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Hedonic Regressions:
A Transaction Economy Approach

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Abstract

In this paper we introduced a descriptive approach to measuring price and volume change. This is based on a framework of a transaction economy, defining the outcome of an economy simply as a set of transactions in price-quality space. We will discuss what theoretical and practical implications our aim of decomposing the aggregate value change into price and volume components have. We also re-introduce the definition of a flexible estimation function that, used for separating the quality change and new goods problem from the index number problem, tries to measure the pure price change of individual transactions. We argue that it can be operationalized and made use of in guiding existing price index programs in statistical agencies.

Key words: Price – volume decomposition, hedonic regression, quality change, index numbers

1. Introduction

We begin with setting up a descriptive transaction economy consisting of three elements; characteristics space that defines all possible transactions, a stochastic economic process defined onto that space, and cumulative joint probability distribution of all characteristics deduced from the process. We finish the second chapter with a hopefully illustrative, however naive example of a simplified orange economy. Our initial aim is to prepare ourselves to find a good or few good ways of decomposing the value change into what will be pure price change and volume change.

Chapter three introduces the problem of decomposing value change into price and volume components and discusses solutions conditional of knowing certain aspects of the underlying population. In four, we propose a family of practical solutions. We feel that being consistent in applying quality adjustment methods, separately from applying any index number formula, is a virtue itself and may also ease to operationalize the use of such methods into index compilation by statistical agencies.

We also discuss some of the issues raised in Diewert [2003], and how these relate to estimation in the transaction economy framework. In particular, we re-raise issues of weighted regression, time indicator models and restricted coefficients. With this respect we generally side, although for reasons stemming from different premises, with Pakes [2003] and make use some of his arguments to justify our stand as well.

Since price index is supposed to measure the price change of same goods, we suggest that only reasonable way of doing this is to mimic the conditions of goods and services transacted in one period and ask how much the price would have been in another period. This is the carrying theme of our paper and we will show different ways of of doing this in the transaction economy framework – first directly in the characteristic space and later with conjunction classification and relating information on the relative importance between the classification groups.

Our approach is by now means an original one. Quality adjustment literature has proposed hedonic imputation methods for a long time. The works of Erwin Diewert, Jack Tripplett, Yrjö Vartia, Ariel Pakes, and Mick Silver, just to name a few, have greatly influenced the author's thinking far beyond the bibliography referred in this paper. We are privileged to have interacted with them. A major difference from mainstream hedonics in our approach is that we are not particularly interested in two somewhat common issues: how the price of a transaction changes as a result of unit change in each characteristics, and on estimating the supply and demand functions for each characteristic. We also acknowledge that the reason for our descriptive statistical approach may well stem more from ignorance rather than full appreciation of economic theory's applicability. Our premises and reasoning are more practical of nature.

2. A Transaction Economy

2.1. Characteristics space

To fully appreciate the questions the transaction economy raises on price-volume decomposition we first need to explain what the hugely complex environment where we observe goods and services – the characteristic space – is all about.

We don't want to restrict anything, neither for practical nor theoretical reasons, in our descriptive economy. We begin by listing all possible goods and services in witch money transactions can occur and all characteristics that are involved in the transactions with these goods and services. For each individual transaction these characteristics will include, among other things, the value of the transaction (price), the quantity that transaction refers to, all physical and non-physical characteristics describing the quality of the good, all characteristics describing the transaction occasion, etc. It is this set of $K+1$ characteristics that span the *Characteristic space.* It combines all possible bundles of all possible characteristics and we call it simply by $\Omega=(X_0, \times X_1 \times \ldots \times X_K)$. These characteristics define what is meant with the same transaction (good) and they are the ones we want to control for in the price index. Then there is another set of *process characteristics* that somehow describe how the economy produces and consumes goods and services. These may include "variables" for individual tastes, technologies, economic growth, market conditions, etc., which do not affect the definition of a transaction and which we allow to change as the economy changes.[1]

We assume, that this set of quality characteristics forms the choice set for all transactions. It fully describes all possible characteristic bundles, and consists of "quality" and price characteristics. In particular, the space can be $\Omega = X \times P$, where P refers to a continuous non-negative nominal scalar value of the transaction and $X=(X_1,\ldots,X_K)$ generally to all other *quality characteristics*.

There are some remarks that should be made. First, so far we haven't paid any attention to actually measuring these characteristics, nor for them being numeric in any way. For example, quality characteristics (or subset of characteristics) describing the shades of color blue or ones indicating the proximity and nature of five closest mid-priced Indian restaurants are as valid characteristic as any others. At this point all we want to claim that all these characteristics are defined in the set $\{X_1,\ldots,X_K\}$.

In sequel, we are not interested in the goods themselves until we get to issues of sampling and classifications. Instead we define everything in the characteristic space. Needless to say, the characteristic space is a very complex, multidimensional, discrete space. It is just a set of characteristics that fully defines the choice set for all possible transactions within the phenomena we are dealing with. The characteristics space defines all possible price-quality combinations within the domain in question, whether it's the economy of Europe Ω_1, consumer purchases in England Ω_2, or HP laptops with 17" wide screen sold to small businesses in East-London Ω_3. These are all just parts of the original characteristic space

1 In this paper we will not discuss further which of the quality, technology, taste or utility characteristics should be treated as quality characteristics in the definition of a transaction and which as process variables. In practice it depends on the use of the index and the theoretical index we are trying to mimic. If we hold everything constant we easily end up just stating that if everything today (period t) was as it was yesterday ($t-1$), then everything would be as it was yesterday and there would be no room left for any pure price change.

and clearly $\Omega_2 \subset \Omega_1$, $\Omega_3 \subset \Omega_2$ and $\Omega_2 \cap \Omega_3 = \emptyset$.

It should be clear that we do not want to simplify anything at the outset – apart from the notation, maybe. The quality characteristics describe not only the physical goods – which themselves are bundles of characteristic having 'attributes' in certain neighborhoods of Ω – but also the conditions describing the transactions. For example, in the retail market information on a rebate campaign of a competitor across the street could be included in the characteristics of transactions made in all other stores as well, if this is seen as characteristic that we want to control for. Similarly, we could also include dimensions defining utility, technology, production costs, tastes – provided that they are conceptually seen as characteristics we want to hold constant in a pure price index. This means that the Ω can also include the amount of utility and cost each transaction poses to the individual participants of any given transaction at any time. These characteristic are clearly something that we will have difficulties to control for and it may not always be clear which are ones that we should hold constant and which should be included as process variables.[2]

Characteristic space also defines what classifications are all about: finding "similar" characteristic bundles together to form individual goods (varieties, models, types, uses, etc.), and further aggregating these into product groups and industries based on the context of the economy. The use of classifications is of course elementarily important in practice.

2.2. Economy and process characteristics

Having defined the complex choice set we next move to the actual process of how transactions are created onto **Ω**. Since we are only interested in the descriptive, though very precise, nature of the economy we simply say that there exists a stochastic data generating process that describes how each transaction comes into existence! We feel it's unnecessary to define this process precisely since it would require several assumptions on the process domains and measurability of variables as well as all the possible paths the process could take. It would mean that the process needed to accommodate for all interactions of all individual demand and supply functions of all economic agents, economic growth, etc. If the characteristic space was a complex construction, this stochastic process would well be described as the mother of all stochastic processes!

We only define the realizations of the economy at time t as a set of transactions occurred during the period. The outcome is thus defined as a set of transactions in **Ω** and, for illustration's sake, let's assume that we could describe all characteristics numerically. Each outcome could then be written as a $(N^t \times K+1)$ matrix $H^t = [p^{t\prime}\ x^t]$ where the rows represent the transactions and columns refer to the characteristics. We call the set of individual transactions the process actually creates each period as *an outcome of the economy at time t*. The outcomes we observe in time is then time-ordered sequence of these outcomes. Further, the nominal *size of the economy* is simply the sum of all transactions associated to each period: $V^t = \sum P_i^t$.

2 It is very difficult to estimate what the price of a pair of 2002 designer jeans would be in 2006 if they were they still in fashion, or what the utility of an old computer would be today if it still were a state-of-the-art computer.

Let us be clear that we are not trying to say that this is a particularly useful approach to economic theory, since it would be, de facto, *a theory of everything!* Knowing the stochastic process would allow us to answer almost any questions in economics and also describe the precision of our answers. Economics would reduce to the study of this stochastic process.

Randomness of the economy. It should be clear that uncertainty of the outcome, the non-systematic part of the stochastic process, of the economy probably increases the further in future we look. In particular, the uncertainty of the next time period, given the outcome of the process up to that point is smaller that the uncertainly of forecasting the outcome ten periods ahead. This is the result of the process responding to endogenous and exogenous factors (competition, legislation, population growth, labor strikes, bird flue pandemics, earthquakes, innovations, etc.) that affect the 'parameters' of the process and the observed transactions (new regions in **Ω**).

2.3. Joint distribution

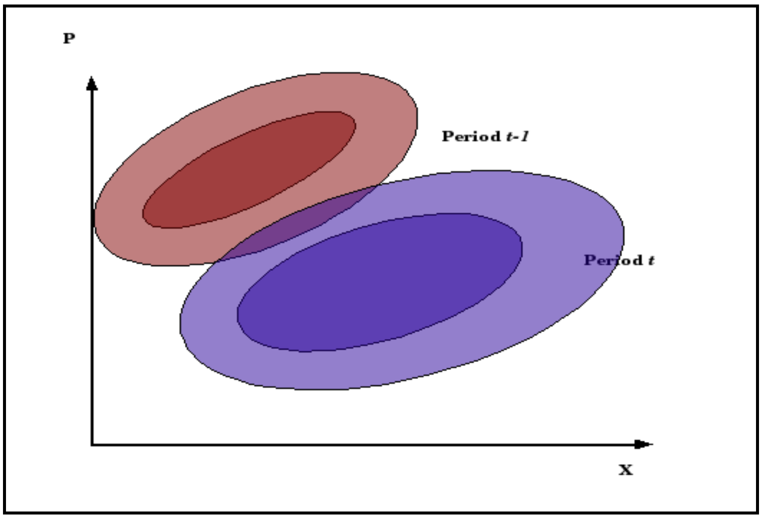
Having defined the outcome of an economy, we claim that the underlying stochastic process sets a joint probability distribution for the individual transactions in **Ω**. It is a very very complex probability distribution and changing in time. We are not particularly interested in the whole joint distribution itself but individual marginal distributions conditional on time. Further, we will actually be most interested only in the first two central moments of very special conditional marginal distributions. The distribution only shows the relative event probabilities of the points in the characteristics space at any given time. All this means is that that the event probability for a neighborhood in the **Ω** defining e.g. brand new T-Fords is zero today, as it was for corresponding area defining Porsche Cayannes 100 years ago. The distribution just defines the reduced form (statistical) dependencies within the characteristics space.

We call this conditional cumulative join distribution simply as

(2.1) $$F(P,X|T=t) \equiv F^t(P,X).$$

The distribution tells us how the possible individual outcomes, the points $(P_i^t, X_i^t) \in \Omega$ are distributed in the characteristic space. Now that's all we need for the moment. As an example and *only* for visual illustration's sake we have used marginal multinormal *density* distributions in picture 1 below showing an imaginary contour plot of two consecutive periods where the dimension of *X* is just 1. This description of the economy is just a reduced form statistical model describing the transactions in terms of their event probabilities. For example Brachinger [2003] develops statistical theory of hedonic price indices based on a similar approach.

Picture 1. Contour plots of density distributions $F^{t-1}(P,X)$ and $F^{t}(P,X)$



From the picture we can point out several aspects of what has happened in this particular – and particularly simple – economy. First, clearly the average (expected) price of all transactions has decreased and the expected 'quality' defined by *X* has increased. Second, we can tell that the dispersion, both in *X* and in *P* have increased. Finally, we also notice that another aspect of the covariance structure has changed: the correlation between *P* and *X* differ in time. Note, that with this information only we cannot tell anything about the change in the nominal size of the economy: it may have stayed exactly the same, increased or decreased.

The marginal distribution in characteristic space can well show that, for example relationship between some quality characteristics and utility changes in time. This is not so far fetched since we are convinced that the author gets only dis-utility from a ten year old computer he paid $2.000 at the time of the transaction. Clearly, the 'preferences' have changed in time! (And so have the budget constraint, technology, variety of computers, etc.) Another example of marginal distribution's time dependency is when new technology is introduced. Whether we want to condition on utility (or production technology) or only on technical quality characteristics is a matter of debate.

2.4. An orange economy

Before going on we want to make a simple example that could clarify the nature of the construction above. Let's take a quick look what this simple formulation holds. Let's assume that our two-period economy consists of only oranges and therefore only transactions in both time periods involve oranges! Now, there are two types of oranges: Class I and Class II. However, let's assume that for some reason we cannot observe the difference between the two and the oranges are the only product in the economy! Our characteristics space is a set of continuous price, and the indicator variables for *CI* and *CII*: $\Omega=(P\times CI\times CII)$ and the characteristic vectors refer to *(P, CI, CII)*. We now let our economy run both periods and observe the outcomes on both periods.

Period 1. 3 Class I oranges all exchanged at price of 5 and 2 Class II oranges at price 8.
Period 2. 2 Class I oranges all exchanged at price of 5 and 3 Class II oranges at price 8.

The outcomes of the economy are the following two sets of transactions in Ω :

$\{(5,1,0),(5,1,0),(5,1,0),(8,0,1),(8,0,1)\}$ for period 1 and
$\{(5,1,0),(5,1,0),(8,0,1),(8,0,1),(8,0,1)\}$ for period 2.

Alternatively, the outcome of the two-period economy is set of outcomes in 4-dimensional space, now including time dimension receiving values 1 or 2 is:

$$\{(5,1,0,1),(5,1,0,1),(8,0,1,1),(8,0,1,1),(8,0,1,1),\\ (5,1,0,2),(5,1,0,2),(8,0,1,2),(8,0,1,2),(8,0,1,2)\}.$$

Now, from the outcome we can estimate the conditional probability density functions (point probability distributions) for each period: { $P(X=(1,0)|t=1)=3/5$ and $P(X=(0,1)|t=1)=2/5$ }for period 1 and { $P(X=(1,0)|t=2)=2/5$ and $P(X=(0,1)|t=1)=3/5$ }. Note, that the probability distribution doesn't seem to be affected by prices, but that is of course not true. It's the properties of the stochastic process (complex process affected by interactions of demand, supply, competition, technology, economic growth, etc.) that determine the shape of the distribution.

Now, let's see what happened in our aggregate "orange economy": the total value of transactions on the two periods were $\sum(p_i^1)=31$ and $\sum(p_j^2)=34$. The economy has nominally grown 3 price units and the value change is $V^2/V^1=34/31$. Since prices of oranges have clearly not changed, a Laspeyres price index should show no price change and we should assign all of this change as volume change. Clearly, making sure and applying a Paasche quantity index we confirm that we get the same result: $p^2\cdot q^2/p^2\cdot q^1=(5,8)\cdot(2,3)/(5,8)\cdot(3,2)=34/31$. However, we have a problem since the index is now defined for two different qualities, instead of the original single good, orange! Clearly the quantity of oranges has remained the same, 5 oranges per period while the average price increased from 6.2 to 6.8.

Although determining between the two classes of oranges does not seem to pose a problem at this simple economy, in reality the changes in the probability distribution is a real and difficult to measure problem. While the value change is relatively easy to measure and compile, what should we do with measuring price change and volume change? In the next chapter we will discuss the problem in more detail in the context of the transaction economy.

3. Defining the Problem

Now that we have defined the joint distribution in the characteristics space we can move to our main theme of volume–price decompositions. Recall that the nominal size of the economy at any time was defined as the sum of each and every monetary exchange transacted during that period. Our aim is to find both theoretical and later practical solutions of describing the evolution of the economy in terms of price and volume. In traditional index number theory price and quantity indices are determined jointly, but we argue that due to the complexity of the characteristic space we are forced to define the volume as a residual. Diewert [2004] also talks about this separation of finding a good price measure from that of quantity measure and we believe that in practice this is the only feasible way of measuring the volume change.

3.1. On volume change

Our aim is to decompose the value change of our transaction economy into price and volume components. This can be done in several ways but for deflation purpose we first define *pure price change* as the change along the price characteristic axis of **Ω** alone from one period to another while holding all other characteristics constant. That change – whether observed or theoretical – is due to the stochastic process and the process variables running the economy. The change in volume is then defined as the residual of value change after controlling for pure price change. Volume then includes the net value effect of changes in everything else in the economy: quantity, quality, (taste, technology, competition, etc).

We think this is a useful approach since there are so many changes that contribute to the volume change that it will not be possible to try to measure them all directly. Also, price is the only common and measurable characteristic to all transactions and basis for the value concepts. Without complicating the picture too much with problems of measuring the price in practice, it can usually be measured more accurately than most of the rest of the characteristics.

We defined the value of the economy as sum of transactions, not the product of prices and quantities, as usually. This sum of course can be written in terms of price and quantity, either with number of multiple transactions at specific points in X or with summation of common quantity measure of otherwise identical transactions. We will get back to this later. Now, the value change can be defined either with original currency prices or in log-prices (relative change):

(3.1)
$$V^t / V^{t-1} = \sum P_i^t / \sum P_j^{t-1} = \frac{\sum (E(P_i^t | X^t) + e_i^t)}{\sum (E(P_j^{t-1} | X^{t-1}) + e_j^{t-1})} \quad \text{or} \quad \ln[V^t / V^{t-1}] \equiv \sum \ln P_i^t / \sum \ln P_j^{t-1} = \frac{\sum (E(\ln p_i^t | X^t) + e_i^t)}{\sum (E(\ln P_j^{t-1} | X^{t-1}) + e_j^{t-1})}$$

and our interest will be how to decompose these into pure price change and volume changes as price and volume indices:

(3.2)
$$V^t / V^{t-1} = P_{t-1}^t (P, X) \times Q_{t-1}^t (P, X) \quad \text{or}$$
$$\ln[V^t / V^{t-1}] = \ln P_{t-1}^t (P, X) + \ln Q_{t-1}^t (P, X) \quad .$$

3.2. "Full information"

We think it's a legitimate question to ask how would we measure the changes in the economy if the statistical agency knew all details of every single transaction in the economy. Hence we first consider two "full information" cases. The reason for this is that, depending on our target index, the way we calculate the index from a sample of observations may be affected[3]. Second reason for examining the underlying probability distribution and the outcomes of the economy is that it will shed light to what we are trying to achieve when we base our index number calculations on a sample.

In the first full information case we assume that we actually knew the conditional marginal probability distribution in two consecutive periods, i.e. that $F^{t-1}(P, X)$ and

3 The basic difference being that the first one produces relative of averages –indices and the second one average of relatives –indices. See Section 4.X.

$F^t(P, X)$ are known. Note, that in this case we have no information on the size of the economy since the distribution doesn't give us any hint of how many transactions the stochastic process produced each period. Nor do we know anything on the individual outcomes of the economy, the actually observed outcomes. However, we do know what is the expected price of any point in *X* on each period since this is just the first moment of conditional distribution of $F^t(P|X)$.

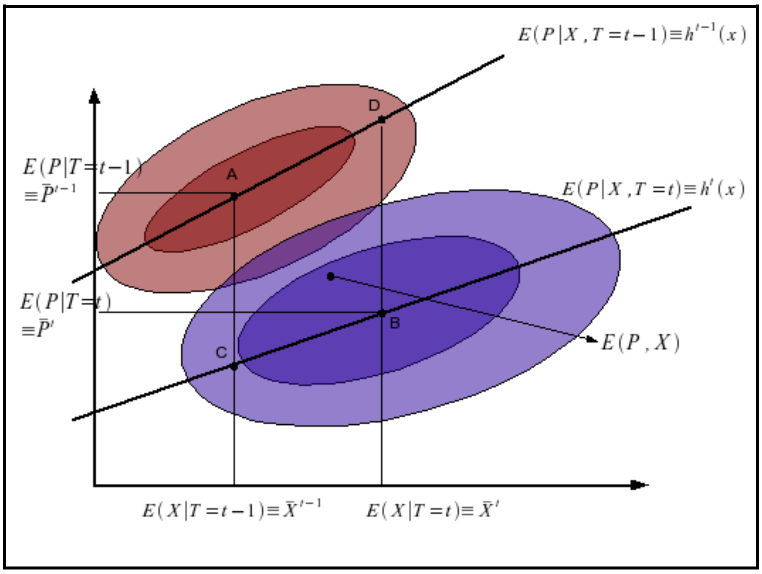
In the second full information case, we assume to also observe all transactions in the economy. Now we assume to know both the outcomes of the economy, i.e. the matrices $H^{t-1}=[p^{t-1}\ x^{t-1}]$ and $H^t=[p^t\ x^t]$ and the conditional expectations[4]. Would our measure for pure price change in this case be any different?

Distributions known. In the first case of know conditional marginal distributions, natural candidate for the measure of overall pure price change would be calculating it based on four conditional “price expectation points”:

(3.3) $$E[P^{t-1}|E(X^{t-1})],\ E[P^t|E(X^t)] \text{ and } E[P^{t-1}|E(X^t)],\ E[P^t|E(X^{t-1})] .$$

In picture 2 below these points refer to A and B for the first set and C and D for the second one. Using the rule of moving along P-axis only for the pure price change, a good choice would be the distance between the conditional price expectations, conditioned on an average quality point. Which one of the two should we choose, or should we take an average of the two? If we want to fix everything at the base period *t-1* , then the pure price index could be the distance (log-difference) or relative of the expected prices: $C-A:\ h^t(\bar{X}^{t-1})-\bar{P}^{t-1}$ or $h^t(\bar{X}^{t-1})/\bar{P}^{t-1}$. Identically, we could fix for the current period *t* quality and $B-D:\ \bar{P}^t-h^{t-1}(\bar{X}^t)$ or $\bar{P}^t/h^{t-1}(\bar{X}^t)$ or take an average of the two differences or relatives[5]. Alternatively, had we information on the joint distribution F(P, X,T) we could use the unconditional expectation E(X,T) as the point in which we measure the pure price change.

Picture (2). Pure price change when distribution $E(P|X, t)$ is known.



4 Assuming that the characteristics can meaningfully be assigned numerical values. Otherwise we just mark them as two sets of known characteristic arguments.

5 These indices will, as one might expect, be the hedonic Laspeyres, Paasche and Fisher indices and their geometric counterparts. We will get to then in more detail in chapter 4.

Above, the function $h^t(x) \equiv E(P|X, T=t)$ is the true conditional expectation of price for period *t*.

Let's assume we don't know the joint distribution in time. Hence, to get the most representative quality point we will propose that the pure price change with known marginal price distribution should be an average of the base and current fixed period measures. For the geometric average of the two relative geometric changes we can write:

$$(3.4) \qquad P_{t-1}^t(Fi) = \sqrt{\frac{\ln h^t(X^{t-1})}{\ln h^{t-1}(X^{t-1})} \times \frac{\ln h^t(X^t)}{\ln h^{t-1}(X^t)}} \quad \text{or}$$

$$(3.5) \qquad \begin{aligned} \ln P_{t-1}^t(Fi) &= \frac{1}{2}[(\ln h^t(\bar{X}^{t-1}) - \ln h^{t-1}(\bar{X}^t)) + (\bar{P}^t - \bar{P}^{t-1})] \\ &= \frac{1}{2}[(\ln h^t(\bar{X}^{t-1}) - \bar{P}^{t-1}) + (\bar{P}^t - \ln h^{t-1}(\bar{X}^t))] \end{aligned} .$$

The arithmetic counterpart using arithmetic average of the two is similarly:

$$(3.6) \qquad P_{t-1}^t(Fi) = \frac{1}{2}\left[\frac{h^t(\bar{X}^{t-1})}{h^{t-1}(\bar{X}^{t-1})} + \frac{h^t(\bar{X}^t)}{h^{t-1}(\bar{X}^t)}\right] .$$

We will get back to the geometric means later and propose that when estimating the function *h* we could actually estimate the logarithmic transformation of price in the first place so that $\ln P = p$.

Transactions known. In the second case we could proceed exactly the same way. However, the situation is different in two accounts. First, since we assumed we knew all transactions as well as the marginal distributions we now have a degree of randomness in the price index reflecting the actual realization of the stochastic process. Should we take this into account or only describe the underlying process? Second, we also know the relative importance of the aggregate economy at the two periods. How, if at all, should this information affect our ideal price measure?

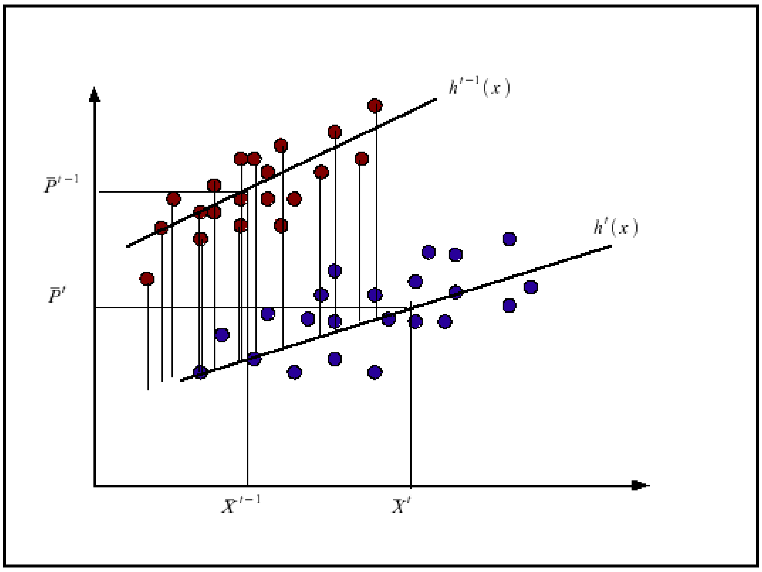
Now, in (3.4) we could simply replace the distribution parameters $E(P^t, X^t)$ with the known occurred population parameters $(\bar{X}^t, \bar{P}^t)$. This may differ from the distribution parameters, while the function $h^t(x)$ stays the same.

Alternatively, we could calculate the index based on the individual transactions. For the base period fixed quality type index we could choose a geometric or arithmetic average change along the price-axis :

$$(3.7) \qquad \ln(P_{t-1}^t) = \frac{1}{N^{t-1}} \sum \frac{\ln h^t(x_{i^{t-1}}^t)}{\ln P_i^{t-1}} \quad \text{or} \quad P_{t-1}^t = \frac{1}{N^{t-1}} \sum \frac{h^t(x_{i^{t-1}}^t)}{P_i^{t-1}} .$$

Picture (3) describes the situation again in the simplistic 2-dimensional multinormal distribution case. For the geometric case imagine that the price axis is measured in logarithms.

Picture (3). Pure price change when all transactions are known



To use all the information we have, we propose that, just as in the first full information case, a good and meaningful measure would be to take an average price changes of all transactions from both periods. In the logarithmic case this would mean:

$$
\begin{aligned}
\ln P_{t-1}^{t} &= \frac{1}{N^{t-1}+N^{t}}\left[\sum \frac{\ln h^{t}(x^{t}_{i^{t-1}})}{\ln P_{i}^{t}} + \sum \frac{\ln P_{i}^{t-1}}{\ln h^{t-1}(x^{t}_{i^{t}})}\right] \\
&= \frac{N^{t-1}}{N^{t-1}+N^{t}} \sum \frac{\ln h^{t}(x^{t}_{i^{t-1}})}{\ln P_{i}^{t}} + \frac{N^{t}}{N^{t-1}+N^{t}} \sum \frac{\ln P_{i}^{t-1}}{\ln h^{t-1}(x^{t}_{i^{t}})} \\
&= \frac{1}{N^{t-1}+N^{t}}\left[\sum \ln P_{i}^{t} - \sum \ln P_{i}^{t-1} + \sum \ln h^{t-1}(x^{t}_{i^{t-1}}) - \sum \ln h^{t}(x^{t}_{i^{t}})\right]
\end{aligned}
\tag{3.8}
$$

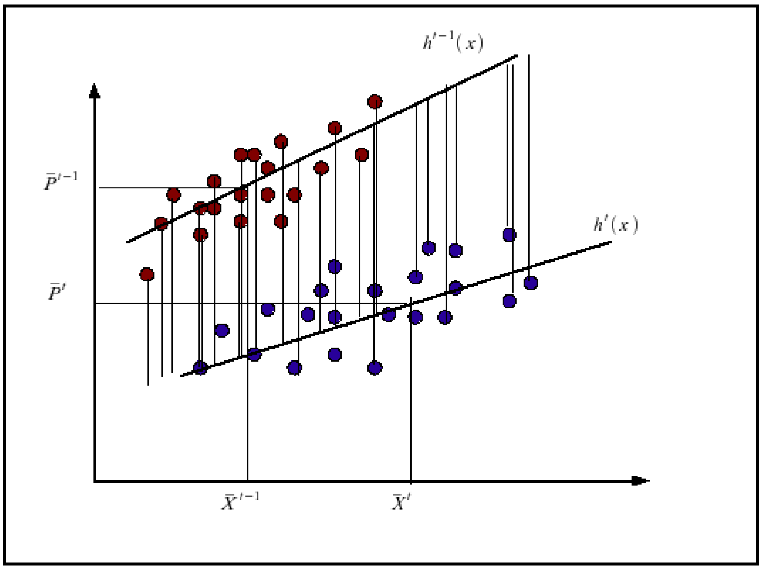
This just says that the grand geometric average uses N^{t-1} price relatives of observations from the base period and N^{t} price relatives from the second. It is relatively straightforward to show that, assuming some simplifications, this is an identical formula to well-known and in may ways recommendable Törnqvist price index formula:

$$
\ln P_{t-1}^{t}(T) = \frac{1}{N} \sum \left(\left(\frac{1}{2}\right)(v_{j}^{t} + v_{j}^{t-1}) \ln \frac{\tilde{P}_{j}^{t}}{\tilde{P}_{j}^{t-1}}\right) \tag{3.9}
$$

, where

the weights are value shares of transactions of J different types of "products" and prices geometric average prices within each "product" in both period: $v_{j}^{t} = \sum_{i} P_{ij}^{t} / \sum_{j}\sum_{i} P_{ij}^{t}$ and $\ln \tilde{P}_{j}^{t} = \overline{\ln P_{ij}^{t}}$. The characteristics space is partitioned and we may observe different number of transactions in all partition cells in both periods but still have observation of all J products in both period. Defined this way, ordinary price index theory concerns non-problematic part of the actual measuring. In picture 4 we illustrate the overall average of $N^{t-1} + N^{t}$ price relatives once more.

Picture 4. Overall average of price relatives



Note, that the idea follows those of the Laspeyres and Paasche indices of reprising the same set of transactions in two time periods. Alternatively, from using average prices we could assume that we have multiple transactions in the "same" characteristics space points x_i^{t-1}. Then we could write it in term of *p*s and *q*s. The *q*s would only tell us about the shape of underlying point probability distribution and is reflected in the formula (3.8) automatically! This, of course takes away the whole idea of quality change both within and between the partition of **Ω** into J products/groups. However, this in fact the case in many real life situations when we need to collect data on prices, quantities and qualities. However, in the transaction economy it will only be a question of sampling a representative sample, not a question of whether weighting the sample or not!

In practice we have an idea (from separate survey data at aggregated level) of the relative importance of products or product groups. These are typically value estimates we use to aggregate elementary indices further and we treat them separately. But products are just regions in *X* and product groups are combinations of these regions. So we have a rough idea of the shape of the (density) distribution of *X* at an aggregate level, where we can't control the detailed characteristics[6]. The question in practice is then more to do with the elementary aggregates and how to control the quality at this level. So, in practice we separate the overall *(P,X)* distribution to individual implicit regions and hope there are no systematic price – quality dependencies *between* the regions other than their relative share. However, provided that we have the information on all transactions of the economy, the formula (3.8) is valid for example for the overall CPI or PPI.

The price indices statistical agencies produce do not have the lone purpose of decomposing the overall value change from period to period. The price index itself and its sub-indices have important uses as such. Second, the procedure of first computing elementary aggregate indices for nearly homogeneous products and then aggregating them together using appropriate weights for their relative importance is the usual way in

6 We will miss some information. For example, there may be a substantial substitution effect between products within a group, that would mean that we can't fix the "quality" at enough detailed level when forming the price relatives.

index compilation – dictated by the available data. This is perfectly compatible with the transaction economy approach. As described before, we partition the characteristic space for products and examine marginal distributions for each partition separately. Using an appropriate formula for aggregation it will not make any difference to our discussion earlier. Only difference in the overall price measure would be result of possible cross-partition dependencies that cannot be accounted for if each partition is examined independently. This is a risk we're taking in practice, say in the CPI, that we control the price change of apples independently of that of cheeses! We take it for granted that the apple index would not react to any changes in cheese market conditions, although the upper level indices would catch some of the possible apple–cheese "substitution" through their weights.

Indeed, the transaction economy approach may be most fruitful (in practical terms) when applied at the elementary aggregate level or at one step more aggregated level. By considering only few products at a time, (why not laptops and PC:s together) we can better try to find the most relevant characteristics to be used in the estimation of $h^t(x)$. The separation of the two products as separate indices is then just a question of conditioning.

3.3. Sampling in Ω

Unfortunately, the assumption of knowing either the marginal conditional price distributions or all transactions of the economy in full detail are far from feasible even in small local regions of Ω. Instead, we need to compromise in two fronts. First, we do not observe prices of all transactions but instead a sample of them. Second, we do not observe the quality characteristics in detail either but only a subset – or a set of proxy variables – of the true characteristics. The methods of choosing the transactions and measuring the characteristics will also affect the degree of quality change we control for the pure price change: change along the price axis controlling for changes in everything else. The actual price index we get will be influenced by the choices we make both about the sample selection and the estimation of the price – quality relation.

Are we actually **sampling transactions or goods?** As far as the transaction economy is concerned, the main task is to measure the price change of transactions. Goods are just more aggregated level "entities" and hence not directly observable in the characteristics space. However, in practice the sampling is often applied in at least two or three stages: first industries, then products, and finally product variants or varieties (the elementary aggregate, still not single points in Ω). (CPI using expenditure survey starting with relevant sub-groups: then products and varieties sampled within; PPI industry survey/census starting with a certain digit-level in a product or industry classification: products – transaction sampled within.)

The transaction specifications we are interested in are in fact observed only after all these sampling steps. Regardless of how good our sample efficiency for a "representative" sample is on the price axis alone or on certain quality characteristics, the sampling frame is most likely not efficient to inference the population parameters of the joint distribution *(P,X)* in the characteristic space. What we observe is a sample of aggregate level observation $(P,\tilde{X})$, where $\tilde{X}$ is hopefully relevant measurable characteristics.

Obviously we cannot measure and control for all relevant characteristics. For example, it it difficult to make sure a utility level from a transaction does not change, or control for weather's effect for ice cream sales and price (if we deem these to quality variables intead of process variables). But, we claim, the best we can do is to forecast the observed transaction's price at the previous or consecutive period. Whether this forecast is best done by "matched model" approach or a large covariance model of several products is irrelevant for now. What we want to underline is the idea that we in fact try to mimic one period's transaction conditions on another period's price axis conditioning over all quality characteristics. We will show that it is not necessary to explicitly condition over all relevant variables. Instead, we claim that by controlling only for characteristics whose *average values* change is enough for the index to be unbiased provided that the sample of goods/varieties provide a representative sample in the characteristics space. That's the sole idea of measuring pure price change in this transaction economy. How alien it is to typical statistical agency approach, will be left as an open question. [*See also section 4.3.]*

Sample attrition. Since we usually first sample the goods and keep them fixed as long as there are transactions associated with that good/variety, we run into problem of not finding similar transactions and eventually even similar goods/varieties. The very similar problem concerns classifications themselves: when is a new transaction deemed a new good, and how should these new goods be classified. Sample attrition refers to sampling at one period and not finding the transactions in another period. Generally, this is just an outcome of changes in the occurred transactions in the characteristics space. The real problem of this type of sampling is that we're trying to estimate population parameters using a sample from another population (using a sample vector x^{t-1} from (P^{t-1}, X) instead of (P^t, X)). Old computers are not produced and sold any more and personal low-flying hydrogen-burning vehicles cannot be observed yet, though they may well be defined in the characteristics space.

In practice, of course, there is other kind of sample attrition as well. Transactions within a region may have a relatively large positive probability but none are still not observed for various practical reasons. A rotating sample, as used by many statistical agencies, tries to tackle this problem. In the short term the problem is biggest with high technology goods but tailored goods and services generally fall into the same category.

4. Finding a Solution

Based on the full information solution, we already have an idea of how to find a good pure price change measure. In this section we first define two types of possible true hedonic price indices and then discuss how to estimate them using a sample of transactions. As already in Manninen [2004] we re-introduce a simple framework based on a flexible *estimation function* that covers most conventional quality adjustment methods. Later, in section 5, we will discuss how this framework could be operationalized in actual price index compilation and how an implementation could instantly bring valuable information to guide the general compilation process.

4.1. The True Hedonic Indices

Whether we start from a set of axioms or from economic theory (simplifying assumptions on the stochastic process) for the true hedonic index, we should be able to verify any definition of a *true hedonic index* with the full information cases in the previous section. Alternatively, we might want to *define* the true index with the full information solutions based on the unknown stochastic process and joint distribution. Either way, we should have a true hedonic index in mind as a target index we want to estimate before beginning to compile an actual index from a sample of transactions.

We first offer several possible alternative definitions for Laspeyres index. Let's first return to the full information case with known $h^t(x)$ and apply Laspeyres' classic idea of holding base period quantity constant. Instead of just quantity, we now hold all characteristics constant at base period, giving us a natural choice for the pure price measure as the expected price change at base period expected/average quality point $E(P^t/P^{t-1}|\bar{X}^{t-1})$ or $E(P^t|\bar{X}^{t-1})/E(P^{t-1}|\bar{X}^{t-1})$. For the first one to be meaningful, we would need to define what is meant by P^t/P^{t-1} since it is not an observable measure itself but instead a function of the joint distribution.

Alternatively, instead of changes in the price level, we may want to only measure relative changes. In this case log–price change would be a good measure. The true log–Laspeyres (or geometric Laspeyres) index can then be defined identically as log-inverse of the distance along log–price axis between the conditional expected log–price between periods *t* and *t-1* at the expected average quality point $E(X^{t-1})$.

In summary, the true ordinary and hedonic log-Laspeyres indices in this case can be defined as:

$$\begin{aligned}P^t_{t-1}(La) &= E[P^t|\bar{X}^{t-1}]/E[P^{t-1}|\bar{X}^{t-1}] \\ &= h^t(\bar{X}^{t-1})/h^{t-1}(\bar{X}^{t-1}) = h^t(\bar{X}^{t-1})/(\bar{P}^{t-1})\end{aligned} \quad \text{and} \tag{4.1}$$

$$\begin{aligned}\ln P^t_{t-1}(La) &= E[\ln P^t|\bar{X}^{t-1}] - E[\ln P^{t-1}|\bar{X}^{t-1}] \\ &= \tilde{h}^t(\bar{X}^{t-1}) - \tilde{h}^{t-1}(\bar{X}^{t-1}) = \tilde{h}^t(\bar{X}^{t-1}) - \overline{\ln P}^{t-1}\end{aligned} \quad , \tag{4.2}$$

where the $\tilde{h}^t(x)$ is the conditional expectation of log–price at $X=x$.

The formula in (4.1) is essentially the same formula as proposed by at least Brachinger [2003] as the true hedonic Laspeyres price index. If we accept this, then Paasche and Fisher indices can be defined similarly (e.g. for log–Paasche and Fisher):

$$P^t_{t-1}(Pa) = \frac{\bar{P}^t}{h^{t-1}(\bar{X}^t)} \quad \text{and} \quad \ln P^t_{t-1}(Pa) = \frac{\overline{\ln P^t}}{\tilde{h}^{t-1}(\bar{X}^t)} \quad \text{for Paasche, and} \tag{4.3}$$

$$P^t_{t-1}(Fi) = \sqrt{\frac{h^t(\overline{X^{t-1}})}{P^{\overline{t-1}}} \times \frac{\bar{P}^t}{h^{t-1}(\bar{X}^t)}} \quad \text{and} \tag{4.4}$$

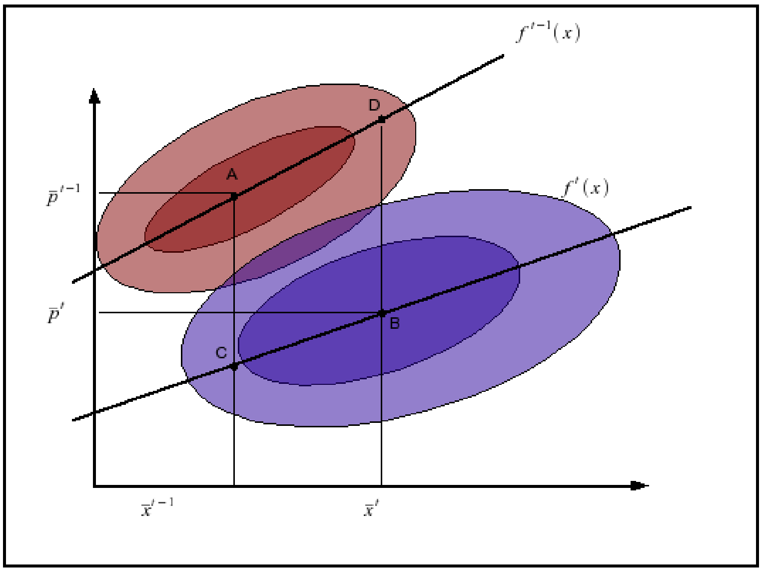
$$\begin{aligned}\ln P^t_{t-1}(Fi) &= \frac{1}{2}[\overline{\tilde{h}^t(x^{t-1})} - \overline{\ln p}^{t-1} + \overline{\ln p}^t - \overline{\tilde{h}^{t-1}}(x^t)] \\ &= \frac{1}{2}[(\overline{\ln p^t} - \overline{\ln p^{t-1}}) + (\overline{\tilde{h}^t(x^{t-1})} - \overline{\tilde{h}^{t-1}(x^t)})]\end{aligned} \quad \text{for Fisher.} \tag{4.5}$$

From (4.5) we see that the log–Fisher, if defined this way, can be interpreted as the average price difference measured at the average quality points of each period[7]. In the picture 5 above it is the average of C-A and B-D or B-A and C-D. The interpretation will

7 Both periods get the same weight in this formulation of Fisher index.

remain the same if we add 'weights' – in the sense of either multiplicative observations in *X* or as an aggregated level with average prices of “goods” – to the formula. The averages will only be weighted averages[8].

Picture (5) Estimation functions



Alternatively, just as when assuming we also knew the outcome of the economy, we could define the two Laspeyres indices as averages of price relatives. As hinted earlier, two versions of Laspeyres index (ordinary and log-Laspeyres) based on the outcome of the economy could be defined. For example:

(4.6) $$P_{t-1}^{t}(La)=\frac{1}{N^{t-1}}\sum\frac{h^{t}(x_i^{t-1})}{p_i^{t-1}}$$ or

(4.7) $$\ln P_{t-1}^{t}(La)=\frac{1}{N^{t-1}}\sum\frac{\tilde{h}^{t}(x_i^{t-1})}{\ln p_i^{t-1}}=\overline{\tilde{h}^{t}(x_i^{t-1})}-\overline{\ln p}^{t-1}$$.

By the same logic we can define Paasche and Fisher indices. As in (3.8) a potential candidate for a good overall measure is the following geometric (Törnqvist type) index:

(4.8) $$\ln P_{t-1}^{t}(Fi)=\frac{1}{N^{t-1}+N^{t}}[\sum\ln P_i^{t}-\sum\ln P_i^{t-1}+\sum\ln h^{t-1}(x_{i^{t-1}}^{t})-\sum\ln h^{t}(x_{i^{t}}^{t})]$$.

Strictly speaking we cannot calculate a Törngvist index using transactions from both periods. However, the log-Fisher index would become a Tornqvist in a situation where the transactions of both periods are observed in exactly the same points in *X*. However, this would reduce to the classical index number problem defined in quality space X instead for goods.

8 In this paper we will not pursue the weighted indices further.

4.2. Estimating the Hedonic Regression in Ω

Finally, the hedonic regression estimation, as already discussed in Manninen [2004], involves estimating the systematic part of the conditional price expectation $E(P^t|X)$ from the distribution function $F^t(P, X)$. To recall, this distribution was induced by the stochastic economic process onto the complex characteristics space. From this joint distribution we can implicitly deduce the systematic and random parts $p_i^t = E(\ln P|X^t) + e_i^t$. We follow Vartia & Koskimäki [1999] and call the estimate of the systematic part simply as:

$$(4.9) \qquad f^t(x) \equiv est\{E(\ln P|X^t)\} = est\{h(x^t)\} \quad .$$

Note, that we do not restrict the estimation method in any way. What follows is very simple. With hedonic regression, or any other estimation method, we try to approximate the distribution's behavior by estimating it with simple parametric functions. To construct the sample versions of the true hedonic price indices, our unknown function of the conditional expectation is replaced by the estimate.

Before applying corresponding sample parameter estimates, three issues of the estimating function needs explaining. First, as discussed in 3.3 about sampling in the characteristics space, we indicate the relevant observable and measurable part of the characteristics space with $\tilde{X}$. Clearly, it is not necessarily a strict subset of the true characteristics space but instead a set of proxy variables that we hope to capture as much of the systematic part of the price–quality relation as possible. For simplicity, we drop off the tilde from *x* but keep in mind that the quality characteristic we actually use may or may not enter the 'true hedonic model' in the first place. And we will never know it either.[9]

Second, we interpret the estimation function in a very wide sense. In particular, we by no means want to restrict to OLS regression but instead also leave room for non-linear models, non-parametric estimation, guesstimates and so forth. The simple notation hides even the most complex and simple estimation methods, as briefly re-introduced in 4.4.

Finally, while *estimating* the $f^t(\tilde{x})$ we do not necessarily restrict ourselves to the use of data on x^t only to the period *t*, while $f^t(\tilde{x})$ is used to forecast prices to that period only. For example, estimating a pooled time indicator model we effectively get several functions $f^t(x)$ from one single estimation . The use of the $f^t(\tilde{x})$ is restricted to forecasting only.[10]

Now, from (4.1) and (4.2), an estimated hedonic Laspeyres price index could be either

$$(4.10) \qquad \begin{aligned} \ln P_{t-1}^t(La) &= \tilde{h}^t(\bar{X}^{t-1}) - \tilde{h}^{t-1}(\bar{X}^{t-1}) = \tilde{h}^t(\bar{X}^{t-1}) - \overline{\ln P}^{t-1} \\ &\rightarrow \; f^t(\bar{x}^{t-1}) - \overline{\ln p}^{t-1} \end{aligned} \qquad \text{or}$$

9 We think this is conceptually an important issue since in reality we will never know the true function. Of course, there also exists the *best* approximation using the $\tilde{x}$, which could be derived from the $E(P^t|\tilde{X})$. However, this is not the same relation induced by the underlying stochastic process.

10 While we maintain the main use of hedonic regression in this concept as just another forecast tool, we do not claim that it is never useful to estimate structural models. For example, studying wage differences between men and women from regression coefficients is perfectly legitimate. It will, however, involve other issues of model specification than pure forecast model we consider here.

$$P_{t-1}^{t}(La)=\tilde{h}^{t}(\bar{X}^{t-1})/\tilde{h}^{t-1}(\bar{X}^{t-1})=\tilde{h}^{t}(\bar{X}^{t-1})/\bar{P}^{t-1} \rightarrow \frac{f^{t}(\bar{x}^{t-1})}{f^{t-1}(\bar{x}^{t-1})}=\frac{f^{t}(\bar{x}^{t-1})}{\bar{p}^{t-1}} \quad (4.11)$$

Defining the target index according to (4.6) and (4.7), and assuming that our estimating function is defined on log-price and is linear with respect to it's parameters we have similarly:

$$\ln P_{t-1}^{t}(La)=\frac{1}{N^{t-1}}\sum\frac{\tilde{h}^{t}(x_i^{t-1})}{\ln P_i^{t-1}} \rightarrow \frac{1}{n^{t-1}}\sum\frac{f^{t}(x_i^{t-1})}{\ln p_i^{t-1}} = \overline{f^{t}(x_i^{t-1})}-\overline{\ln p}^{t-1}=\widehat{\overline{\ln p}}^{t}-\overline{\ln p}^{t-1} \quad (4.12)$$

and

$$P_{t-1}^{t}(La)\rightarrow\frac{1}{N^{t-1}}\sum\frac{f^{t}(x_i^{t-1})}{p_i^{t-1}}=\frac{1}{N^{t-1}}\sum\frac{\hat{p}_{i^{t-1}}^{t}}{p_i^{t-1}} \quad (4.13)$$

With Paasche defined respectively, we get for example a log–Fisher index either as relative of averages

$$\ln P_{t-1}^{t}(Fi)=\frac{1}{2}[\overline{\ln p}^{t}-\overline{\ln p}^{t-1}+f^{t}(\bar{x}^{t-1})-f^{t-1}(\bar{x}^{t})] \quad (4.14)$$

or average of relatives

$$\ln P_{t-1}^{t}(Fi)=\frac{1}{2}[(\overline{\ln P^{t}}-\overline{\ln P^{t-1}})+(\overline{\tilde{h}^{t}(x^{t-1})}-\overline{\tilde{h}^{t-1}(x^{t})})] \rightarrow\frac{1}{2}[(\overline{\ln p^{t}}-\overline{\ln p^{t-1}})+(\overline{f^{t}(x^{t-1})}-\overline{f^{t-1}(x^{t})})] \quad (4.14)$$

4.3. Weights and regression estimation

The questions is now simply how to get a representative picture of the (P^t, X) joint distributions and, in particular, the conditional expectation of the marginal distribution $E(P^t|X)$ for both periods. With simple random sampling of transactions, we clearly cannot have any weight information other that having multiple identical observations in *X*. Using PPS sampling (using population *Q*:s as size measure) would be more efficient if we wanted to estimate the population mean price. However, it may not be the best way to choose data for the estimation of the price – quality relation itself.

Without getting into regression estimation yet, we illustrate this point with an example. In an extreme case where, for example we knew that the *p* – *x* relation *must* be linear, estimating the regression coefficient efficiently would mean that we divide our sample at relevant minimum and maximum points in *X*, e.g. equally at $(min\, x_i, max\, x_i)$, collect multiple prices for these quality points and estimate the coefficient. This would be arguably be the most accurate way of estimating the linear regression coefficient. Here, estimating the relation with weighted least square (WLS) having either the quantities or value shares as weights would not make any sense, however the population is distributed in *X*!

Our point is that the regression estimation (hedonic regression) to be used in price forecasting should be separated from the index number compilation. That would mean that the weights reflecting the shape of our quality distribution should be used cautiously in regression estimation in the price index context. Hence we take a more skeptical view on weighted regression than e.g. Diewert [2003] and deHaan []. Diewert proposed that weighted regression should be used on *representativity* grounds, but why should we, at least a priori, believe that weights describing the probability distribution of *X* should improve our estimation of expectation $E(P|X)$? In some cases this may be true but we have difficulty of finding the benefit of it since the weights could still be used in the actual index compilation using the tow sets of now identical transactions – in our view a separate issue from the quality adjustment procedure. At least we should be clear what we believe to achieve when using weighted regression.

What we mean by **weights** in the transaction economy can only mean multiple observations at single quality point or single measurable quality point, as mentioned earlier. Since our original characteristics space could be seen almost infinitely fine (we can't step to the same river twice), no same transactions can occur!

To be more realistic, let's assume that we actually have multiple observations in *X* at least in the observable and measurable world, if not necessarily in the true characteristics space. Let's also assume that we only observe prices for the very same quality points in the two periods but the number of observation in each quality point differ – just as in the "Orange economy" example. We can write this as a 4n vector $(\tilde{p}^{t-1}, Q^{t-1}x, \tilde{p}^t, Q^t x)$ or $(\tilde{p}^{t-1}, q^{t-1}, \tilde{p}^t, q^t)$, where *Q*:s are *(n x n)* diagonal matrices of quantities, *x* is the same vector defining the quality points in both periods, and $\tilde{p}^t = A^t p^t$ (ordering the p-vector, a *(n x n^t)* matrix of ones and zeros that picks the the first "same quality" price from the original p-vector). This is exactly the typical starting point of price index theory! The difference to our transaction economy is only notational. Where we would have had *p*-vectors of different lengths referring to different quality points (p_i^t, x_i^t) in *Ω*, the traditional index number theory restricts it's scope to changes of point probability function parameters (the event probabilities of fixed quality points in *X*) and allows prices not to vary at the same quality point within a time period. Otherwise the analysis is the same but not elaborated further. Starting from the formulas in 2.2. and writing them in terms of *Q* one should end up in more familiar index number formulas for Lapseyres, Paasche and Fisher.

4.4. Different estimation functions re-introduced

As already discussed in Manninen [2004], we list the different kind of estimation methods anew. As before, all of these methods are special cases of forecasting the prices of one period transactions to another period. The forecasting do not have to be based on regression analysis, but we can use all the quality adjustment methods know and fit them into the same framework.

Unit-value indices are often used at the elementary aggregate level. These can either be grand unit-values over all elementary aggregate observations or unit-values for a classification within a particular elementary aggregate:

$$f^t(x) = a^t \quad \text{or} \quad f^t(x) = a^t + b_1^t D_1 + \cdots + b_C^t D_C = a_c^t \quad ,$$

where *D*:s indicate whether quality point *x* belongs to a class *c*. These methods are perfectly useful as long as the average quality does not change within the class, in which case no quality adjustment is needed in the first place. The unit-value method relies on proper classification and requires little information on the transaction characteristics but one should note that the unit value methods can also be interpreted as regression based forecast models.

In practice, the method is used with imputing missing observations when no matched transaction can be found. In that case we base our price (change) estimate on the average price (change) within the classification group, or the next aggregation level, depending on our regression model. The estimated price for missing price for transaction *i* in period *t* would be simply $\hat{p}_i^t = a^t$. The unit value method can also be generalized to include the full classification system used for an overall index in question at the most detailed level.

Matched model can also be seen as an estimation. As its name suggest, matched model follows the prices of same transactions. This can be presented in terms of estimation function f returning the observed price for each period t. A possible notation of this is $f^t(x) = p_i^t$ for all transactions for which x_i is the same for both periods. The method does not utilize data for transactions that do not have matches. In an extreme case (or if the "quality" could be measured to a fine enough degree) it may be impossible to construct a price measure if there are not matching transactions at all. Matching – since it implicitly controls for a part of unmeasurable part of characteristic space *X* – can be very useful practice and is usually the cornerstone method of statistical agencies compiling price indices.

Patched match model is a matched model index with imputed missing values: $f^t(x) = p_i^t$, if a match can be found and $f^t(x) = x'\hat{\beta}^t$, if not. Pakes [2003] regards this type of quality adjustment in one way as the most accurate. Pakes argues that using the matches where possible decreases the variance of the overall index but also acknowledges the possible pitfalls. If the conditional expectation for matching (surviving) transactions/goods differ from the rest of the population, then our price measure is potentially biased. We could, however, try to estimate the extent and direction of this bias.

Other methods that use matches whenever possible and other methods for the missing prices fall into this category as well. The international price index manuals for CPI and PPI recommend this kind of method for temporarily missing prices. However, some methods that are in use are fundamentally flawed. For example, in the presence of changed quality of a transaction, we should not base the price change estimate on an estimate of how much *of the price change* is due to change in quality. If the price does not change, there cannot be any quality change either. Nor can there be positive quality change associated with price decline.

Regression models were discussed already. The important thing to notice is that any regression model can use the idea of imputing all one period's transaction prices on another period, thus creating n^t price pairs of equal quality. The index compilation is thus separated from the estimation of regression models and hence from quality changes and problems associated with new and disappearing transactions/goods. The same rule can be applied to pooled regression models as well as adjacent period models, although one could derive the price index directly from regression coefficients as well. The resulting index numbers will of course stay exactly the same.

5. On practicalities of the approach

We actually believe that the estimation function presented above can have practical utility. In Statistics Finland there's a project on building a price index engine, a conceptually coherent yet workable system for compiling and maintaining various index number programs. We hope that the estimation function will be, albeit a small detail of the whole system, in the core of operationalising the link between sampled elementary aggregate item/transaction price–quality data sets from two periods into a meaningful constant quality price measure. The price index engine begins with a conceptual framework describing the real world phenomena and links it with actual data collection under suitable classification systems. Using consecutive period matched model approach – likely in the form of estimation function – as its basis it defines and associates two constant quality elementary aggregate level price data sets for each first level aggregation. Since the project is still at a development phase and we have not been closely collaborating for a while we refer to Statistics Finland representatives for additional information on the actual project's progress.

5.1. Operating with the estimation function in practice

As briefly described in the Statistics Finland case, the estimation function could be used as a bridge between the data collection and elementary aggregate index calculation. Estimation function always returns "matching pairs", whether using period *t* function $f^t(x)$ on period *t-1* transactions, $f^{t-1}(x)$ on period *t* transactions or both simultaneously (perhaps retroactively after gathering later period aggregate weights as well). The derivation of individual estimation functions or larger functions for groups of elementary aggregates can be programmed into the index application's data manipulation applet and be integrated into the data validation process. A major benefit using the estimation function is that this way one can separate the treatment of quality changes, missing prices and new goods, as well as sample representativity issues, from the rather straightforward actual aggregation. A default method of matched model "estimation" together with proper implicit adjustment methods for missing items and replacements could probably be used for most product groups. However, regression based estimations could be used as an option the index compiler can choose for some more difficult goods. While the basic application stays the same hedonic models could be included into the estimation function by the user defining the variables included in the regression[11].

5.2. To log or not to log?

Which functional form the hedonic regression takes place is determined by the join distribution of $F^t(P,X)$ since that also describes the relationship of $E(P|X,T=t)\equiv h^t(x)$, the hedonic regression surface. However, to determine the functional form requires information on all relevant characteristics which we don't have. If we accept that in practice we are interested in the forecast model, then all we need to do is to find a suitable estimate. We fully agree with Pakes [2005, p. 10] who states that "Any

11 Of course, initial research on the set of relevant explanatory variables is needed for specifying the regression model. However, we believe that this may not need to be very labor intensive nor requiring collection of a large number of quality characteristics.

sufficiently rich functional form will do, and all sufficiently rich functional forms will generate approximately the same result." Similarly there should not be any constraints on the coefficients. Further, if − and often when − the data suggest a logarithmic transformation of price to provide a good fit, and practical index compiler's elementary index is a geometric average, we could well proceed with forecasting logarithmic prices received from OLS estimation. By no means is this necessary but it will give us some direct quantitative information on the effect of quality corrections. By using a decomposition similar to the Oaxaca decomposition we separate the direct quality change, possible covariance effects between weights (if we use a combination of classification and continuous quality characteristics in the regression) and the pure price change.

5.3. Other issues

We will continue with the transaction economy approach into a few particular directions. First, we will combine the classification and regression estimation. Second, we hope to discuss the economic approach of index numbers and show what restrictions economic theory may pose on the joint distribution $F^t(P,X)$ and the conditional expectation $E(P|X,T=t)$. With that respect, we will discuss the role of process variables and how changes in utility and technology could show up in the probability distribution and what that means for a theoretical target index. Further, we hope to illustrate how low level product indices can be derived from the overall joint distribution describing the overall economic frame in question (e.g. CPI, PPI) just by defining the conditioning set. We also hope to build a multi-product regression model from real data to demonstrate the use of the estimation function in practice.

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