Using Scanner Data to Explore Unit Value Indexes

Room document at the Sixth Meeting of the International Working Group on Price Indices

Canberra, Australia
2-6 April 2001

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This paper reports work-in-progress. The author(s) would welcome comments, which can be sent to **malti.jain@abs.gov.au**

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March 2001

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1 Introduction

The unit value concept has a long history. Early contributions, as outlined in Diewert (1995), included the following:

- Segnitz (1870) proposed a unit value index for homogeneous commodities;
- Drobisch (1871) proposed a unit value index for heterogeneous commodities; and
- Walsh (1901) and Davies (1924) proposed unit values as an aggregation formula at the first stage of aggregation for a price index.

Until quite recently, unit values were restricted to the level of theoretical studies or to small-scale practical applications for which sufficiently rich datasets were available. Now, as vast amounts of scanner data become available, unit values can be calculated and assessed across a wide range of commodities. During the past decade, statistical agencies and academics around the world have increased the research effort devoted to potential applications of unit values.

This paper reports ABS investigations of unit values based on a moderate-sized set of scanner data (the "13-week dataset") obtained from AC Nielsen. We have recently begun further investigations based on a larger set of data (the "52-week dataset").

The key question addressed by this paper is:

> If unit values are to be used in index construction, at what level should they be aggregated (over time, outlets and/or commodities)?

The answer to this question hangs largely on the degree of homogeneity of commodities across those dimensions.

Our reference problem is testing the usefulness of unit values for constructing indexes at the elementary aggregate level of a Consumer Price Index.

Section 2 of the paper gives an overview of analyses that may be undertaken with scanner data. Section 3 describes the scanner dataset obtained by the ABS; the commodities and stores used in our analyses are briefly described. Section 4 defines the indexes we have constructed and explains our measures of various biases. Section 5 some of the issues related to unit values. Section 6 describes the methodology used and unit values constructed. Section 7 summarises our results. Our conclusions are presented in Section 8, and limitations of the study and possibilities for future work are discussed in Section 9. The attachments show the index formulae used and provide more details of the results.

2 Applications of Scanner Data and the ABS Analysis Program

Potential Applications of Scanner Data

It is conceivable that scanner data might eventually be used directly in some index construction, because they provide both:

- Price data — so the size of price samples might, for example, be increased substantially at relatively low cost.
- Quantity data — so it might be possible to construct, say, unit prices or superlative indexes. (It must be kept in mind, however, that some of the sales covered by the scanner data may not be sales to our indexes' target populations, say households in the case of the CPI. There is some evidence, for example, that small stores buy items from large stores when the latter offer attractive "specials".)

Even in the shorter term, however, scanner data can be useful for informing and enhancing the current index construction practices of statistical agencies. Scanner data might, for example, be a source of:

- An up-to-date listing of commodities being sold through the outlets covered by the dataset.
- (Approximate) commodity and outlet weights.
- Price, quantity and expenditure information to assist analyses of consumer behaviour.
- Information to assist hedonic modelling and hedonic-based quality adjustments.
- An early indication of new items and items going off the market.

Analyses being Undertaken by the ABS

This paper is based a small experimental Australian scanner dataset covering just 13 weeks. This was the first such dataset that the ABS obtained.

This paper reports analyses undertaken to evaluate the effect of different groupings of items and outlets when constructing unit values. The analysis is based on only those items and outlets that were common to all time periods covered by the year-long dataset. The scanner data have, in effect, been treated as the total "population" of items and stores. This assumes away all sampling errors. We define a "true" or benchmark index; systematic over- or under-estimation of a given index relative to the benchmark index gives us an indicative measure of the performance of various unit value indexes.

A number of other analyses based on scanner data are being conducted or are planned by the ABS. Some of these relate to current ABS practices and may have implications in the short term, for example, imputation for missing values, splicing, and sampling of items and stores. Other analyses may have a longer term application, for example, unit values, and the sampling of items and stores (if scanner data were used directly in index construction).

Other ABS analysis projects using scanner data include:

- Biases Scanner data provide prices and quantities that can be used to measure the biases that arise due to the inability of fixed weighted indexes to take into account the substitutions that consumers make as a reaction to price movements. An accompanying paper discusses early findings of these analyses, based on the 52-week scanner dataset.

- Imputation The scanner data lend themselves to analysis of the effects of not including new items and outlets in indexes and the best method for imputing for discontinued items and outlets. Under current CPI practices, for example, new items and outlets are introduced into the sample fairly infrequently and only if an existing sampled item or outlet needs to be replaced.

- Missing prices Missing prices are a fairly common problem for index compilers. Items temporarily missing are imputed and items that go out of production may also be imputed for a period before being replaced. This project is analysing the effect of observations that are not included in the index and the best method for imputing for missing prices.

- Sampling Currently only a few statistical agencies (such as the US Bureau of Labour Statistics and the UK Office of National Statistics) use probability sampling to select the item and outlet samples for their CPIs; the ABS, like most other agencies, uses purposive sampling. Although the ABS has no intention of moving to probability-based sampling for its CPI in the near future, scanner data provide a better basis for determining appropriate sample sizes and purposive sample allocation.

- Different designs Scanner data can be used to guide the drawing of boundaries around the elementary aggregates so that more representative samples could be drawn.

3 The Scanner Data

The scanner dataset used in this analysis relates to 18 weeks of which 5 weeks were from February - March 1997 and 13 weeks from February to April 1998.

The dataset shows weekly sales (quantity and prices) of each item (and details of brand, size, packaging, etc.) in each commodity group.

The dataset includes sales by all stores of the four supermarket chains in one capital city. These four chains (with over 100 stores) account for over 80 per cent of the grocery sales in that city.

The dataset includes sales of 7 commodities. All items within those commodities have a unique 13-digit Australian Product Number (APN).

3.1 Time Period

Five weeks in February-March 1997 were chosen as the "base period" for all our index construction. The 13 weeks from February to April 1998 are referred to as the "current period".

The base period price (Po) was computed as the unit value over the 5 weeks of the base period. The base period quantity (Qo) is the average weekly quantity of the item sold in the base period.

The current period prices (Pt) were the weekly unit values and the quantities (Qt) were weekly quantities sold at each store.

3.2 Stores (or Outlets)

The scanner data relate to stores belonging to four supermarket chains in one city. They account for about 15 per cent of all food stores in that city, but over 80 per cent of employment and sales in food stores.

Stores were divided into three types according to their presence in different time periods. A store could be:

- continuing (same store in all weeks covered by the dataset),
- closed down (during the current period), or
- new store (opened during the current period).

In the base period, 101 operating stores belonged to the four chains. One store closed down and seven new stores opened during the current period. Thus 100 stores were common between the base period and all current periods; only these "continuing" stores have been used in our analyses. Stores that closed down and reopened in the same location during the intervening months between the base and current periods were treated as continuing.

3.3 Commodities (and Items)

The following 7 commodities were selected for the analysis:

- butter
- cereals
- coffee
- detergent
- margarine
- oil

- toilet paper.

Most of these commodities corresponded to an "expenditure class" (EC) in the 12th Series Australian CPI. But the structure of Australian CPI has changed considerably in the last two years -- with the introduction of 13th Series in September 1998 and the 14th Series in September 2000.

For example, butter which was an EC in the subgroup Dairy Products (with a weight of 0.055) in the 12th Series was moved to the EC Fats and Oils in the subgroup Other Food in the 13th series.

Most of the commodities analysed cover only a part of the EC in the 14th series.

Some of the commodities include a large number of items (APNs), which do not always represent distinct products. For example, toilet papers of different colours are given different APNs. Such fine classification of items increases the incidence of missing values in the scanner data because every APN may not be sold every week. However, in our analysis, they have been treated as different items.

Items have been classified into those:

- sold in the base period and all current periods
- sold in the base period and some current periods
- sold in the base period only
- sold in the current period only.

A small number of APNs disappeared from the market but a very much larger number of APNs came into the market during the study period. Only the items sold in the base period and each of the current weeks could be included in the analysis. These items have been called "continuous items". Thus only about 50 per cent of the APNs that were available at some stage have been analysed. For most of the commodities the continuous items contributed 50-80 per cent of the total sales. In about half of the commodities, the total sales of the continuous items were marginally less than the sales in the base period. Presumably some of the current period sales is shifted to the new items appearing in the market.

All indexes reported in this study were calculated using only items and stores that were continuous throughout both the base and current periods.

4 Analytical Framework

The ABS uses a base-weighted (Laspeyres) index as its basic formula for calculating the CPI. The Laspeyres index measures the change in the total cost of a basket of goods (with quantity weights fixed at those observed in the base period) between the base period and the current period; it is thus a measure of pure price change.

4.1 Population index formulae

For our analyses, the scanner data are assumed to represent the whole population of items and outlets. Therefore the indexes defined below are postulated to be "population indexes" rather than sample-based estimates.

At the lowest level, index compilation involves two levels of aggregation, across items and across outlets. Different aggregation methods can be used for the two levels.

Throughout our scanner data analysis, we have named all our weighted indexes with the same naming convention.

In the index name "XY":

- the "Y" refers to the initial of the formula used to aggregate across outlets, ie producing summations at the item level and
- the "X" refers to the initial of the formula used to aggregate across items, ie producing an index at the commodity level

  unit value indexes have been denoted as LUx, where x denotes on the grouping used.

Index formulae are given in Attachment 1.

Each commodity has been treated as independent and we have not tried to aggregate across commodities.

In this study, all indexes have been calculated as direct indexes.

4.1.1 Laspeyres and Paasche indexes

The scanner data show quantity information for items at the outlet level. Therefore it is possible to use quantity weights in compiling an item index. This can be either a Laspeyres (base period) or a Paasche (current period) weight.

We have denoted a fixed weighted index as LL_t to denote an index which is calculated using fixed base period weights to aggregate across both item and outlet levels and PP_t to denote a current weighted index at both levels.
In general the Laspeyres price index at both commodity and item levels (LL_t) exhibits higher values than the corresponding Paasche index (PP_t) — a relationship that holds whenever price and quantity relatives are negatively correlated. This is a typical behaviour in a market economy where consumers react to changes in relative prices by moving consumption away from those products which have become relatively more expensive and towards those which have become relatively less expensive. As the Laspeyres index assumes quantities remain constant and equal to those in the base period, it is likely to be higher than the true index of price change, while the Paasche index is likely to be lower.

The primary advantages of the LL_t which explains its wide acceptance, are its minimal data requirements, and the ease of understanding what the index measures. Its interpretation as the change in the price of a fixed basket of products and services is relatively straightforward and understood easily as a pure price change. The Paasche index has greater data requirements because current weights are required and such data are usually not available (in the absence of scanner data).

4.1.2 Fisher index

A Fisher index is a geometric mean of Laspeyres and Paasche indexes. A Fisher index uses information on values in both the base period and the current period for weighting purposes. Equal importance is attached to the two periods being compared. It also satisfies various tests that are considered important, such as the "time reversal" and "factor reversal" tests. Diewert (1976) has shown that the Fisher index is also a superlative index since it equals or approximates the true theoretical index corresponding to a family of flexible functional forms.

In our analyses, we have used the Fisher index as a proxy for the cost of living index. It is our benchmark for measuring the overall bias of an index.

4.1.3 Unit values

A unit value at any level is in effect a weighted average of prices. An index that is calculated as a unit values across outlets and then aggregated using fixed base period weights across items has been denoted as LU. For LU(1), the price relative consists of unit values for each item for periods 0 and *t*. Using Laspeyres weights they are then weighted and aggregated across all items. LU(2) is calculated as a unit value across all outlets within each chain and then the chains and the items are aggregated using fixed base period weights. In LU(3), the order of aggregation has been reversed and a unit value is calculated for each outlet across all items. Outlet indexes are then aggregated using Laspeyres weights.

4.1.4 Indexes computed in this study

A number of indexes were computed, representing combinations of indexes. These various combinations are summarised below. The first letter denotes the index used to aggregate across items and the second letter to denote the aggregation across outlets.

Table 1. Summary of Price Indexes and Formulae Used

Index	Formula for aggregation across items	Formula for aggregation across outlets
LL_t	Laspeyres	Laspeyres
PP_t	Paasche	Paasche
FF_t	Fisher	Fisher
LU_t	Laspeyres	Unit value

In calculating the weighted indexes, the quantity weights down to the outlet level for each APN were used. Different aggregations were used when calculating unit value indexes

4.2 Sources of Bias in Consumer Price Indexes

Diewert (1996) has identified several possible sources of bias in consumer price indexes at the low level of aggregation. These are biases in the sense that a concept of a "true index" exists. Diewert's concept of a "true" or "unbiased" index is a social cost of living index. This index allows consumers to change their baskets of goods in response to changes in relative prices.

Diewert identifies five sources, namely:

(commodity) substitution bias
outlet substitution bias
elementary index bias
quality adjustment bias; and
new goods bias.

The ABS, like most statistical agencies, uses a Laspeyres index as its basic formula for calculating the CPI. The Laspeyres index measures the change in the total cost of a basket of goods between the base period and the current period (with quantity weights fixed at those observed in the base period). A Laspeyres index can be expressed as a weighted sum of indexes at a lower level. At the lowest level of aggregation (ie, at the outlet level, where quantity weights are not available), the index is calculated using price data only.

We have analysed the first two sources of bias (commodity substitution and outlet substitution) in this study.

5 Unit Value Indexes

A unit value is simply a weighted average price, defined as

$$\overline{P_{tij}} = \frac{\sum_{k}^{N_{tij}} P_{tijk} Q_{tijk}}{\sum_{k}^{N_{tij}} Q_{tijk}} \qquad t = 0,1,2......$$

where P_{tijk} denotes price k of item i in outlet j in time t , Q_{tijk} denotes quantity k of item i in outlet j in time t and N_{tij} denotes the number of different prices that item is sold at in that time period.

In other words, it is the total sales divided by the total quantity sold.
If the time period is very short, then there will be only one price for each commodity ie $N_{tij} = 1$. In our scanner data the prices actually are average weekly prices since they are calculated from the total sales and total quantity sold in the week.

In other words, a unit value is obtained by summing sales, where sales equal price times quantity sold, and dividing this by the total quantity sold.

The summation above can be across time, item and/or outlet.

For our scanner dataset, a unit value will generally aggregate over time to some extent. Even if one had a dataset observed in "continuous time", this would pose considerable processing problems due to the large size of the dataset and missing values because there was no sale of the item. In general, the smallest unit of time used in these summations is weekly.

Currently, at the lowest level of aggregation, the Australian CPI is compiled using sampled price data. Because quantity information is not available, a microindex formula (Jevons index) based on price data only is used to aggregate the prices at the elementary aggregate level.

Diewert (1995) suggests that a price relative obtained using unit values:

$$U_t(i) = \frac{\overline{P}_{ti}}{\overline{P}_{0i}}$$

is a better measure of an item index than the traditionally used microindexes that rely only on price information.

In the past, it has been difficult to calculate unit values in practice because of the lack of current quantity data. The use of scanner data, however, provides a valuable opportunity to test the theory and see how the "ideal" elementary aggregate behaves in a real index.

Our scanner data provide an average weekly price (unit value) for an item, obtained by averaging all sales for the week. This weekly unit value is the lowest level of aggregation (with respect to time) available to us, which means that any price variations occurring within the week will be missed. The data provides a unit value for each item for each store.

An "item" is defined as those products covered by a discrete APN (Australian Product Number). Each item is regarded as unique. To illustrate this point, consider the example of toilet paper. Two packages of toilet paper —

of identical number of toilet rolls,number of sheets, thickness, brand, and softness— will have separate APNs if they are of different colours.

Unit values must be aggregated using a weighted index formula. In our methodology, two levels of aggregation are involved: firstly across outlets; and then across items:

> In general, a fixed base-weighted Laspeyres index at both the item and outlet level provides the upper bound for indexes used to estimate the cost of living (COL). This index, LL, is defined previously. Because our LL indexes have been calculated using weekly unit values per item per store, in a sense they represent the finest level of unit value indexes. They are expected to provide the upper bound for unit value indexes under normal consumer behaviour. Our empirical results show this to be the case.
>
> On the other hand, a weekly unit value index calculated across all stores and items within a commodity would provide the most aggregated unit value index. Such a unit value assumes perfect homogeneity of stores and items within a commodity. This assumption is of course unrealistic, but it serves to provide a lower boundary, below which we would not expect more realistic unit value indexes to fall. This is discussed in more detail in Sections 5.2 and 6.2.4

5.1 Unit values assessed from the axiomatic perspective

The axiomatic approach to evaluating index formulae dates from Fisher (1922). It was then taken up by Eichhorn and Voeller (1983) and Dalen (1992) and further developed by Diewert (see Diewert (1995)).

Over time, a set of axioms has been compiled against which a prospective index may be tested. In choosing a particular index formula, we may look at the number of axioms that the index satisfies or how many axioms the index satisfies approximately. We could also look at the purpose for which the index is being compiled and whether a particular axiom is relevant in the situation that the index is being used. This is an important issue theoretically and goes to the heart of whether or not unit values can be regarded as being useful in the context of indexes.

Prima facie, a unit value index can violate the identity, dimensionality and/or the proportionality axioms (although in practice, the price statistician would apply unit values in such a way as to conform with or very nearly conform with the principles underlying those axioms).

> The identity axiom states that if all prices remain constant the value of index equals 1, irrespective of any changes of quantities. A unit value index clearly violates this axiom since:

$$u = \frac{\dfrac{\sum_i P_{0i} Q_{ti}}{\sum_i Q_{ti}}}{\dfrac{\sum_i P_{0i} Q_{0i}}{\sum_i Q_{0i}}} \neq 1 \qquad unless \quad \frac{Q_{ti}}{\sum_i Q_{ti}} = \frac{Q_{0i}}{\sum_i Q_{0i}} \quad \forall i$$

> In other words, even if all the prices stay the same between base period and current period, a unit value index would not necessarily be equal to 1.
>
> The dimensional invariance (commensurability) axiom states that a price index must be independent of the units of measurement of the quantities. Theoretically a unit value index:

$$u = \frac{\dfrac{\sum_i P_{ti} Q_{ti}}{\sum_i Q_{ti}}}{\dfrac{\sum_i P_{0i} Q_{0i}}{\sum_i Q_{0i}}}$$

is not equal to the unit value index

$$u = \frac{\dfrac{\sum_i \lambda_i P_{ti} \left(\dfrac{Q_{ti}}{\lambda_i} \right)}{\sum_i \left(\dfrac{Q_{ti}}{\lambda_i} \right)}}{\dfrac{\sum_i \lambda_i P_{0i} \left(\dfrac{Q_{0i}}{\lambda_i} \right)}{\sum_i \left(\dfrac{Q_{0i}}{\lambda_i} \right)}} \qquad unless \ \lambda_i = \lambda \quad \forall i$$

In practice, unit value indexes would be constructed across items whose quantities are measured in the same units.

The proportionality axiom states that a proportional change in all prices should result in the same proportional change in the index. In general this is not the case for a unit value index. It can be shown that:

$$u = \frac{\dfrac{\sum \lambda P_{0i} Q_{1i}}{\sum Q_{1i}}}{\dfrac{\sum P_{0i} Q_{0i}}{\sum Q_{0i}}} \neq \lambda \qquad unless \ \frac{Q_{1i}}{\sum_i Q_{1i}} = \frac{Q_{0i}}{\sum_i Q_{0i}} \quad \forall \ i$$

The identity axiom is a special case of the proportionality axiom, and the price statistician's practical application of unit values would obviate the violation of the axioms.

In 1923, in relation to the axiomatic testing of indexes, Fisher said

> But, from a practical point of view, we want to know how *near* to agreement [.. with the axiom] the formula comes. We find that in some cases the disagreement is great and, in others, negligible so that the mere fact of non-agreement is of little practical value. (Fisher: 1923, page 421.)

Fisher was making the point that an index may in theory fail a test or axiom but in practice almost pass, so that the degree of failure is also important in choosing a given index for a given purpose.

There is no denying that unit values could prima facie fail some of the axioms. However, they may yet be useful in price index consideration. One important consideration is that the commodities being aggregated should be exactly the same or very nearly homogeneous (very close substitutes for one another).

If the price statistician were to contemplate using unit values in mainstream compilation of price indexes, the problems would include:

- how to define "homogeneity", and
- how to organise the data, so that each unit value is computed across items that are closely substitutable.

This issue is further discussed in Section 6.

5.2 Unit value bias

Analysis of unit values against axiomatic tests is only one way of evaluating their suitability. In practice it is probably more useful to evaluate their actual performance by measuring their divergence from a benchmark index. The divergence will constitute a measure of the bias inherent in the unit value index.

Unit value bias is one way of determining the suitability of an index composed of unit values. Balk (1995) points out that if the unit value bias is equal to zero then the unit value index may be used instead of the Fisher price index. This means that it can be regarded as an approximation of a COL index in this circumstance. Balk defines unit value bias thus:

> "Consider the ratio of the unit value index to a price index. The difference between this ratio and 1 is called the unit value bias."

Under certain conditions, this unit value bias will be equal to zero. This is desirable and worth striving to achieve if it renders the unit value indexes more useful. Using the Bortkiewicz theorem, developed to decompose the differences between Paasche and Laspeyres indexes, Parniczky (1974) analyses the difference between a Paasche price index (the benchmark) and a unit value index. He expresses the unit value bias in terms of the covariance between the base period price level and the quantity relatives, and derives the conditions under which the unit value bias would be zero.

Balk (1995) has generalised Parniczky's theorem to obtain the conditions for bias with respect to a Fisher price index. A zero unit value bias occurs when at least one of the following are met:

- all base period prices P_{0i} are equal to each other and all comparison period prices P_{ti} are equal to each other.
- all quantity relatives $\frac{Q_{ti}}{Q_{0i}}$ are equal to each other - meaning the amount purchased in each time period relative to the base period remains the same; and
- there is, both in the base period and in the comparison period, no correlation between prices and quantity relatives - meaning that a normal inverse price/quantity relationship does not hold (there is no downward sloping demand curve)

  ie $\rho\left(P_{0i}, \frac{Q_{ti}}{Q_{0i}}\right) = 0$ and $\rho\left(P_{ti}, \frac{Q_{ti}}{Q_{0i}}\right) = 0$

It is clear that in a real world situation, none of the above conditions would hold.

There are strategies that can be adopted to reduce a unit value bias if it is not identically zero. Balk (1995) suggests dividing the commodity into sub-groups, calculating unit values for each of these subgroups and then aggregating using a price index (Balk suggests a Fisher index). Presumably these subgroups would need to contain items that were as near as possible to be completely substitutable ie as homogeneous as possible, in order to satisfy one or more of the conditions outlined above.

Parniczky (1974) has shown that there are two components to a unit value bias:

- a within-subgroups component. This is caused by consumers showing a preference for certain items over other items within the same sub-group (ie, they are not perfectly substitutable).

- a between-subgroups component. This is caused by consumers switching from items within one sub-group to items in other sub-groups (ie, there is some substitutability between sub-groups).

Dividing a group into various subgroups will get rid of the between-subgroups component. But if the two components have biases of opposite sign, removing one of the sources of bias may result in a larger bias (Balk, 1995).

It is therefore important, if such a strategy is to be employed to reduce unit value bias, to ensure that there is adequate knowledge of the causes of the bias.

Our exploratory analyses suggest that:

- a pure Laspeyres index which treats each item and store as unique has no within-group bias but a large positive substitution (between-group) bias
- a unit value computed across all items and stores has no between-group bias but a large negative within-group bias.

Various groupings of items and stores would have different levels of within- and between-group biases. The total bias would depend on the extent the two biases cancel each other.

The relative sizes of bias components depend on the particularities of the data being analysed. It is not yet apparent how one would define a general rule.

When contemplating practical applications of unit values, both issues must be considered:

- the grouping of items and outlets at the lowest level (across which the unit values are computed) and
- the aggregation formula used to combine the unit values.

6 Methodology

A unit value is the average price of a group of homogeneous items. Despite its seeming simplicity, the term "homogeneous" requires interpretation. The way data is organised into groups has the potential to influence the degree to which the resulting index approximates the ideal index. It takes careful analysis and thought to obtain a grouping that avoids potential bias, over-simplification and over-complication. Some examples of groupings are: in a specific outlet; and across all outlets within a certain area.

A classic definition by Debreu, of a unique commodity, is given in Balk (1995):

> A commodity is characterised by its physical properties, the date at which it will be available and the location at which it will be available.

As regards our scanner datasets, these characteristics translate to item, time and outlet.

6.1 Level of aggregation for calculating unit values

In principle, scanner data are observable in continuous time for the most finely divided commodity (item) specifications and for all outlets. In practice, some aggregation of the data available is usually required before price index construction can begin. Indeed the commercial owners of the datasets may make them available only at a slightly aggregated level, usually over time.

> As an example, our scanner dataset contains weekly aggregations in that we received, for each item, the price averaged over each week in the target period and the quantity sold during that week. Of course this involves assuming that the quality of each item remains the same over the whole week.

It may be the case that the lowest possible level of aggregate is at the item level across outlet type. If this is too detailed, or data is unavailable, then item level across all outlets might be the lowest level (effectively treating all outlets as the same).

For this reason we tested specific aggregation methods, compiling indexes from these elementary aggregations and comparing the results with the Fisher ideal, in order to see the effect of aggregation on the index. The aim was to determine which aggregations most closely equate to the Fisher ideal and most effectively reduce bias.

To aggregate unit values, Walsh (1901) and Davies (1924) recommended a superlative index number formula such as a Fisher with prices replaced by unit values :

$$\left[\left(\frac{\sum_i \overline{P_{ti}} Q_{0i}}{\sum_i \overline{P_{0i}} Q_{0i}}\right)\left(\frac{\sum_i \overline{P_{ti}} Q_{ti}}{\sum_i \overline{P_{0i}} Q_{ti}}\right)\right]^{\frac{1}{2}}$$

where $\overline{P_{0i}}$ is the unit value for item i in time 0, Q_{0i} is the quantity sold of item i in time 0, $\overline{P_{ti}}$ is the unit value for item i in time t and Q_{ti} is the quantity sold of item i in time t, whereas Saglio (1994) uses Laspeyres for aggregation.

Notwithstanding the fact that Walsh and Davies suggested the use of a superlative formula, we decided to aggregate the elementary indexes using Laspeyres weights. This was to ensure that our analysis and results would be dovetail with current CPI practice.

Having produced indexes comprising unit values with Laspeyres weight, they could then be analysed in comparison with the previously calculated Fisher and Laspeyres indexes.

6.2 Unit values computed in this study

Indexes were calculated using unit values at different levels of aggregation. An index that is calculated as a unit value across outlets and then aggregated using fixed base period weights across items has been termed LU.

6.2.1 Unit values computed by grouping outlets — LU(1) and LU(2) Indexes

Initially we looked at grouping across stores. Unit values were calculated using two groupings: across all outlets and across outlets within each supermarket chain. For the purposes of this paper, these will henceforth be known, respectively, as LU(1) and LU(2). For LU(1), the price relative consists of unit values for each item for periods 0 and *t*. Using Laspeyres weights they are then weighted and aggregated across all items. For LU(2), the price relative consists of unit values for each item for each chain for periods 0 and *t*. They are then weighted together using expenditure weights for each item x chain.

In our study, the index LU involves an elementary aggregate comprising a unit value for each item within a commodity. These are then aggregated across items using Laspeyres weights. LU(1) has no between group (between outlet) bias but would have a within group bias since the outlets are not completely substitutable. It also has an item substitution bias. LU(2) on the other hand would have a between group (between chain) bias, a within group bias as well as the item substitution bias. Within group bias tends to be negative and underestimates the price movement whereas between group bias which is similar to the substitution bias is positive and overestimates the price movement. Thus LU(2) is always higher than LU(1) and in fact is very close to LL.

Compared to benchmark Fisher index FF, LU(1) shows a very small bias which means that the within group (outlet) bias is negative and almost cancels out the positive item substitution bias.

6.2.2 Unit values computed by grouping by brand, package size, and product type — LUB, LUS and LUT Indexes

Next we wanted to look at the effect of combining items into groups of similar items. The motivation for this was purely practical. Should scanner data, and therefore unit values, be used in the compilation of the CPI in the future, it will be necessary for the data to be organised in some way that reduces the overall size of the datasets to manageable proportions. It would also reduce the problem of missing observations in the scanner data. If an item is not sold in a certain period, its price is not recorded. It is desirable that new items coming into the market can also be incorporated in the index and items that are discontinued can easily be removed. One way of doing this is to group items that have similar characteristics. However this must be done carefully to keep biases resulting from heterogeneity to a minimum.

Bradley (2000) disagrees with this approach. He has found that New York cereal data violates all the conditions necessary for a unit value to be unbiased. He suggests modelling the selection effects to derive "virtual prices" at the lowest level of aggregation, and computing appropriate price indexes that can be used at higher levels of aggregation.

The data that we have used for this analysis excluded all the new and discontinued APNs and therefore we do not have any missing prices. We have analysed the bias empirically for various groupings of items within commodities into a small number of groups. Initially we used size, brand, and type. A unit value was obtained for each group (this compares with LU(1), for example, where a unit value was obtained for each item). These were then weighted together, using group weights, into a Laspeyres index. The resulting indexes for each commodity are called LUB (brand), LUS (size) and LUT (type).

We also compiled an index for cereals using a more detailed grouping.

6.2.3 LU(3) Indexes — Unit values computed across all items within the store

We then experimented with an index whereby a unit value was obtained for each store (for each commodity). This is based on the premise that consumers, in making purchasing decisions, are far more likely to choose from the items available within the store they are in, than seek further choices in competing stores. We must stipulate that this premise, though intuitively appealing for grocery items, is unlikely to hold for many other commodities, particularly big one off purchases.

This index has been called LU(3).

6.2.4 UU Indexes — Unit values computed across all items and stores for a commodity

The broadest grouping that was used was across all items and outlets. This assumes perfect substitutability between items and outlets. It is of course unrealistic but has been found to provide a lower bound of unit value indexes. There is no between group bias, only within group bias which is usually negative.

6.2.5 Unit values computed across multiple time periods

This aspect of unit value compilation will be analysed at a later date with the larger "52-week" scanner dataset.

6.3 Total bias of unit value indexes

We have assumed a Fisher index to be our benchmark index. Thus the difference between a Fisher and LU(1) or LU(2) represents the total bias of unit value indexes computed across all outlets and across all stores within each chain respectively. These have been compared with bias of the corresponding Laspeyres index.

$$biasLU(1) = \frac{LU(1) - FF}{FF}$$
$$biasLU(2) = \frac{LU(2) - FF}{FF}$$
$$biasLL = \frac{LL - FF}{FF}$$

7 Results

The relationship between different unit value indexes for various commodities show a similar pattern. Therefore only butter and cereal have been discussed here in detail. All other commodities have been presented in the Attachments 2 - 5.

7.1 Unit value bias

As mentioned in Section 5.2, Balk (1995) defines the unit value bias as the difference between a ratio of the unit value index to the price index, and 1. In our analysis we have constructed indexes which comprise unit values at the elementary level, aggregated across items using Laspeyres weights and compared them to FF ie an index comprising Fisher at the elementary level and then aggregated using Fisher across items.

We have found that any grouping of items or outlets introduces a within group bias which is always negative. The larger the group, the larger is this bias. A larger group is expected to be more heterogeneous, thus the larger bias in a larger group is probably caused by the larger heterogeneity within the group. On the other hand the between group bias is caused by the substitution between groups and is expected to be positive.

An LU index which fell closest to the FF line would contain balanced amounts of upward or positive substitution bias caused by between group substitution and downward or negative bias caused by within group bias and therefore a small overall or net bias. However, the precise grouping necessary to achieve this ideal will vary from commodity to commodity.

Our exploratory analyses suggest the following conclusions regarding the various biases affecting unit value indexes:

- LL contains positive outlet substitution bias or between-group (outlet) bias, no within-group bias and positive item substitution bias.
- LU(1) contains negative within-group (outlet) bias, no between-group bias ie no outlet substitution bias and positive item substitution bias and
- LU(2) contains negative within-outlet bias, positive between-chain substitution bias and positive item substitution bias.
- LUS (and LUB and LUT) contains negative within-outlet bias, negative within-group (item) bias and positive between-group substitution bias.
- UU contains negative within-outlet bias and negative within-item bias.

The following text and charts illustrate these conclusions.

7.2 Groupings across outlets — LU(1) and LU(2) Indexes

The table below shows the total bias of a Laspeyres index and the two unit value indexes LU(1) and LU(2) with respect to a Fisher index. The biases have been averaged across the 13 weeks (current period) and represent the bias over a one year between the base and current period.

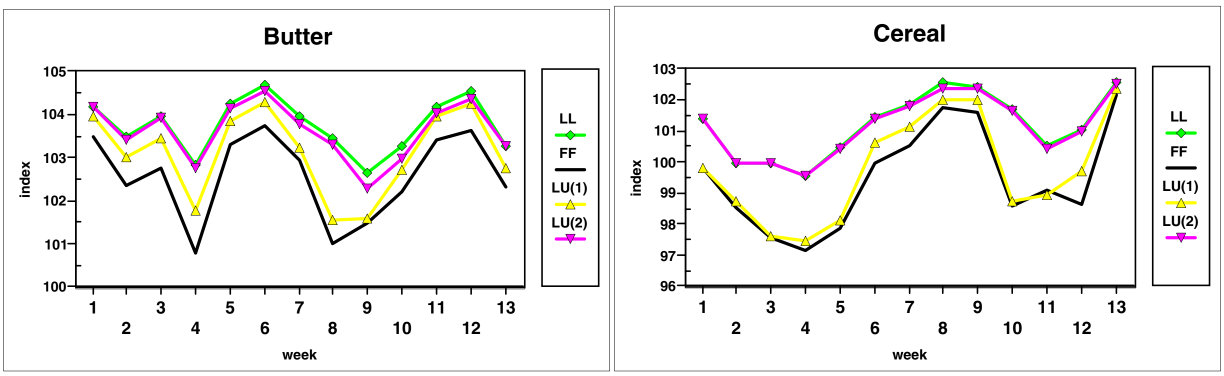
Table 2 : Total Bias in Laspeyres, LU(1) and LU(2) Indexes

Commodity	Total bias	Total bias1	Total bias2
	(LL - FF)/FF	(LU1 - FF)/FF	(LU2 - FF)/FF
Butter	0.0115	0.0052	0.0102
Cereal	0.0173	0.0031	0.0167
Coffee	0.0303	0.0131	0.0300
Detergent	0.0282	0.0250	0.0342
Margarine	0.0458	0.0107	0.0413
Oil	0.0121	0.0056	0.0119
Toilet paper	0.0352	0.0093	0.0317

The total bias in LU(1) is much smaller than a Laspeyres index. Bias in LU(2) is almost same as Laspeyres except for detergents where it is much higher.

The following charts show the two unit value indexes LU(1) and LU(2) along with weighted indexes LL and FF. We have included only Butter and Cereal. For other commodities, see Attachment 2.

Figure 1 : LU(1) and LU(2) Indexes for Butter and Cereal

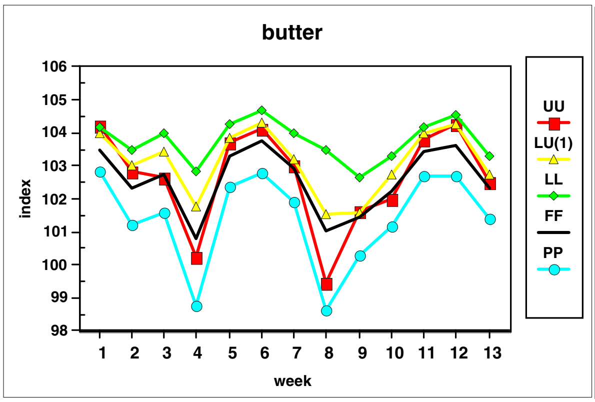


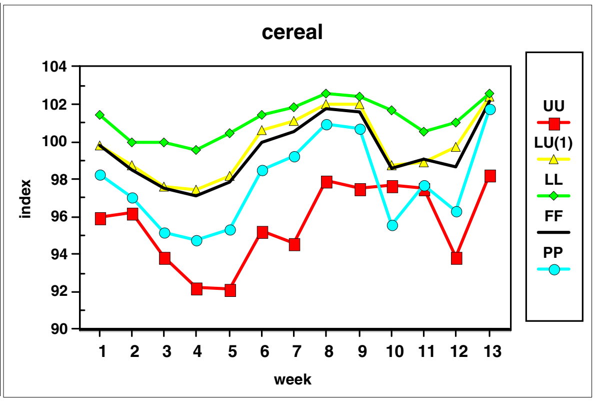
It can be seen in both these cases that LU(2) is virtually identical to LL while LU(1) is very close to FF. Since LU(1) has an upward bias due to item substitution, we conclude that the unit value bias must be negative and is due to within group (outlet) bias. LU(2) on the other hand has a smaller within group (within chain) bias and between group (between chain) bias which is same as the between chain substitution bias. These two biases cancel out each other to some extent and we are left with mainly the item substitution bias.

7.3 Grouping across all items and stores — UU Indexes

We have also calculated unit values across all items within a commodity. These indexes show a large negative bias with respect to Fisher and in fact lie below Paasche in some cases. The results for butter and cereal are shown below. Charts for other commodities are given in Attachment 3.

Figure 2 : UU Indexes for Butter and Cereal





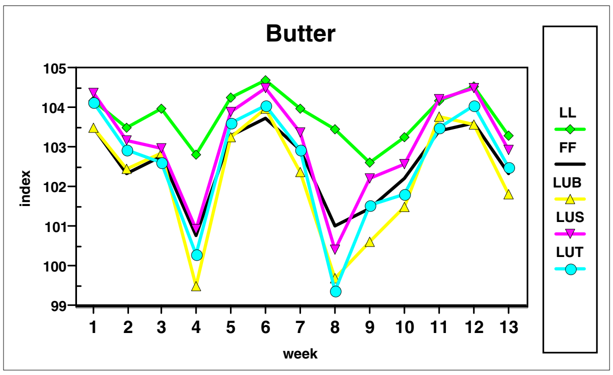
We have concluded that these large negative biases are mainly due to within group bias. By calculating each commodity as one large unit value we have effectively assumed that all items within a commodity are homogeneous. To use one of the examples below, we have assumed that a consumer standing in a street ready to purchase breakfast cereal, will be equally indifferent between the local corner store and the supermarket, and between say a packet of WeetBix and a packet of CocoPops. By making this assumption we have been able to produce a lower bound of unit value indexes for each commodity in which all between group bias has been removed and only within group bias remains. On the other hand,the index LL which has no within group bias or between group bias, provides an upper bound of the unit value indexes.

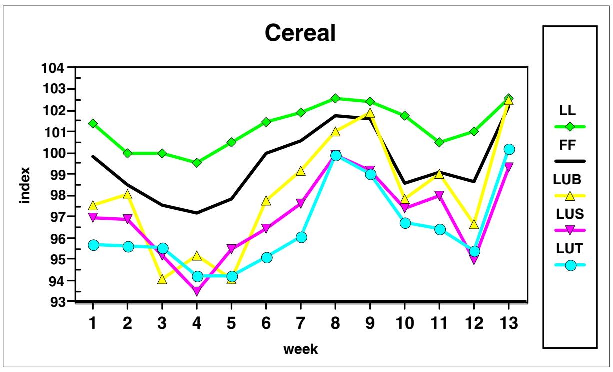
These two commodities show that the level of UU relative to the other indexes can vary quite considerably. We have concluded that the greater the diversity within the items that constitute a commodity, the greater the tendency for UU to fall relative to a Paasche (PP) index for the same commodity. Butter, within which there is little diversity beyond salt levels and container size, has a UU index above PP. Cereal, within which there is a great variety from barely processed grains through to very expensive specialised items has a UU falling below PP. We think this reflects the level of heterogeneity or within group bias.

7.4 Groupings across items — LUB, LUS and LUT Indexes

We next looked at various groupings of homogeneous items, the aim of this strategy being to reduce the number of unit values. If we are to use scanner data directly, this would be one of the options that we may consider. Here are the results for butter and cereal. Results for other commodities can be found in Attachment 4.

Figure 3 : LUB, LUS and LUT Indexes for Butter and Cereal





Initially we used no more than five groupings within a given commodity. The reason for restricting the number of groups was that in the real world of CPI compilation adding more groups would increase the processing and other costs. Within our seven commodities the degree to which items fell neatly into this number of groups varied enormously. In most cases we had one conglomerate group, described as "other", consisting of all the items that could not be classified into a distinct group.

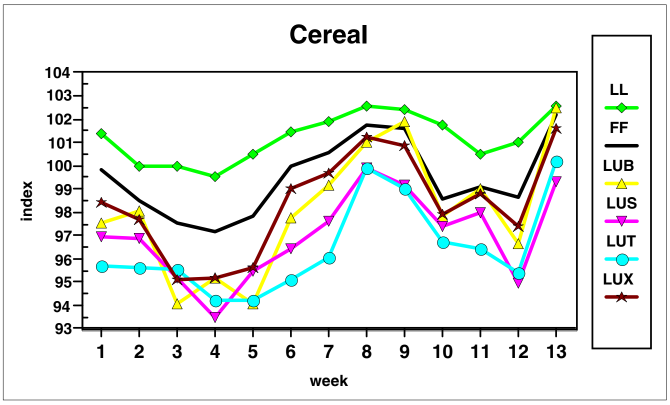
We found that greater the degree of heterogeneity within a grouping, greater the downward bias. In some cases this was quite extreme. This is consistent with our hypothesis that downward bias is caused by within group bias, as a more heterogeneous group would in all probability contain more within group bias.

Again the negative bias due to grouping of items in cereals was much larger than in grouping of items in butter. On the other hand these indexes also contain some upward or positive bias attributable to between group substitution bias. For this reason they do not fall as low as the UU indexes in which all upward bias has been removed.

7.4.1 Finer groupings

We found that the more homogeneous the grouping, the less the negative bias. We therefore tried a finer grouping. We tested it with our data for cereal. We produced an index, LUX, based on 9 groups that the *Choice* magazine, published by Australian Consumers' Association ("Breakfast cereals" in *Choice* magazine March 2000), used to classify the cereals that they were testing. The index LUX compared favourably with those of our earlier, rougher groupings of cereal but had a smaller downward bias.

Figure 4 : LUB, LUS, LUT and LUX Indexes for Cereal



Our conclusion, across all commodities, was that grouping by items is probably not the best approach to employ in using unit values in the CPI.

7.4.2 Grouping across all items — LU(3) Indexes

Finally, in contrast to our previous unit values, we constructed indexes by store, across items (previous work involved unit values across stores by item) and aggregated these using outlet level base period weights. We have called these indexes LU(3).

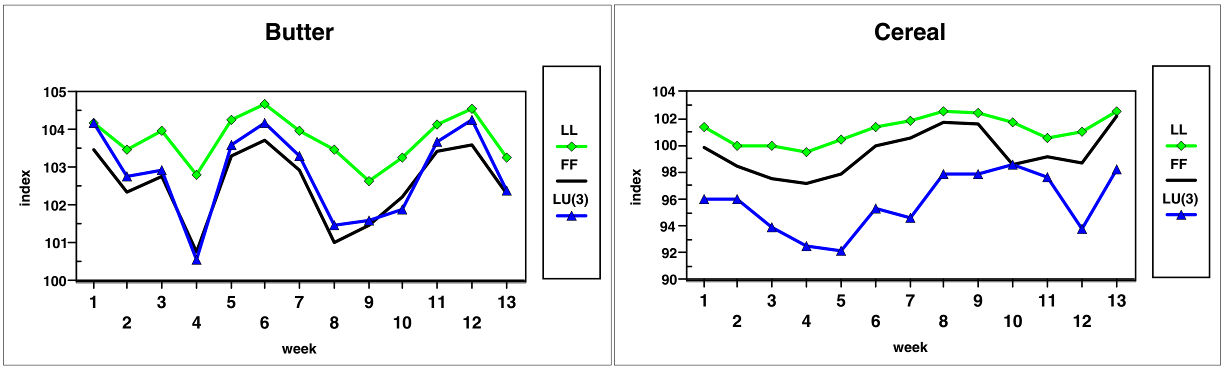
Our aim was to reduce substitution bias as much as possible. We needed to get our groupings ie the items across which we obtain a single unit value, as close as possible to those items between which a consumer is indifferent. In making a purchasing decision for, say, butter, a consumer will not normally regard as potential for purchase, all items within a given category regardless of location. Yet our previous unit values were effectively making this assumption.

In reality, consumers are more likely to make a choice between the array of brands, sizes, flavours etc offered by a single store in which they are already making other purchases. As previously pointed out, this logic would probably not hold for larger, lumpier purchases.

The unit values themselves, of course, still introduce downward bias. It can be seen that LU(1) and LU(3) both fall close to FF at times. As mentioned earlier, we think this downward bias is due to the fact that the averaging that occurs in calculating a unit value generally results in items being grouped together which would not really be seen by consumers as substitutes. So in LU(1) we are assuming that consumers are indifferent between, for example, any 250gm pack of Devondale salted butter, regardless of location. In LU(3) we are assuming that consumers are happy to buy any butter as long as it is, for example, in Coles in Woden. Neither assumption is realistic.

Cereal shows a greater degree of downwards within group bias in the LU(3) index, relative to butter. This is consistent with our previous observations regarding the effect of heterogeneity within a group or unit value.

Figure 5 : LU(3) Indexes for Butter and Cereal



For results of unit values by store, LU(3), of other commodities see Attachment 4.

7.5 Outliers

Coffee is an interesting case study. We know from checking the data, that in week 7, one item had a reduced price and consequently huge sales. This blip seems to be smoothed out in LU(1) (Coffee by item). When the unit value is obtained by item, the effect of the price reduction/sales increase is restricted to only one unit value. The effect is therefore lost in the weighting process because of the use of base weights which have the effect of under weighting that item in that period.

Figure 6 : LU(1) and LU(2) Indexes for Coffee

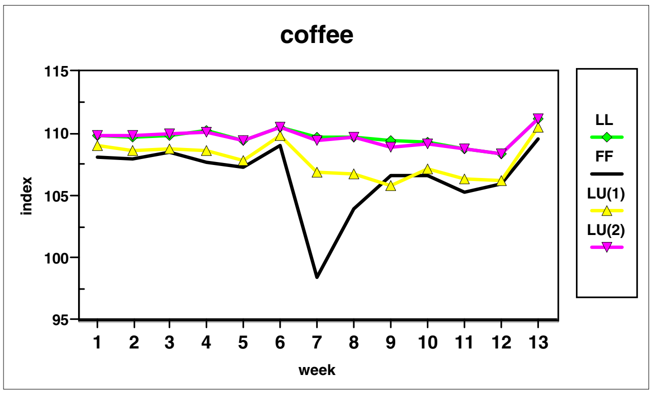
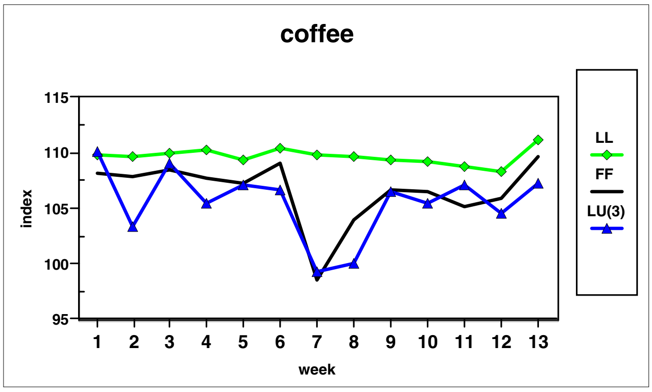
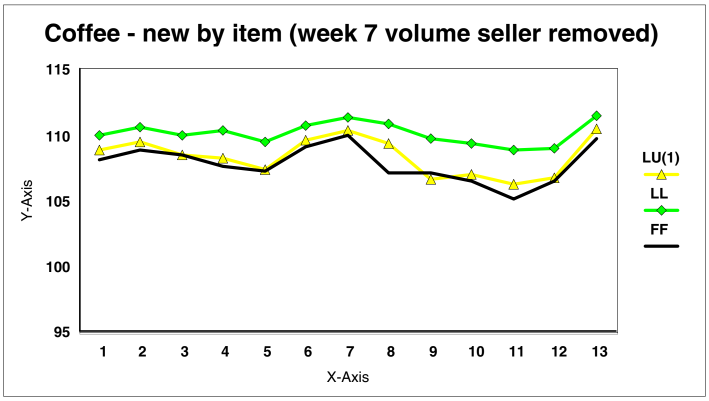


Figure 7 : LU(3) Index for Coffee



However, when the unit values are computed by store, a small effect from the price reduction/sales increase is contained in almost every unit value. This means that the weighting process does not smooth out the blip in the same way as it does in the previous example. This suggests that the weights of each store have not changed markedly in this period relative to the base period. However, LU(3) shows unacceptable levels of volatility from week to week. Monthly calculations (using the newer, bigger dataset) may smooth this effect.

Figure 8 : LU(1) Index for Coffee After Removing the Outlier



The indexes above were recalculated with the outlier removed. All the indexes showed similar movements confirming that the unusual movement was caused by just one item. Clearly, extraordinary sales of a particular item have the potential to radically alter the shape of an index.

8 Conclusions

Faced with a rich scanner dataset, the price statistician might contemplate exploiting the quantity data to compile, say, superlative indexes or unit value indexes.

A superlative index, for example a Fisher index, would take account of the substitution effects.

However, a Fisher index requires special treatment and assumptions when the statistician must incorporate new items or stores.

Moreover, the price and quantity data are required at the finest possible level of aggregation.

A unit value index could be a more tractable alternative. It can be calculated even with data aggregated to a higher level.

But aggregation across heterogeneous or non-substitutable items, stores or time periods would introduce a unit value bias. This can, to some extent, be offset by substitution between the groups. If the groups could be formed in such a way that the two biases —within- and between-groups— could be balanced, then the index would be free of overall substitution bias.

A unit value can also handle new items and stores if they could be grouped with similar items and stores already included in the unit value index compilation. Items not sold during the period can also be handled provided another item in the group was sold.

Our aim in these analyses was to test the usefulness of unit values as the basis of an index at the elementary aggregate level of CPI compilation. We also planned to test which is the best method of aggregation — across items; across outlets; and/or across time.

In this paper, we have also looked at a number of ways of grouping items/stores in order to condense the information in scanner datasets without losing important detail. Our rationale is that, in its raw state, a typical scanner dataset is very large and could pose considerable processing problems for the price statistician.

We have experimented with a number of groupings to see how the resulting index compares to our "ideal" index (FF) and what sort of biases are introduced by the use of unit values.

In summary:

- the LL index can be regarded as our upper bound (under normal consumer behaviour), containing as it does only between-group bias;
- the UU index is our lower bound where there is effectively only one group and where all between-group bias has been removed leaving only within-group bias; and
- the LUx indexes, with some degree of grouping —across brand, size and/or type— usually fall between LL and UU.

Looking to possible practical applications of unit values to price index compilation, we have considered the questions:

Is it possible to define a general schema for grouping the price and quantity data, so that our unit value indexes approximate the benchmark (FF) index as closely as possible?

Is it possible to minimise the number of groupings, in order to balance the need for accuracy (bias reduction) with the need to keep processing costs down?

Our exploratory study suggests that:

- Most of the ways of grouping items with which we experimented so far introduce appreciable levels of unit value bias.

- It is not apparent how one would define a general schema for grouping items. The price statistician would have to consider each commodity case-by-case. The judgments made during such a grouping procedure would be similar in kind and in scale to the judgments made by prices statisticians when they are applying traditional approaches to selecting price samples.

- There appears to be a potential for grouping outlets. Instead of using scanner data at the individual outlet level, data might be obtained at the supermarket chain level or perhaps aggregated across all outlets. This could reduce the size of the dataset considerably.

We would wish to test these interim findings using the larger scanner dataset now available to us.

There are, however, a number of advantages to using unit values. Non-responses and new items and stores are easily accommodated. Also, missing values can be absorbed without the need for explicit imputation. These matters are being addressed in other ABS studies based on the scanner datasets.

9 Limitations of Our Analysis and Future Work

There are a number of limitations to our analysis:

- Our dataset covers only one calendar quarter and there may be cyclical and seasonal variations in price and quantity which do not show up.

- The indexes in this paper were calculated at weekly intervals, which leads to quite volatile price movements when unit values are the basis of the index.

Analyses planned for the future include:

- Calculating unit value indexes for a much longer dataset of 65 weeks.

- Making use of the longer dataset to calculate monthly indexes. It is hoped that this will remove a lot of the volatility inherent in the weekly unit value indexes.

- Future work could involve looking at the variances of various groupings of items within a commodity as a way of determining the best ones.

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Attachment 1

Formulae for "Population Indexes"

For the analyses reported in this paper, the scanner data is taken to represent the whole population of items and outlets. Therefore the indexes defined below can be considered to be "population indexes" rather than a sample-based estimates.

Weighted Index Formulae

Laspeyres and Paasche indexes

We have denoted a fixed weighted index as LL_t to denote an index which is calculated using fixed base period weights to aggregate across both item and outlet levels and PP_t to denote a current weighted index at both levels. We have also calculated indexes that use fixed weights to aggregate across items but current weights to aggregate across outlets and vice versa. These indexes have been denoted as LP_t and PL_t respectively.

The commodity index computed as a Laspeyres to aggregate across item and outlet levels (LL_t) is given by

$$LL_t = \frac{\sum_i \sum_j P_{tij} Q_{0ij}}{\sum_i \sum_j P_{0ij} Q_{0ij}}$$

This can also be written as

$$LL_t = \sum_i W_{0i} \sum_j W_{0ij} R_{tij}$$

where R_{tij} is the price relative for item i and outlet j in period t, i.e. $R_{tij} = P_{tij} / P_{0ij}$,

$$W_{0ij} = \frac{P_{0ij} Q_{0ij}}{\sum_j P_{0ij} Q_{0ij}}$$

is the expenditure weight for item i and outlet j in the base period (t = 0) and

$$W_{0i} = \frac{\sum_j P_{0ij} Q_{0ij}}{\sum_i \sum_j P_{0ij} Q_{0ij}}$$

is the expenditure weight for item i in the base period (summed across outlets).

An index calculated as Paasche at both commodity and item level (PP_t) can also be expressed in terms of expenditure weights as

$$PP_t = \left(\sum_i W_{ti} \sum_j \frac{W_{tij}}{R_{tij}} \right)^{-1}$$

where

$$W_{tij} = \frac{P_{tij} Q_{tij}}{\sum_j P_{tij} Q_{tij}}$$

is the expenditure weight for item i and outlet j in period t ; and

$$W_{ti} = \frac{\sum_j P_{tij} Q_{tij}}{\sum_i \sum_j P_{tij} Q_{tij}}$$

is the expenditure weight for item i in period t (summed across outlets).

Fisher index

A Fisher index is a geometric mean of Laspeyres and Paasche indexes. Thus FF_t is the Fisher index at both item and outlet levels:

$$FF_t = (LL_t * PP_t)^{1/2}.$$

An index can also be computed which is Fisher index at one level but a Laspeyres or Paasche index at another. Thus an index computed as Fisher at the commodity level and Laspeyres at the item level (FL_t) is given by

$$FL_t = (LL_t * PL_t)^{1/2},$$

while an index computed as Laspeyres at the commodity level and Fisher at the item level (LF_t) is given by

$$LF_t = (LL_t * LP_t)^{1/2}.$$

Unit values

LU(1): Unit value computed across all outlets

$$LU(1)_t = \sum_i W_{0i} \frac{\overline{P}_{ti}}{\overline{P}_{0i}}$$

where $\overline{P_{ti}}, \overline{P_{0i}}$ are unit values for item i and $\overline{P_{ti}}$ is given by

$$\overline{P}_{ti} = \frac{\sum_j P_{tij} Q_{tij}}{\sum_j Q_{tij}}$$

and W_{0i} is the fixed Laspeyres weight for item i.

LU(2): Unit value computed across all outlets within each chain

$$LU(2)_t = \sum_i W_{0i} \sum_c^4 W_{0ic} \frac{\overline{P}_{tic}}{\overline{P}_{0ic}}$$

where $\overline{P_{tic}}, \overline{P_{0ic}}$ are unit values of item i in chain c.

$$\overline{P}_{tic} = \frac{\sum_{j\in c} P_{ticj} Q_{ticj}}{\sum_{j\in c} Q_{ticj}}$$

and W_{0ic} is a fixed Laspeyres weight defined as:

$$W_{0ic} = \frac{\sum_{j\in c} P_{0icj} Q_{0icj}}{\sum_{c} \sum_{j\in c} P_{0icj} Q_{0icj}}$$

LUB: Unit value computed across different brands and all outlets

$$LUB_t = \sum_b W_{0b} \frac{\overline{P}_{tb}}{\overline{P}_{0b}}$$

where $\overline{P}_{tb}$, $\overline{P}_{0b}$ are unit values of brand group b.

$$\overline{P}_{tb} = \frac{\sum_{i\in b} \sum_{j} P_{tbij} Q_{tbij}}{\sum_{i\in b} \sum_{j} Q_{tbij}}$$

and W_{0b} is a fixed Laspeyres weight defined as:

$$W_{0b} = \frac{\sum_{i\in b} \sum_{j} P_{0bij} Q_{0bij}}{\sum_{b} \sum_{i\in b} \sum_{j} P_{0bij} Q_{0bij}}$$

LU(3): Unit value computed across all items

$$LU(3)_t = \sum_j W_{0j} \frac{\overline{P}_{tj}}{\overline{P}_{0j}}$$

where $\overline{P}_{tj}$, $\overline{P}_{0j}$ are unit values for outlet j and $\overline{P_{tj}}$ is given by

$$\overline{P}_{tj} = \frac{\sum_{i} P_{tij} Q_{tij}}{\sum_{i} Q_{tij}}$$

and W_{0j} is a fixed Laspeyres weight for outlet j.

UU: Unit value computed across all items and outlets

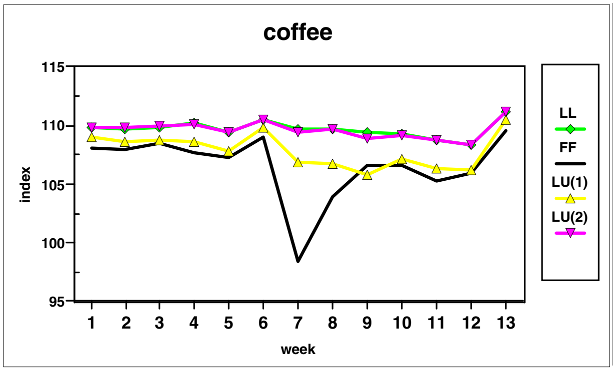
$$UU_t = \frac{\overline{P}_t}{\overline{P}_0}$$

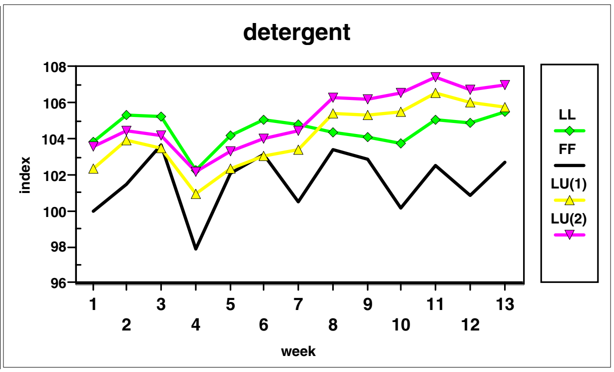
where $\overline{P}_t$, $\overline{P}_0$ are unit values across all items and outlets

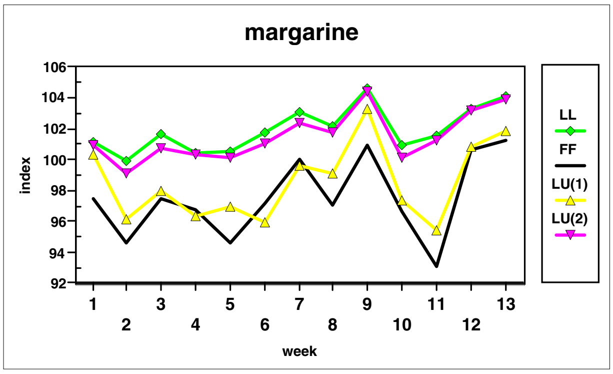
$$\overline{P}_t = \frac{\sum_i \sum_j P_{tij} Q_{tij}}{\sum_i \sum_j Q_{tij}}$$

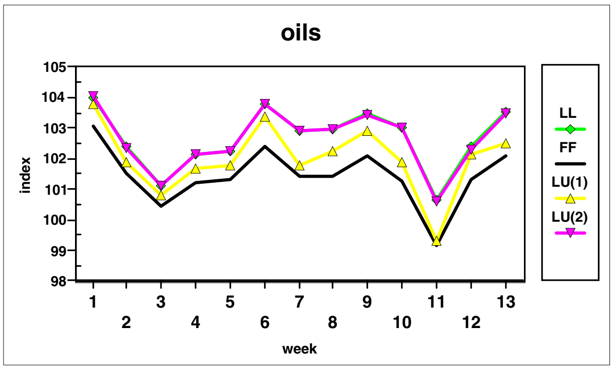
Attachment 2

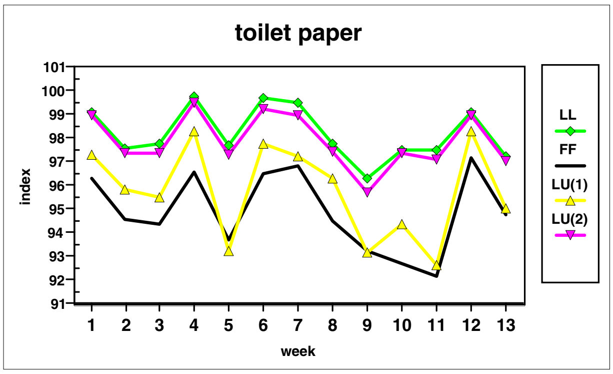
Unit Values Across Outlets and Across Outlets within Each Chain — LU(1) and LU(2) Indexes





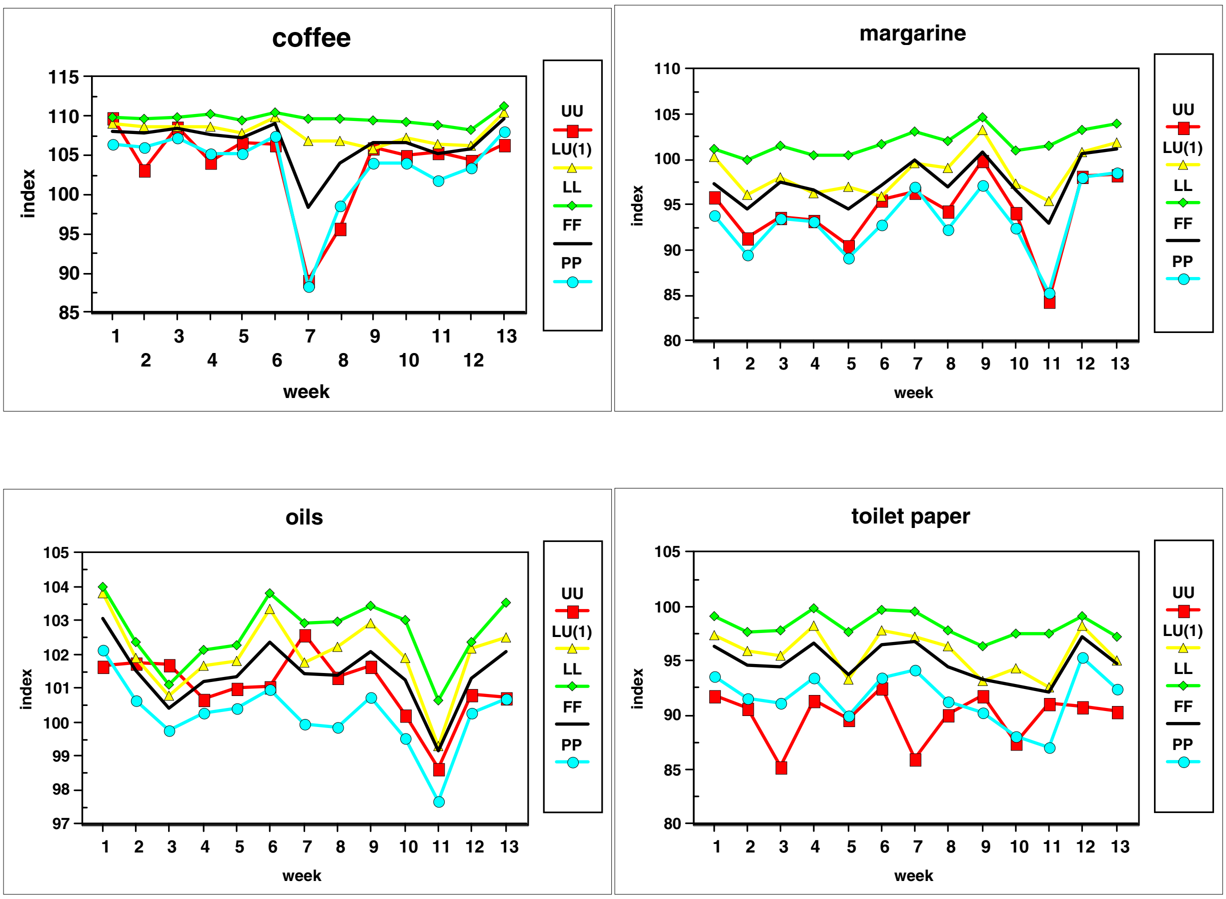






Attachment 3

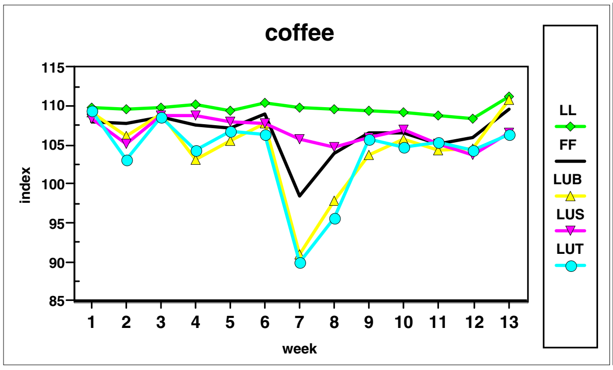
Unit Values Across All Items and Outlets —
UU Indexes

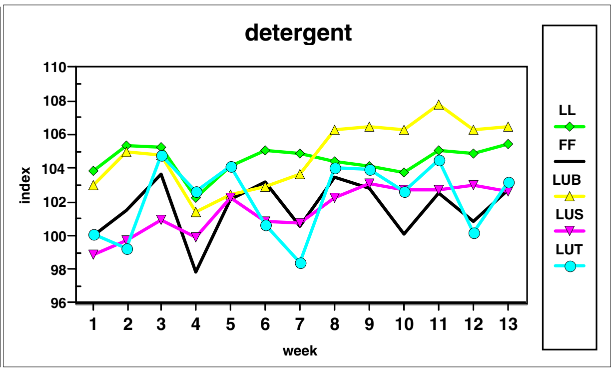


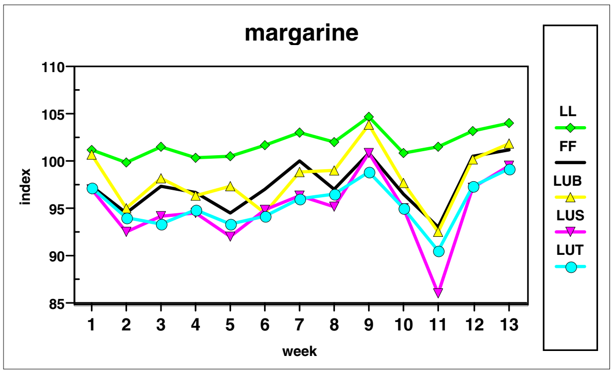
Most of the dishwashing detergents are liquid but some are powders. Detergents also come in different strengths, some are normal strength and some are concentrates. A unit value across powders measured by weight and liquids (of different strength) measured by volume could not all be combined into one unit value and therefore UU was not calculated for the detergents.

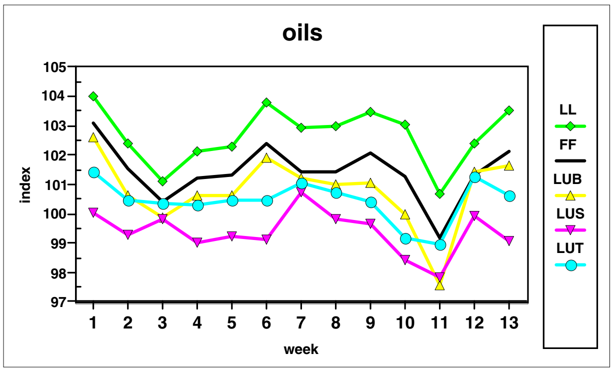
Attachment 4

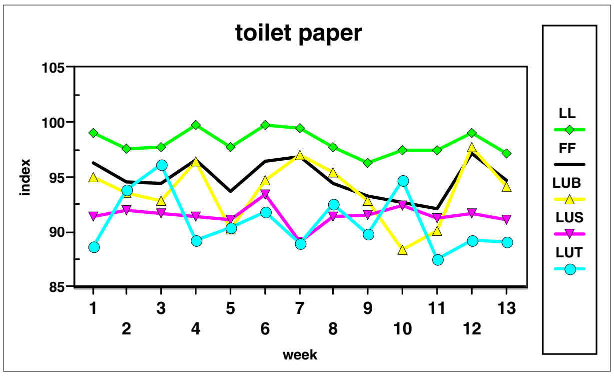
Unit Values Across Items, Brands, Sizes and Types — LUB, LUS and LUT Indexes



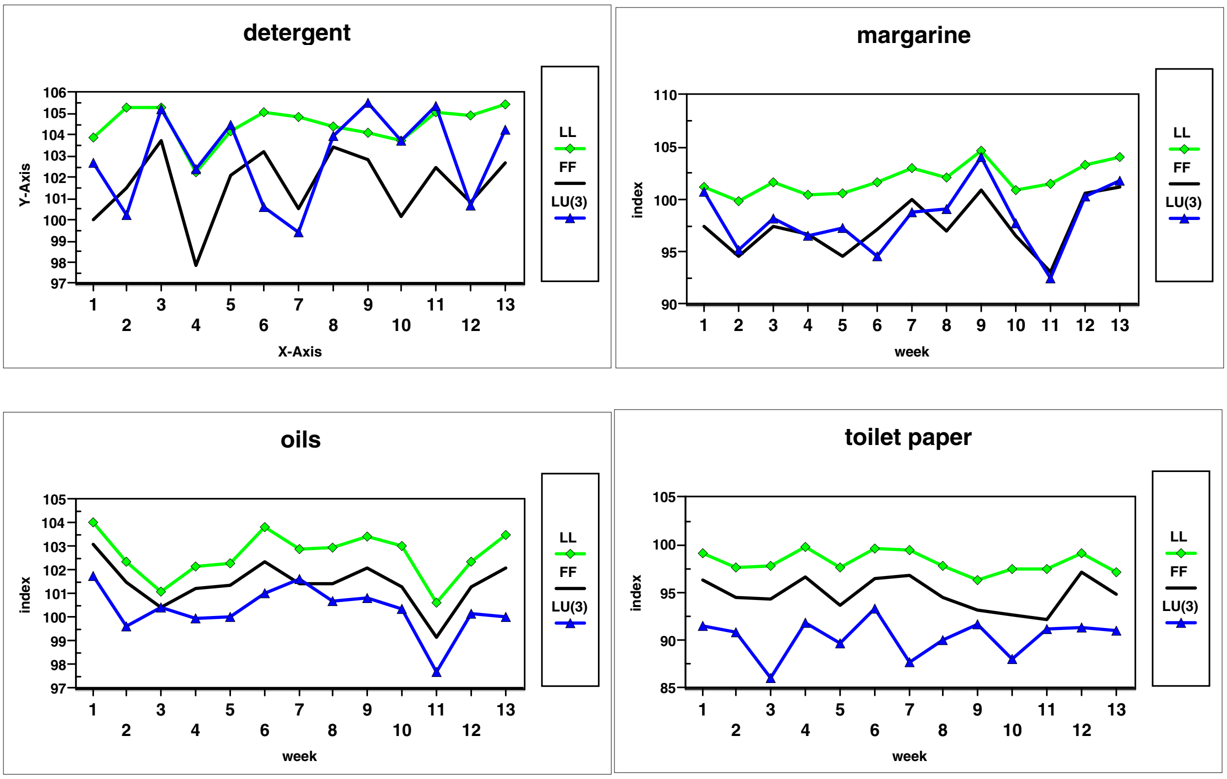








Unit Values Across All Items — LU(3) Indexes



No LU(3) was calculated for the detergents (see Attachment 3).

Attachment 6

Item Groupings for Butter in LUB, LUS and LUT Indexes — An Example

Group type	Groups	Comments
Brand	Devondale	
	generic	this category includes all generic brands, from each of the four chains in the data, grouped together
	other	this category includes all brands with one or only a small number of items
	Western Star	
Size	125 grams 250 grams 375 grams 400 grams 500 grams	
Product type	blend 250 grams garlic 375 grams light 400 grams low salt 500 grams normal	