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Some Empirical Experiments with CES Functions

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Remarks:

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SOME EMPIRICAL EXPERIMENTS WITH CES FUNCTIONS

Summary: Balk (2000) suggests a CES-based approximation of a cost of living index, that accounts for substitution and new goods. In this paper empirical evidence is presented based on Dutch Household Expenditure Survey data and supermarket scanner data.

Keywords: substitution bias, new goods bias, cost of living index

1. Introduction

In 1996 the Advisory Commission to Study the Consumer Price Index, chaired by Michael J. Boskin, concluded that changes in the U.S. consumer price index (CPI) would overstate changes in the true cost of living. The Commission estimates the size of the upward bias by 1.1 percentage points per year, with plausible values ranging from 0.8 to 1.6 percentage points per year (Boskin (1996)).

The Commission identified four categories of potential bias:

- (Upper and lower level) substitution bias
- Outlet substitution bias
- Quality change bias
- New products bias

The upward bias would contribute about \$148 billion to the U.S. deficit in 2006 and \$691 to the national debt by then. In 2008, these contributions would be \$202 billion and \$1.07 trillion, respectively (Boskin (1996)). Early 2000 Bert M. Balk (Statistics Netherlands) attempted to attack the upward bias of the CPI. He proposed a CES-based cost of living index formula that, under certain assumptions, accounts for substitution and new goods (Balk (2000)). He also provided formulas for calculating this index "in real time".

In this paper Balk's recipe is tested on real data. In section 2 the underlying theory is summarised. Section 3 deals with empirical studies and section 4 concludes.

2. Theory

2.1 One commodity group

In Balk (2000) variable (but overlapping) commodity sets are considered. It is assumed that

- the representative consumer's preference structure exhibits homotheticity;

- for any pair of commodities the elasticity of substitution is the same¹;
- the unit cost (expenditure) function $C_\sigma(p^t, u|I^t)$ is of the CES type, that is

$$C_\sigma(p^t, u|I^t) = u \cdot \left(\sum_{n \in I^t} b_n (p_n^t)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \quad (1)$$

where $u > 0$ is a utility level, I^t is the set of available commodities in period t , $\sigma \geq 0, \sigma \neq 1$ is the (demand) elasticity of substitution, $b_n > 0 (n \in I^t)$ are quality or taste parameters and p^t is the vector of prices $p_n^t > 0 (n \in I^t)$ in period t ;

- σ and $b_n > 0 (n \in I^t)$ are time-invariant.

The cost of living index for period 1 relative to period 0 is defined as

$$P_\sigma^C(p^1, p^0|I^1, I^0) \stackrel{def}{=} \frac{C_\sigma(p^1, u|I^1)}{C_\sigma(p^0, u|I^0)}, \quad (2)$$

which is independent of u , by homotheticity.

Define $I^{01} = I^0 \cap I^1 \neq \emptyset$ as the set of ongoing commodities. Let $s_n^{t*} = p_n^t x_n^t / \sum_{n \in I^{01}} p_n^t x_n^t$ denote the expenditure shares relative to I^{01} ($n \in I^{01}, t = 0, 1$),

$$\lambda^t \stackrel{def}{=} \frac{\sum_{n \in I^{01}} p_n^t x_n^t}{\sum_{n \in I^t} p_n^t x_n^t} \quad (3)$$

the fraction of the expenditure in period t that is attributable to the ongoing commodities, and $x_n^t \geq 0$ the quantities consumed ($n \in I^t, t = 0, 1$). Then Balk (2000) shows that, under the further basic assumption that the actual expenditure shares are equal to the optimal shares, the cost of living index is given by

$$P_\sigma^C(p^1, p^0|I^1, I^0) = \left[\frac{\lambda^0}{\lambda^1} \right]^{\frac{1}{1-\sigma}} \left[\sum_{n \in I^{01}} s_n^{0*} \left(\frac{p_n^1}{p_n^0} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \quad (4)$$

We will denote the right hand side of (4) by the *CES Laspeyres index*.

Then Balk (2000) shows that²

$$\left[\sum_{n \in I^{01}} s_n^{0*} \left(\frac{p_n^1}{p_n^0} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = \left[\sum_{n \in I^{01}} s_n^{1*} \left(\frac{p_n^1}{p_n^0} \right)^{-(1-\sigma)} \right]^{\frac{-1}{1-\sigma}}, \quad (5)$$

¹ Hence the name "CES": constant elasticity of substitution.

² x^t is the vector of quantities consumed $x_n^t (n \in I^t)$ in period t .

$$\left[\sum_{n \in I^{01}} s_n^{0*} \left(\frac{p_n^1}{p_n^0} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = P^{SV} (p^1, x^1, p^0, x^0 | I^{01}) \quad (6)$$

and

$$\left[\sum_{n \in I^{01}} s_n^{1*} \left(\frac{p_n^1}{p_n^0} \right)^{-(1-\sigma)} \right]^{\frac{-1}{1-\sigma}} = P^{SV} (p^1, x^1, p^0, x^0 | I^{01}). \quad (7)$$

$P^{SV} (p^1, x^1, p^0, x^0 | I^{01})$ is the Sato (1976) - Vartia (1976) price index, based on the set of ongoing commodities. This index is defined as

$$P^{SV} (p^1, x^1, p^0, x^0 | I^{01}) \stackrel{def}{=} \prod_{n \in I^{01}} \left(\frac{p_n^1}{p_n^0} \right)^{w_n^{01}}, \quad (8)$$

where³

$$w_n^{01} \stackrel{def}{=} \frac{L(s_n^{0*}, s_n^{1*})}{\sum_{n' \in I^{01}} L(s_{n'}^{0*}, s_{n'}^{1*})} \quad (n \in I^{01}). \quad (9)$$

Clearly

$$P_\sigma^C (p^1, p^0 | I^1, I^0) = \left[\frac{\lambda^0}{\lambda^1} \right]^{\frac{1}{1-\sigma}} \left[\sum_{n \in I^{01}} s_n^{1*} \left(\frac{p_n^1}{p_n^0} \right)^{-(1-\sigma)} \right]^{\frac{-1}{1-\sigma}}. \quad (10)$$

We will denote the right hand side of (10) by the *CES Paasche index*.

Equations (5), (6) and (7) constitute a practical recipe for obtaining the elasticity of substitution σ . Due to the well known monotony of any generalised mean $\left(\sum_n a_n (z_n)^k \right)^{\frac{1}{k}}$ in its parameter k (Hasenkamp (1978), Theorem 2), the three equations each have at least one solution⁴. The three computed σ 's may differ, as a result of numerical instability.

2.2 More commodity groups

The preceding theory was presented for one commodity group. Along the same lines of reasoning equations can be developed for a set of commodity groups.

³ The logarithmic mean $L(a, b)$ of $a \neq b$ ($a > 0, b > 0$) is defined as

$$L(a, b) \stackrel{def}{=} (a - b) / (\ln(a) - \ln(b)). \quad L(a, a) \stackrel{def}{=} a.$$

⁴ When all price relatives p_n^1 / p_n^0 ($n \in I^{01}$) are identical, equations (5), (6) and (7) become identities, having an infinite number of solutions. When all price relatives are nearly identical, numerical solutions may show pathological behaviour, as is the case for the vitamins data in section 3.2.

Let I^t be the union of disjoint commodity groups I_g^t ($1 \leq g \leq G, t = 0, 1$). We assume that the set of commodity groups is time-invariant. Then for any $1 \leq g \leq G$ and any choice of commodity group subindex⁵ $\Pi_g^{01}(p_g^1, x_g^1, p_g^0, x_g^0 | I_g^{01})$, and under similar assumptions as the foregoing, the following equations hold:

$$\left(\sum_{n \in I_g^{01}} s_{ng}^{0*} \left(\Pi_g^{01}(p_g^1, x_g^1, p_g^0, x_g^0 | I_g^{01}) \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} = \left(\sum_{n \in I_g^{01}} s_{ng}^{1*} \left(\Pi_g^{01}(p_g^1, x_g^1, p_g^0, x_g^0 | I_g^{01}) \right)^{-(1-\sigma)} \right)^{\frac{-1}{1-\sigma}}, \quad (11)$$

$$\left(\sum_{n \in I_g^{01}} s_{ng}^{0*} \left(\Pi_g^{01}(p_g^1, x_g^1, p_g^0, x_g^0 | I_g^{01}) \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} = \prod_{n \in I_g^{01}} \left(\Pi_g^{01}(p_g^1, x_g^1, p_g^0, x_g^0 | I_g^{01}) \right)^{w_{ng}^{01}} \quad (12)$$

and

$$\left(\sum_{n \in I_g^{01}} s_{ng}^{1*} \left(\Pi_g^{01}(p_g^1, x_g^1, p_g^0, x_g^0 | I_g^{01}) \right)^{-(1-\sigma)} \right)^{\frac{-1}{1-\sigma}} = \prod_{n \in I_g^{01}} \left(\Pi_g^{01}(p_g^1, x_g^1, p_g^0, x_g^0 | I_g^{01}) \right)^{w_{ng}^{01}}, \quad (13)$$

where $s_{ng}^{t*} \stackrel{def}{=} p_{ng}^t x_{ng}^t / \sum_{n \in I_g^{01}} p_{n'g}^t x_{n'g}^t$ ($n \in I_g^{01}, t = 0, 1$), $w_{ng}^{01} \stackrel{def}{=} L(s_{ng}^{0*}, s_{ng}^{1*}) / \sum_{n \in I_g^{01}} L(s_{n'g}^{0*}, s_{n'g}^{1*})$ and $\sigma \geq 0, \sigma \neq 1$ is the (constant) between-group substitution elasticity.

Now the cost of living index becomes

$$P_\sigma^C(p^1, p^0 | I^1, I^0) = \left[\sum_{g=1}^G s_g^{0*} \left(\Pi_g^{01}(p_g^1, x_g^1, p_g^0, x_g^0 | I_g^{01}) \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad (14)$$

or

$$P_\sigma^C(p^1, p^0 | I^1, I^0) = \left[\sum_{g=1}^G s_g^{1*} \left(\Pi_g^{01}(p_g^1, x_g^1, p_g^0, x_g^0 | I_g^{01}) \right)^{-(1-\sigma)} \right]^{\frac{-1}{1-\sigma}}, \quad (15)$$

where s_g^{t*} is the period t expenditure share of group g ($t=0, 1$). Note that there is no factor $(\lambda^0/\lambda^1)^{1/(1-\sigma)}$, as in (4) and (10), since there are no new or disappearing groups, by assumption.

Balk (2000) notices that "maintaining (1) with commodity set being variable through time is only sensible when the (overall) elasticity of substitution is larger than 1". As he further states, most empirical work points to a value between 0 and 1. This is also one of our observations (section 3.1.2). As a way out, Balk (2000) suggests to assume a two-level CES structure in the consumer's preferences. He arrives at a cost

⁵ $I_g^{01} \stackrel{def}{=} I_g^0 \cap I_g^1 \neq \emptyset$. p_g^t is the subvector of prices p_{ng}^t ($n \in I_g^{01}$) and x_g^t the subvector of quantities x_{ng}^t ($n \in I_g^{01}$) for group g in period t ($t = 0, 1$).

of living index of the form (14) with $\Pi_g^{01}(p_g^1, x_g^1, p_g^0, x_g^0 | I_g^{01}) = P_{\sigma_g}^C(p_g^1, p_g^0 | I_g^1, I_g^0)$ (as defined in (4)), where σ_g is the (constant) elasticity of substitution for group g ($1 \leq g \leq G$).

3. Empirical evidence

3.1 At the upper level

3.1.1 Data

As a first exercise upper level substitution was studied. From the Dutch Household Expenditure Surveys (HES's) in the period 1990-1995 yearly expenditure shares were computed for the 186 commodity groups for which Statistics Netherlands had published Laspeyres type subindices P_g^{t*} ($1 \leq g \leq 186, 1991 \leq t \leq 1995$) relative to base year 1990. These subindices were used to compute commodity group year-to-year indices

$$\Pi_g^{t-1,t}(p_g^t, x_g^t, p_g^{t-1}, x_g^{t-1} | I_g^{t-1,1}) \stackrel{def}{=} P_g^{t*} / P_g^{(t-1)*} \quad (1 \leq g \leq 186, 1991 \leq t \leq 1995). \quad (16)$$

Table 1 summarises the HES expenditure shares.

Table 1. Summary of Dutch HES expenditure shares (× 10000)

	1990	1991	1992	1993	1994	1995
Minimum	2.04	1.64	2.20	1.41	2.00	2.13
Maximum	1192.96	1192.77	1252.81	1251.74	1320.25	1244.36
Mean	53.76	53.76	53.76	53.76	53.76	53.76
Standard deviation	113.43	114.33	117.41	117.36	121.78	116.97

3.1.2 Computations and results

Solving equations (11), (12) and (13) using (16), elasticities $\sigma_t^{(k)}$ ($1 \leq k \leq 3, 1991 \leq t \leq 1995$) were obtained. They are presented in figure 1.⁶

Next, for the overall elasticity $\hat{\sigma}$ the 10% trimmed mean of all $\sigma_t^{(k)}$'s was chosen ($\hat{\sigma} = 0.369$). This value was used to compute CES Laspeyres (14) and CES Paasche (15) year-to-year indices. They are presented as a chained index in figure 2. As a reference, the chained Fisher index, based on ongoing commodities, has been added,

⁶ The labels in the legend at the bottom of figure 1 refer to the equations in a natural way: "Laspeyres=Paasche" to (11), "Laspeyres=Sato-Vartia" to (12) and "Paasche=Sato-Vartia" to (13).

being a well-known competitor of both chained CES indices in the cost of living index race.

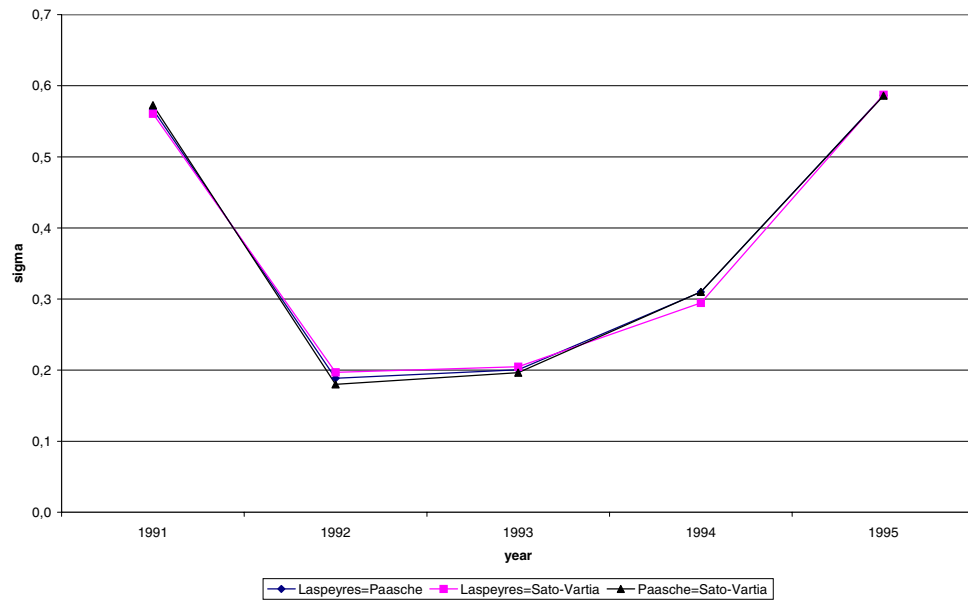


Figure 1. Elasticities for Dutch HES and CPI data

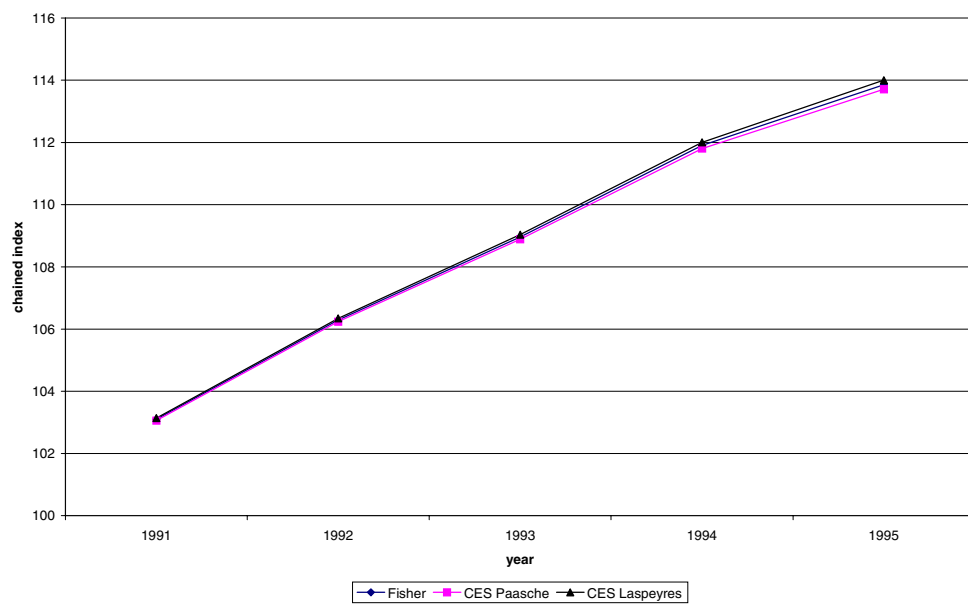


Figure 2. Chained indices for Dutch HES and CPI data

3.2 A two level approach

3.2.1 Data

Statistics Netherlands receives scanner data from two of the largest Dutch supermarket chains on a weekly basis. To study lower level substitution, a selection was made from the vast amount of data, covering expenditure and quantity data for 10 commodity groups and for some 80 outlets during week 26, 1999, up to and including week 49, 2000. These weekly data were aggregated to 19 four-week periods. Commodities are identified by a European Article Number (EAN). Table 2 summarises the number of different EAN codes (representing different commodities sold) in each of the 19 four-week periods.

Table 2. Distribution of number of different EAN codes

commodity group	mean	st. dev.	min	max
cake snacks	73.5	2.7	70	79
cereal	18.4	0.7	17	19
crisps	37.6	3.9	33	43
detergents	50.3	3.1	45	56
disposable baby's napkins	38.8	6.6	21	45
scents	22.4	1.9	20	26
soft drinks	101.3	2.6	98	107
tea	138.7	10.6	115	150
vitamins etc.	26.9	13.7	18	51
yoghurt with additives	59.8	5.7	54	70

3.2.2 Computations and results

At the lower level, for each commodity group, substitution elasticities $\sigma_{gt}^{(k)}$ ($1 \leq k \leq 3$, $2 \leq t \leq 19$) were computed, using (5), (6) and (7) with

$$p_n^\tau \stackrel{\text{def}}{=} \frac{e_n^\tau}{x_n^\tau}, \quad (17)$$

where $e_n^\tau > 0$ and $x_n^\tau > 0$ are the expenditure (turn over rate) and the quantity sold, respectively, for commodity n in period τ ($n \in I^{t-1,t}$, $\tau = t-1, t$). The $\sigma_{gt}^{(k)}$ are presented in the "Elasticities for..." figures⁷.

Next, for the overall group elasticities $\hat{\sigma}_g$ the 10% trimmed means (over k and t) of all $\sigma_{gt}^{(k)}$'s were chosen (table 3). These values were used to compute CES Laspeyres (4) and CES Paasche (10) period-to-period group indices. They are presented as a

⁷ "Laspeyres=Paasche" refers to (5), "Laspeyres=Sato-Vartia" to (6) and "Paasche=Sato-Vartia" to (7).

chained index in the "Chained indices for ..." figures below. As a reference, the adjusted Fisher price index⁸ has been added, as considered by De Haan (2001).

Table 3. Summary of group elasticities

commodity group	10% trimmed mean	st.dev.	min	max
cake snacks	4.45	1.17	2.63	6.57
cereal	2.47	1.89	-5.40	4.01
crisps	4.38	0.97	2.62	6.74
detergents	4.97	1.42	2.33	8.36
disposable baby's napkins	6.40	1.91	1.32	9.18
scents	2.11	2.98	-3.49	7.31
soft drinks	3.52	0.63	2.41	4.70
tea	4.98	1.14	2.60	7.52
vitamins etc.	-21.16	182.99	-604.74	405.70
yoghurt with additives	3.90	1.14	1.78	5.42

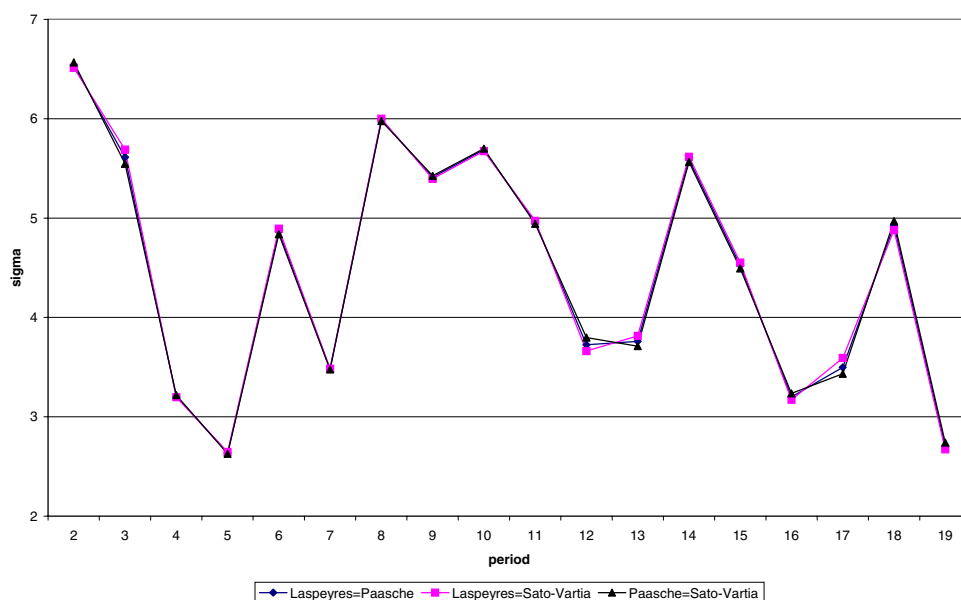


Figure 3. Elasticities for cake snacks data

⁸ The adjusted Fisher price index is simply the usual Fisher price index, based on ongoing commodities, multiplied by $(\lambda^{t-1}/\lambda^t)^{1/(1-\sigma)}$, thus adjusting for new and disappearing commodities.

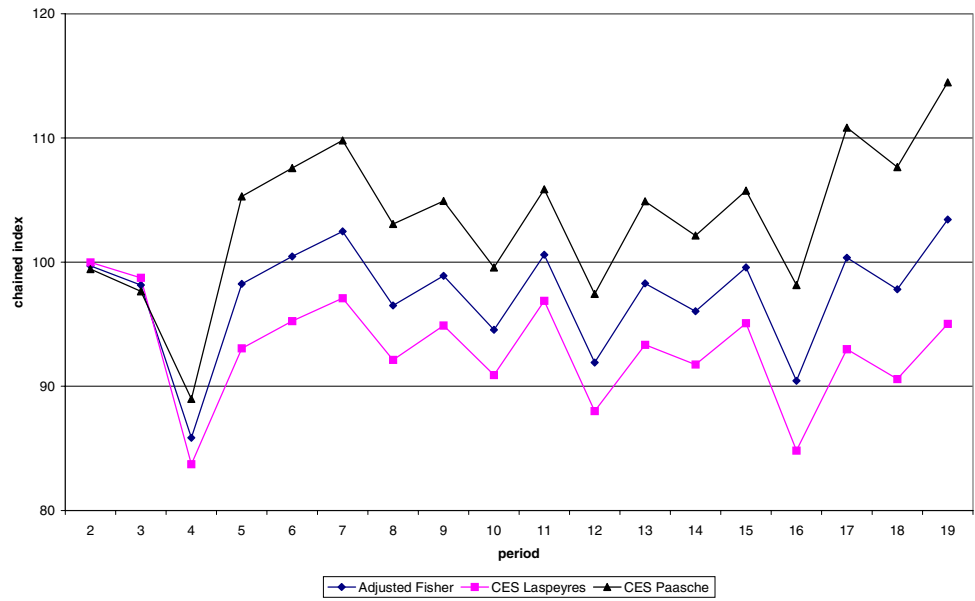


Figure 4. Chained indices for cake snacks data

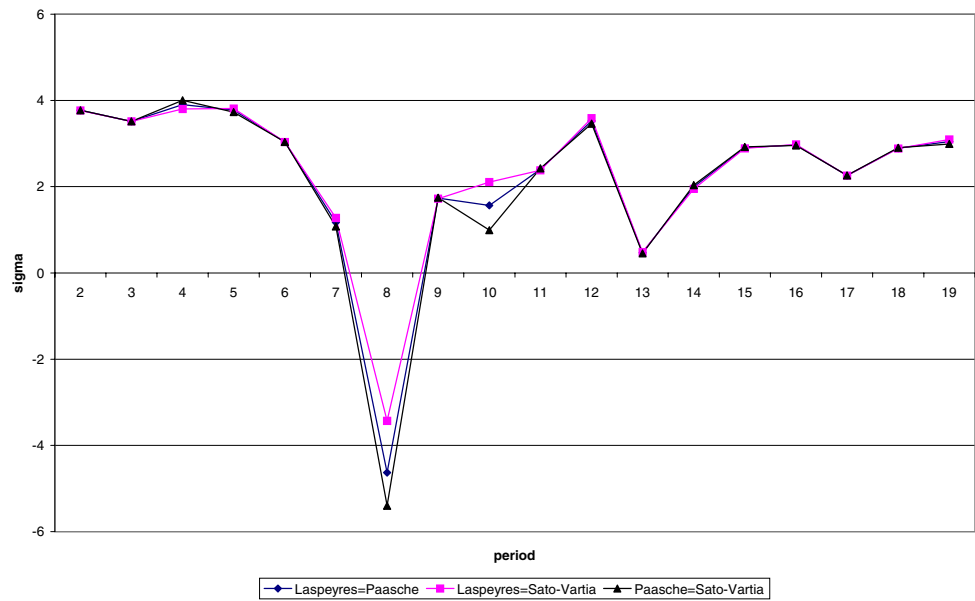


Figure 5. Elasticities for cereal data

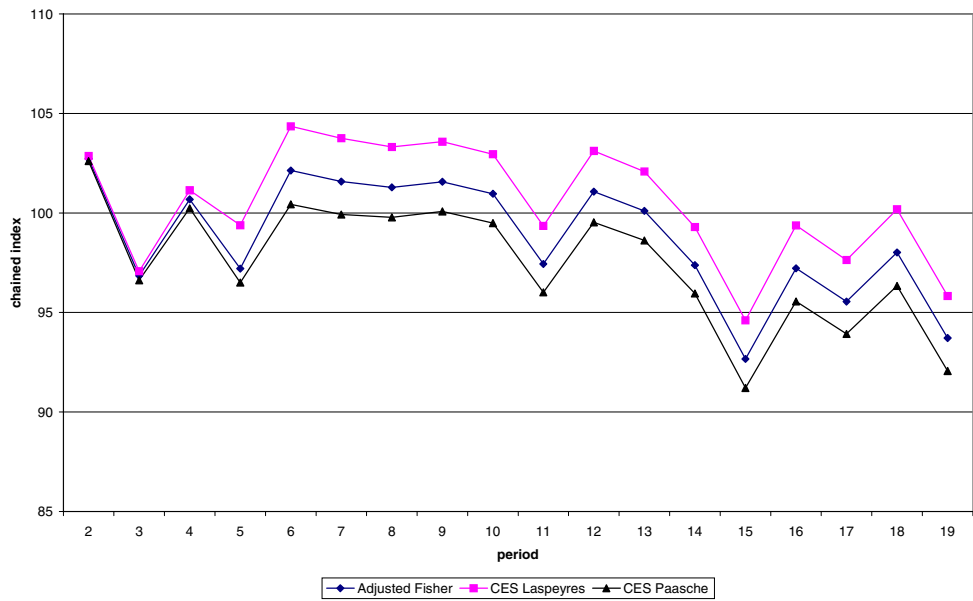


Figure 6. Chained indices for cereal data

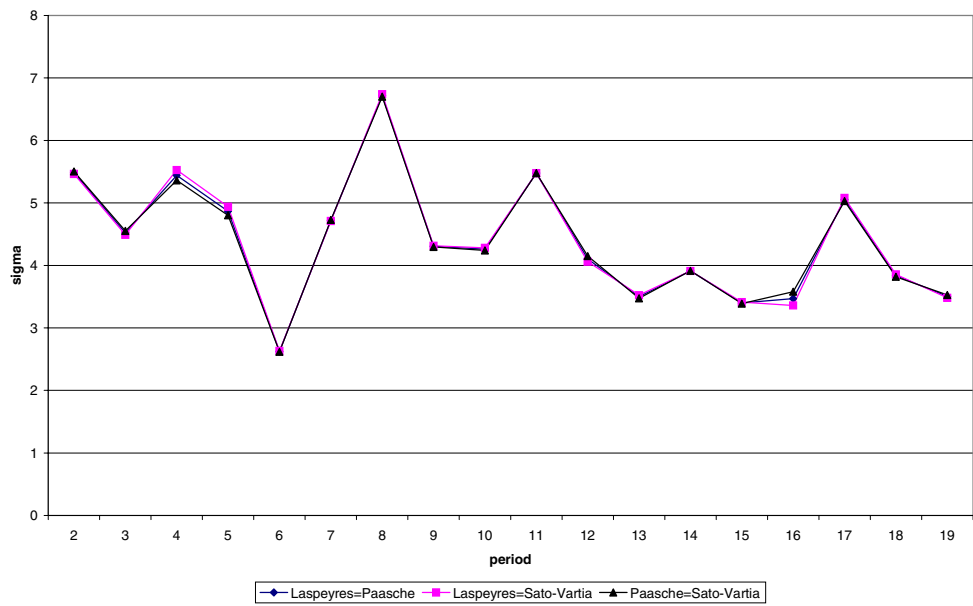


Figure 7. Elasticities for crisps data

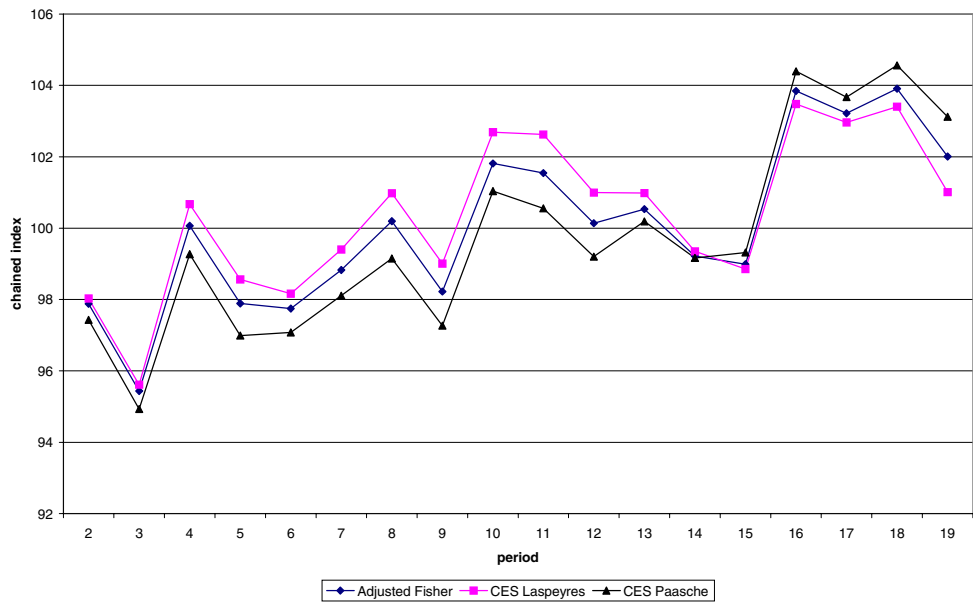


Figure 8. Chained indices for crisps data

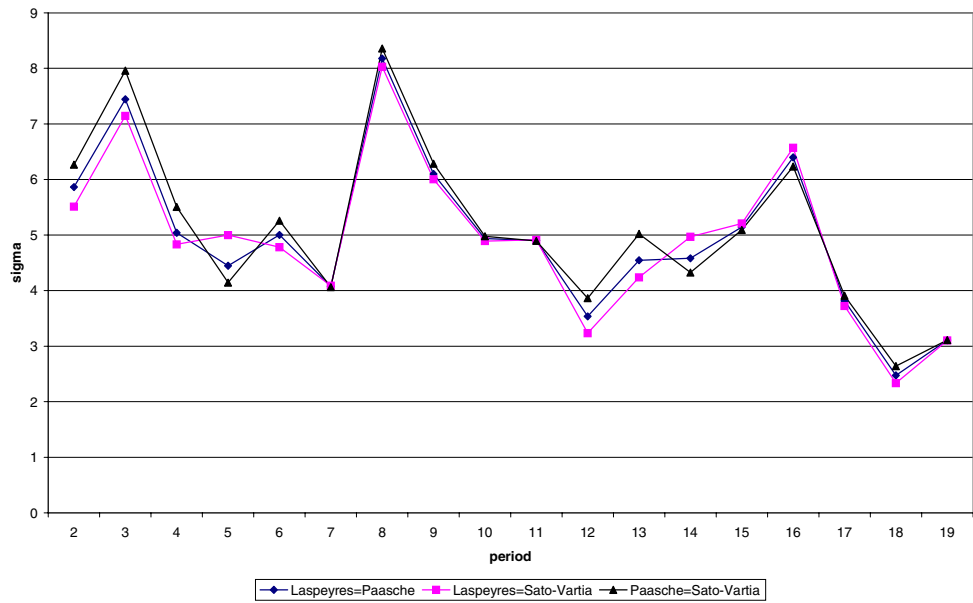


Figure 9. Elasticities for detergents data

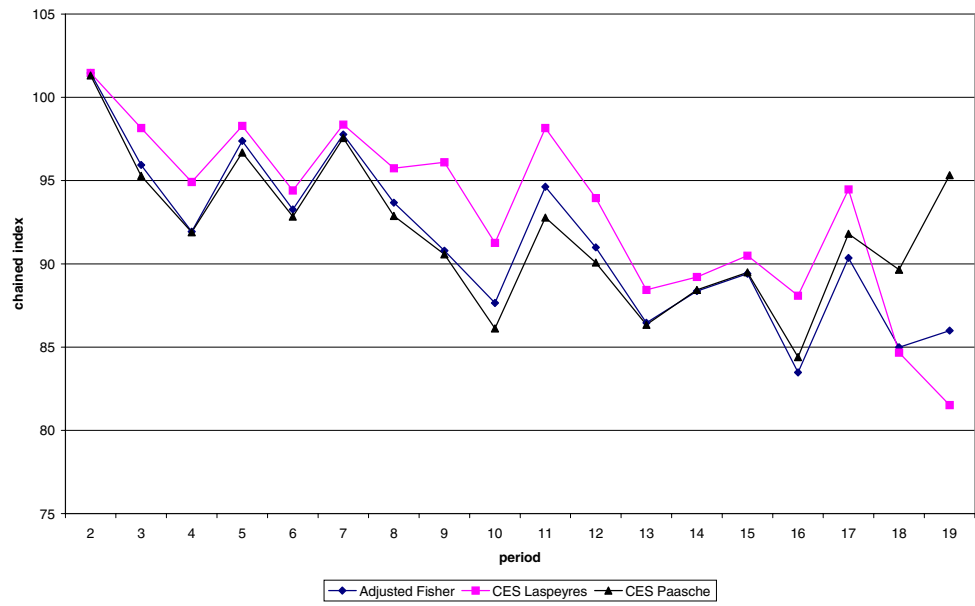


Figure 10. Chained indices for detergents data

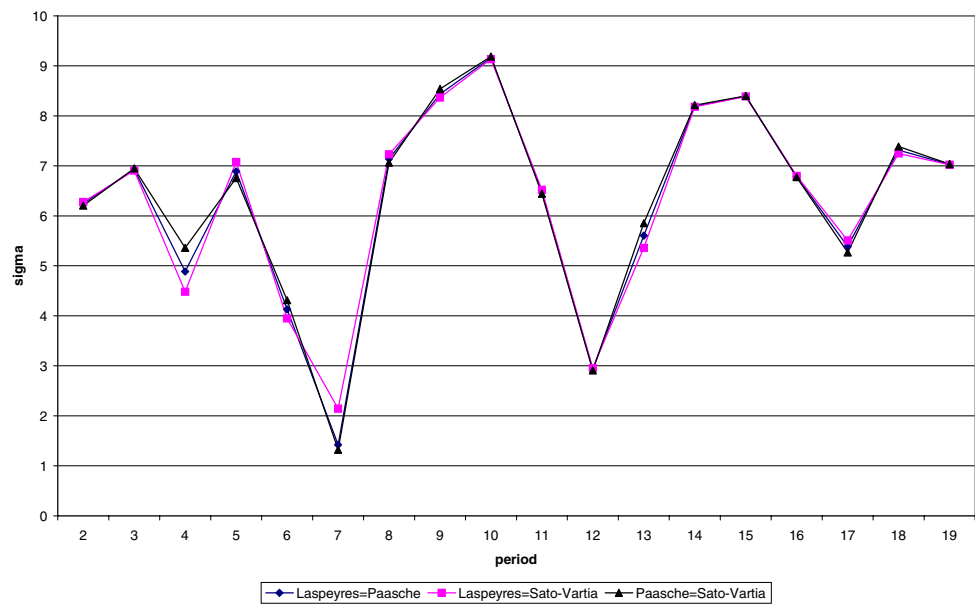


Figure 11. Elasticities for disposable baby's napkins data

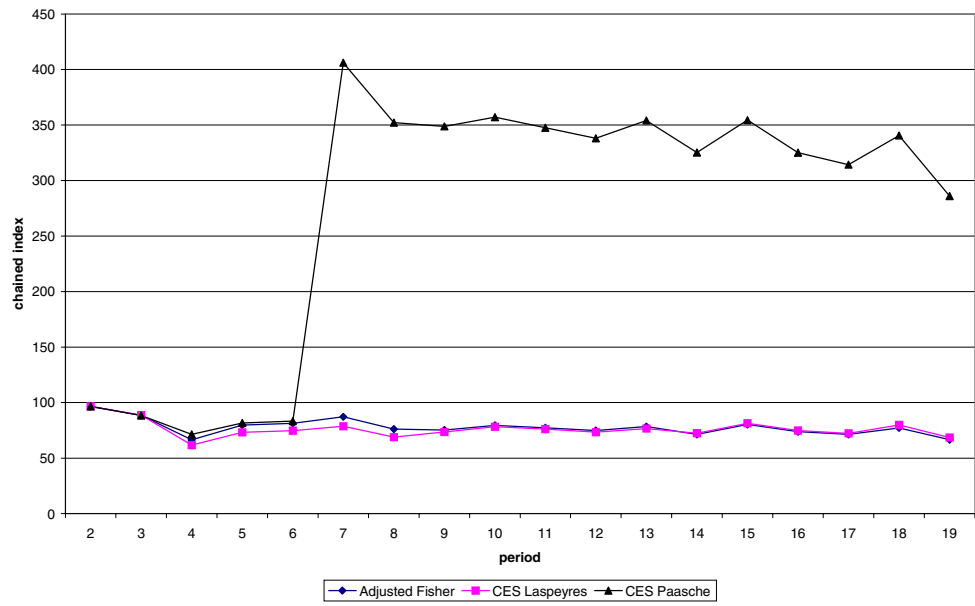


Figure 12. Chained indices for disposable baby's napkins data

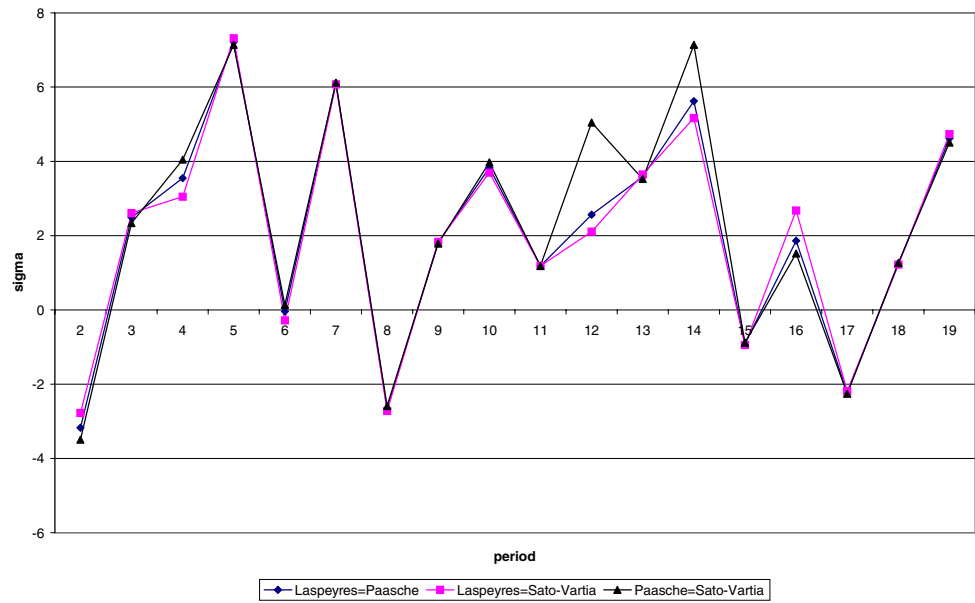


Figure 13. Elasticities for scents data

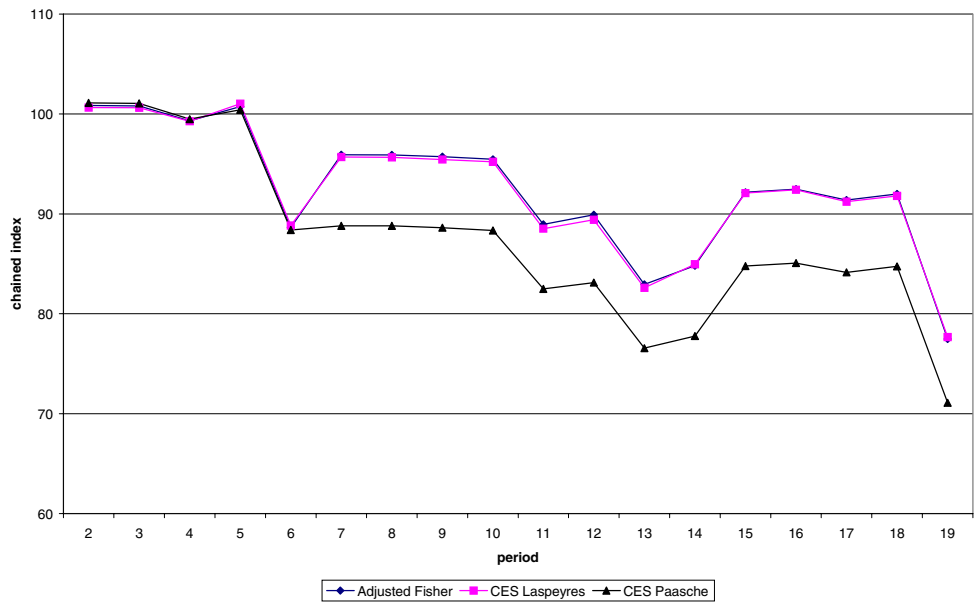


Figure 14. Chained indices for scents data

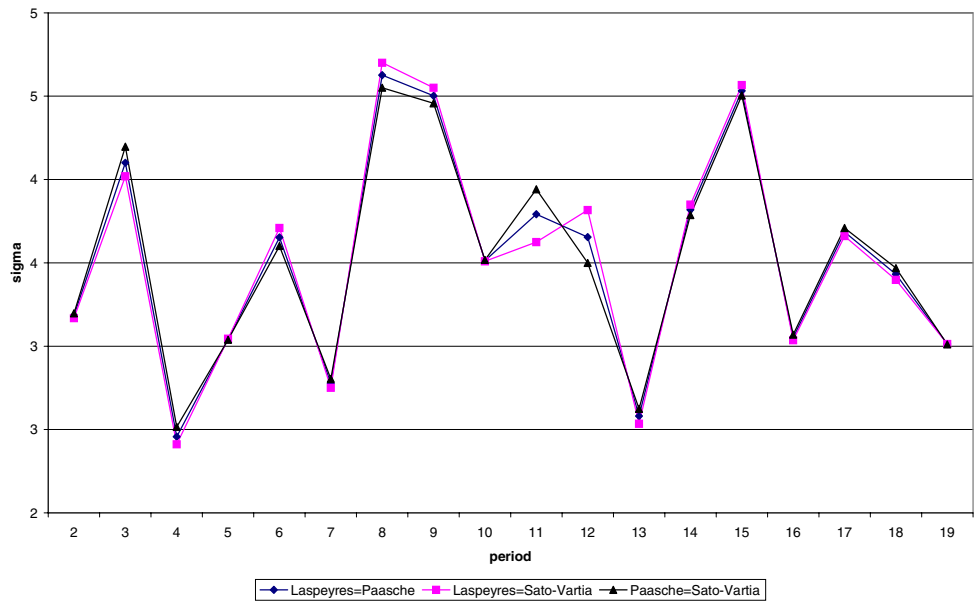


Figure 15. Elasticities for soft drinks data

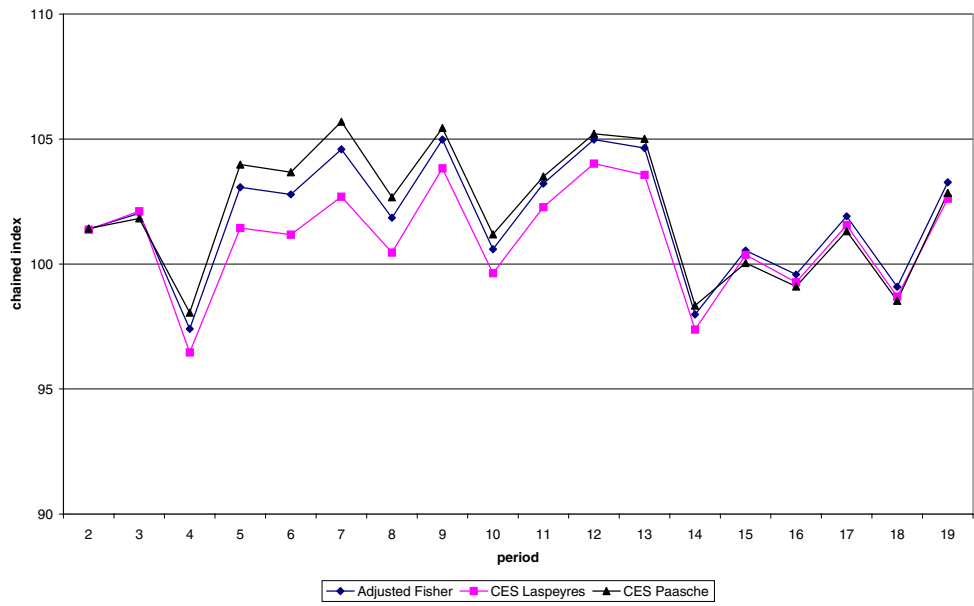


Figure 16. Chained indices for soft drinks data

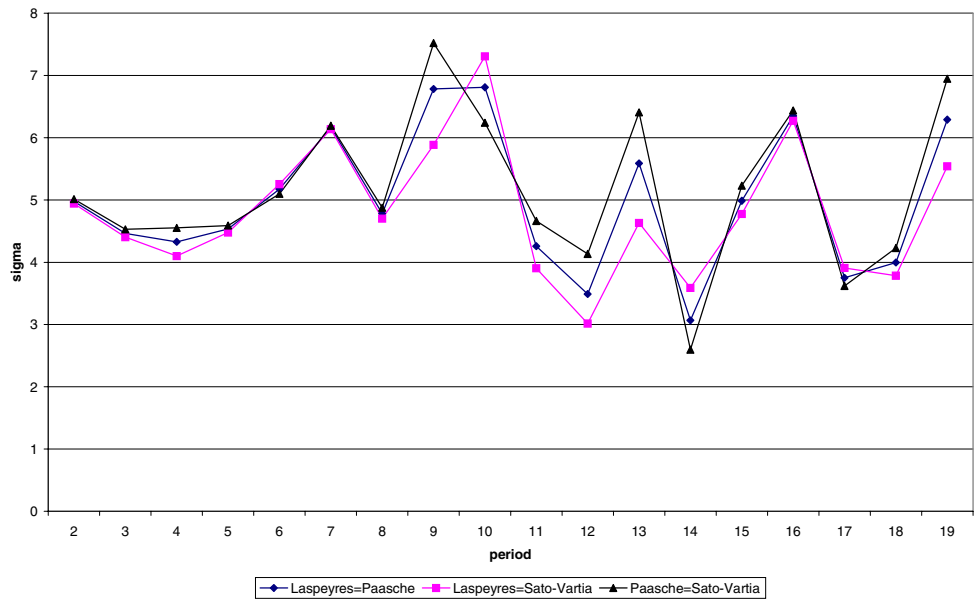


Figure 17. Elasticities for tea data

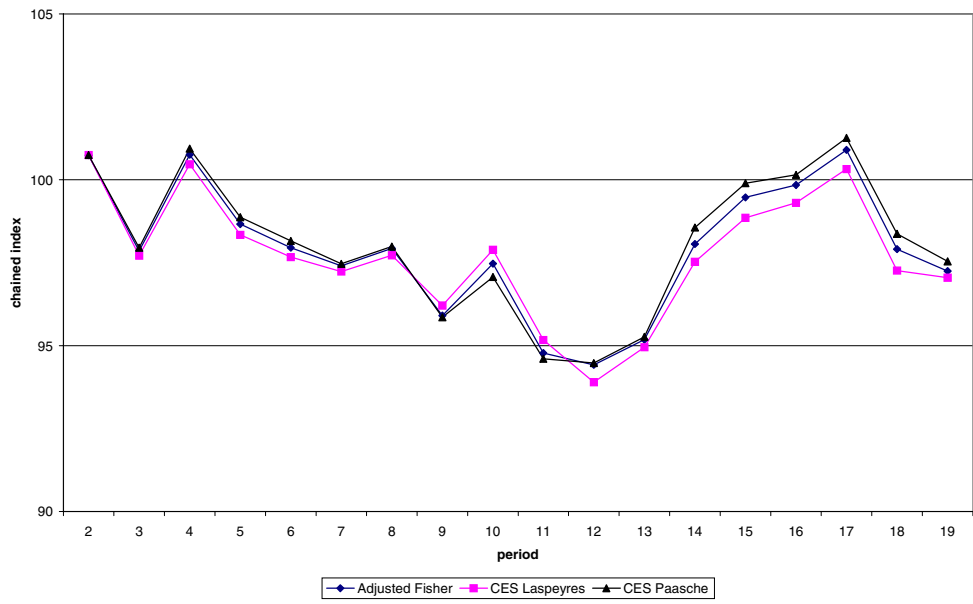


Figure 18. Chained indices for tea data

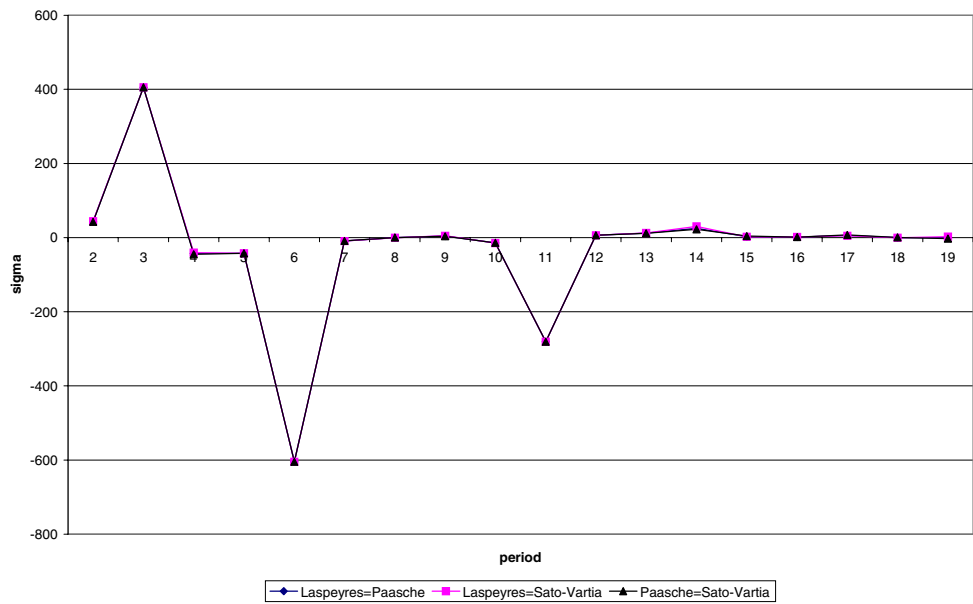


Figure 19. Elasticities for vitamins data

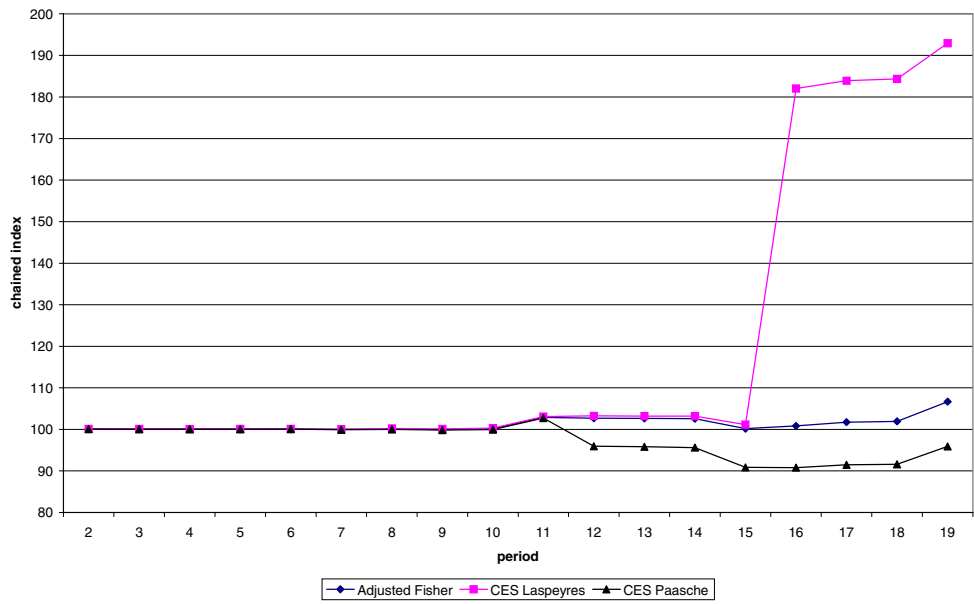


Figure 20. Chained indices for vitamins data

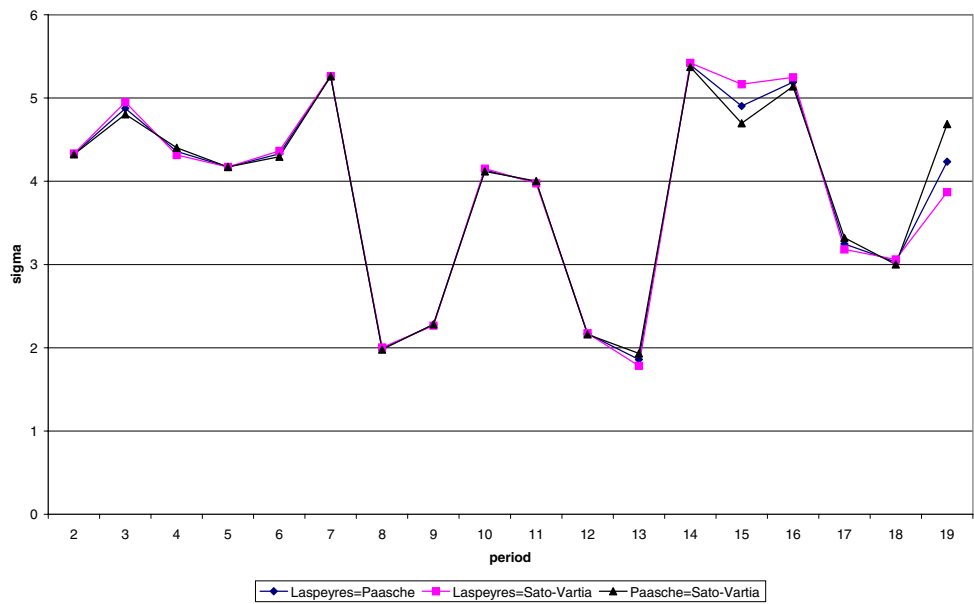


Figure 21. Elasticities for yoghurt with additives data

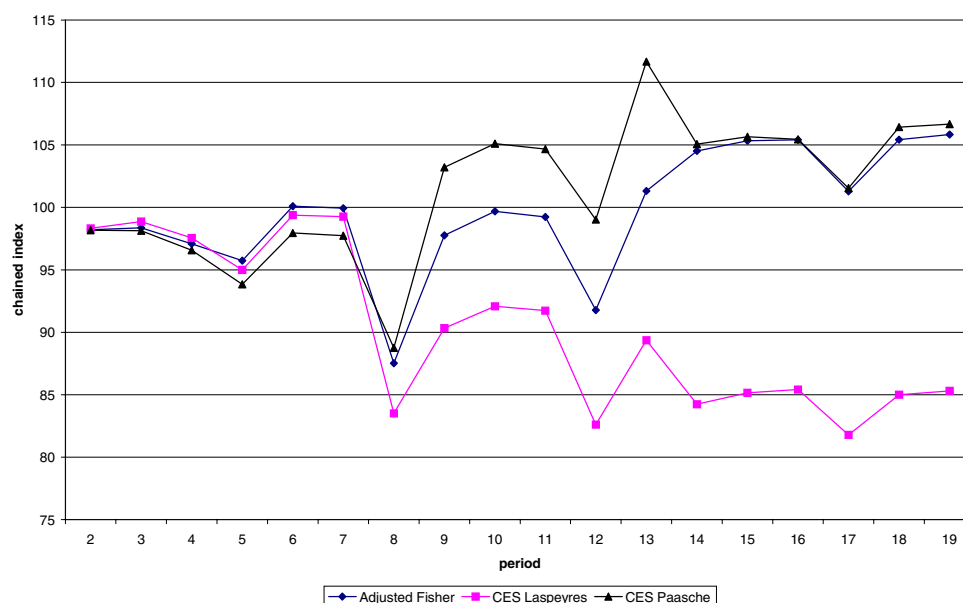


Figure 22. Chained indices for yoghurt with additives data

Finally, following Balk (2000), the process was repeated at the upper level, both with the CES Laspeyres index and the CES Paasche index as group price index, in (14) as well as in (15). Again $\sigma_t^{(k)}$'s were obtained and in both cases their 10% trimmed mean was taken as an overall elasticity $\hat{\sigma}$ (table 4, figure 23 and figure 25). Again these $\hat{\sigma}$'s were used to compute CES Laspeyres (14) and CES Paasche (15) indices (figure 24 and figure 26).

Table 4. Summary of overall elasticities

group period-to-period price index used	10% trimmed mean	st.dev.	min	max
CES Laspeyres	4.03	1.17	1.75	6.58
CES Paasche	3.93	1.73	0.70	8.56

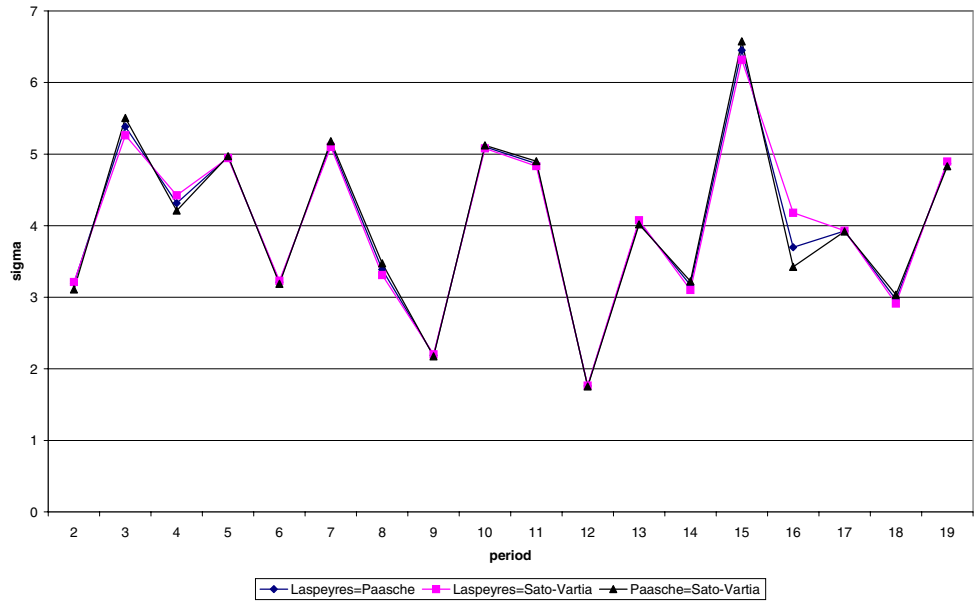


Figure 23. Elasticities for all commodity groups, using CES Laspeyres indices at the lower level

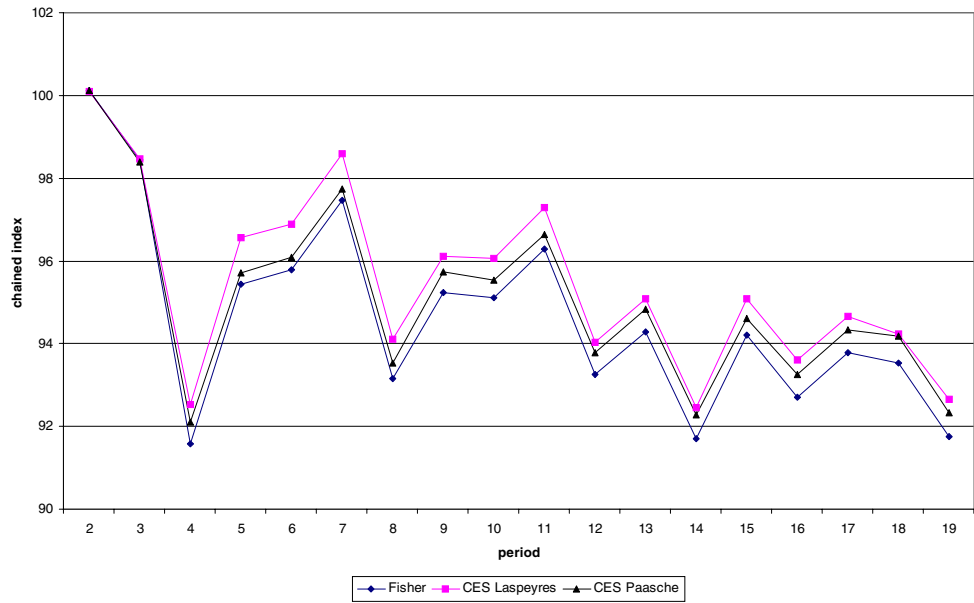


Figure 24. Chained indices for all commodity groups, using CES Laspeyres indices at the lower level

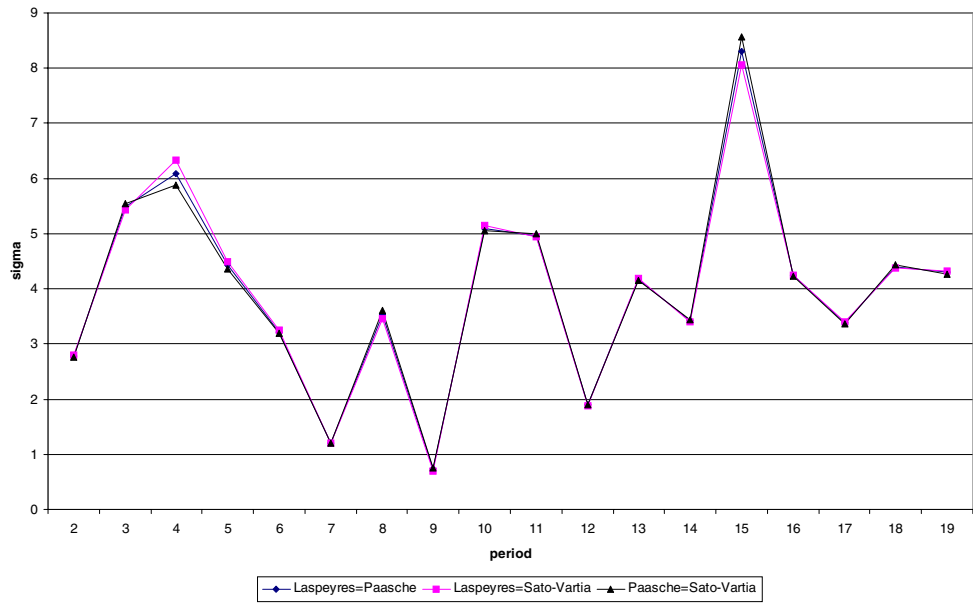


Figure 25. Elasticities for all commodity groups, using CES Paasche indices at the lower level

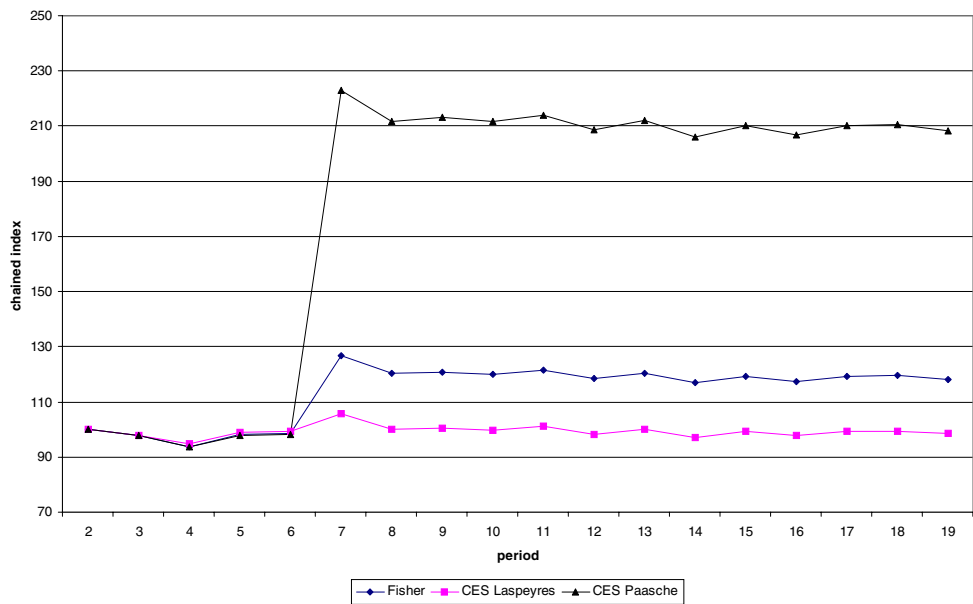


Figure 26. Chained indices for all commodity groups, using CES Paasche indices at the lower level

As a final exercise, the union of all commodity groups was considered, in order to illustrate the significance of distinguishing commodities into different groups. Again

$\sigma_t^{(k)}$'s were obtained and their 10% trimmed mean⁹ (= 4.56) was taken as an overall elasticity $\hat{\sigma}$. This $\hat{\sigma}$ was used to compute CES Laspeyres (4) and CES Paasche (10) indices.

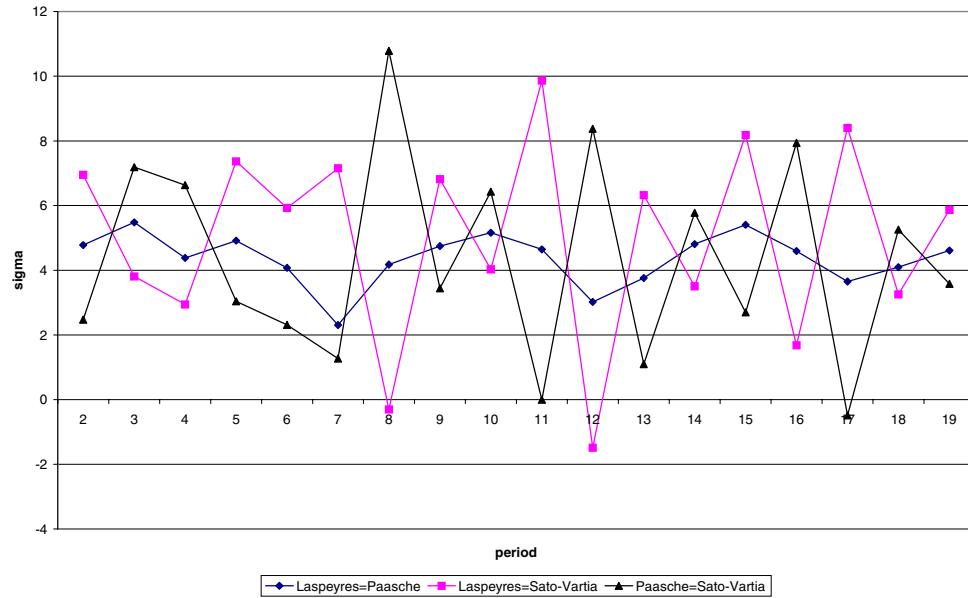


Figure 27. Elasticities for the union of all commodity groups

⁹ Standard deviation was 2.52; values ranged from -1.49 to 10.79.

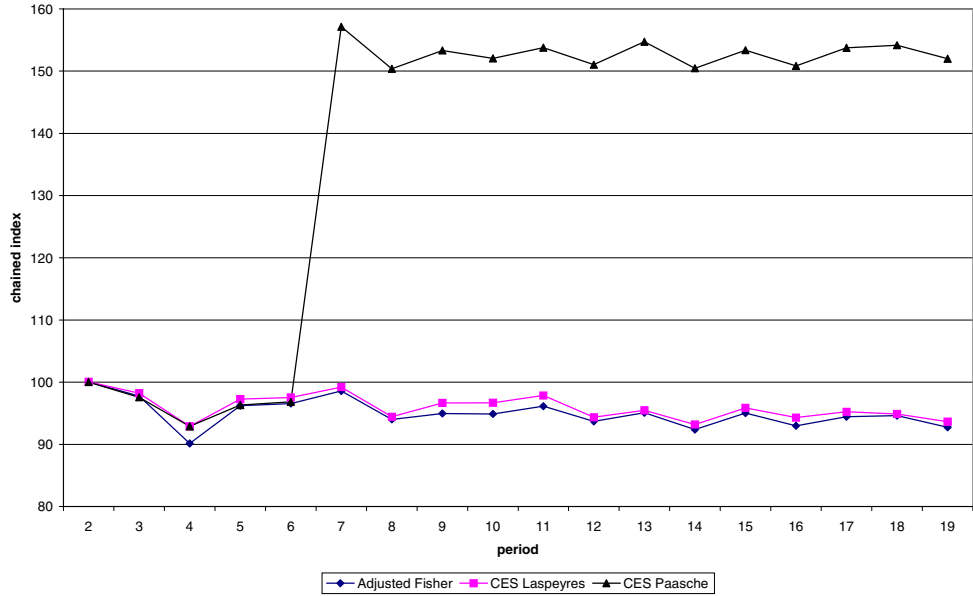


Figure 28. Chained indices for the union of all commodity groups

4. Discussion

4.1 Restrictions

The vitamins data show that the use of chained month-to-month indices, as in the foregoing theory, is inappropriate for seasonal products, since they are wrongly treated as disappearing and new goods. Diewert (1999) and Dalén (1999) suggest some ways to tackle the problem.

As already noted (footnote on page 3), the vitamins data also provide an example of pathological behaviour of equations (5), (6) and (7). When the price relatives of the ongoing commodities are (nearly) equal, say to π^{01} , the CES Laspeyres index (4) and the CES Paasche index (10) both reduce to

$$P_{\sigma}^C(p^1, p^0 | I^1, I^0) = \left[\frac{\lambda^0}{\lambda^1} \right]^{\frac{1}{1-\sigma}} \pi^{01} \quad (18)$$

Since, due to numerical instability, it is (almost) impossible to determine the elasticity of substitution σ , a meaningful CES based cost of living index cannot be computed.

Outlying price relatives may lead to a substantial difference between the CES Laspeyres index (4) and the CES Paasche index (10), as the disposable baby's napkins data show (figure 12). Inspection of the data reveals that one commodity, baby's tissues, had been delivered for free most of the time during the introduction period (period 6), which led to an increase in turnover rate (weight) in the next period and an outlying price relative of 12.98, the price relatives of the other

commodities being close to 1. After removal of the commodity involved in period 6, the difference vanishes (table 5).

Table 5. Effect of removing baby's tissues from period 6

chained index	before removal	after removal
CES Laspeyres	105.38	105.31
CES Paasche	487.06	104.50
adjusted Fisher	107.11	104.84

4.2 Feasibility

Suppose, for the sake of the argument, that the above restrictions have been removed. Then the question still remains: How feasible is the theory? Is it possible to construct and maintain a CPI of the CES Laspeyres type?

First, estimates of the overall elasticity of substitution σ and group elasticities σ_g (for each commodity group g) are needed. They may stem from an independent source, but they may also be estimated using previously recorded expenditure information and individual price relatives. In the latter case, (adjusted) Fisher price indices¹⁰ could be computed instead.

Second, a good estimate of the ratio λ^0/λ^1 in (4) must be available for each commodity group. The quality of the estimate is crucial, since the CES Laspeyres index (4) turns out to be quite sensitive for the choice of λ^0/λ^1 . When no good independent estimate of the ratio is available, expenditure information still remains needed for the object period.

One of the assumptions of the present theory is time-invariance (see page 2). The previous empirical evidence, however, suggests that it would be good practice to re-estimate the elasticities on a regular basis. Again, when independent estimates are lacking, expenditure information still has to be collected for every intermediate period.

Another assumption, that of constancy of elasticities, may also be violated in practice. To tackle this problem, Balk (2000) suggests to cluster commodities into groups with constant within-group elasticity. The resulting groups, however, may not coincide with commodity groups for which CPI's must be published. For new commodities he suggests to allocate them provisionally to a group. In the next period, when they have become ongoing commodities, they may be reallocated. Eventually, this may lead to a recalculation of the cost of living index for the present period. Since the CPI's usually are used as short-term indicators, this may encounter some practical opposition.

¹⁰ Or Sato-Vartia price indices, Törnqvist price indices, etc.

5. References

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