983	110.04	109.91	109.94	110.14	109.95
83	110.237	110.235	110.235	110.236	110.238	110.26	110.21	110.23	110.28	110.12
84	108.417	108.416	108.416	108.420	108.463	108.49	108.42	108.41	108.52	108.47
85	107.153	107.153	107.153	107.154	107.147	107.17	107.11	107.13	107.14	107.06
86	104.454	104.446	104.455	104.457	104.436	104.55	104.25	104.43	104.59	104.57
87	104.767	104.775	104.762	104.763	104.816	104.87	104.74	104.63	104.87	104.66
88	105.902	105.900	105.900	105.900	105.884	105.91	105.85	105.81	105.91	105.63
89	106.562	106.557	106.558	106.558	106.537	106.56	106.50	106.54	106.57	106.44
90	110.000	109.996	109.996	110.000	109.967	110.10	109.78	110.00	110.16	110.11
91	109.267	109.277	109.268	109.267	109.108	109.25	108.90	109.25	109.26	109.02
92	101.738	101.740	101.738	101.739	101.704	101.76	101.62	101.70	101.77	101.77
93	105.026	105.020	105.019	105.020	105.011	105.10	104.88	104.96	105.08	105.06
94	102.885	102.886	102.887	102.887	102.895	102.92	102.86	102.95	102.96	102.88
95	102.765	102.765	102.766	102.766	102.783	102.81	102.74	102.74	102.87	102.79
96	100.990	100.990	100.989	100.989	100.987	101.03	100.92	100.99	101.05	100.95
97	102.055	102.055	102.055	102.055	102.111	102.16	102.04	102.06	102.21	102.16
98	100.849	100.849	100.849	100.850	100.862	100.88	100.84	100.86	100.96	100.72
MEAN	105.472	105.471	105.470	105.472	105.467	105.52	105.39	105.45	105.55	105.43