Challenging the CES assumption with scanner data – pitfalls of the fixed basket

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Abstract

A theoretical motivation for applying the geometric mean formulation (Jevons) to elementary aggregates is the embedded account for elasticity of substitution, i.e. a conception that the consumer collective behaves as theoretical economic agents and consider relative prices in any given situation and for a fixed set of commodities. A fringe benefit of having scanner data in CPI is the opportunity of empirically analyzing elasticities in the presence of a changing product universe as data reveals the empirics of reality. Since the Jevons is the standard index formula for elementary aggregates in the European HICP as well as the Swedish CPI, empirics should support theory. In this analysis, substitution elasticities are estimated on a set of presumably substitutable products in scanner data in the simplest setting possible – direct matching of the current period to the base period for a fixed basket of items and stores.

Keywords: consumer price index, constant elasticity of substitution, CPI, Jevons, CES, scanner data

1 Introduction

The advent of electronic point of sales data[1], scanner data, has evoked an adaptation of index formulations to large-scale price and sometimes quantity information, as opposed to traditional sparse sample data of presumably well-representative spot shelf prices. Method development is ongoing concerning the exhaustion of information from obtained data, as promoted by e.g. Boskin et al. (1997) in order to capture cost of living, abbrev. COLI, timely. However, a major challenge for conventional index methodology, adhering to the philosophy of “less is more”[2], has been the inconvenience of direct comparisons over time. The inconvenience is due to the changing product universe, i.e. a vivid market, observed almost in real-time with scanner data. This challenge has transferred to development of methods, like GEKS, c.f. e.g. van der Grient and de Hahn (2011) or change-in-price-levels, CPL, c.f. von Auer (2011) and for instance though adaptations of the Geary-Khamis method, c.f. the QU method by Chessa (ÅÅÅÅ). All of these are in place to circumvent monthly chaining at elementary levels, i.e. the Jevons formulation adapted to a changing product universe. von der Lippe (YYYY) argues stringently against such high-frequency chaining approaches, simply with the pitfalls of transitivity/circularity beliefs and the effect on chained index series. The Jevons formulation has otherwise been, at least for direct comparisons (fixed basket), a well promoted and theoretically justified method, with appealing properties and relying on a theoretical framework.

In this analysis, the underlying assumption of constant elasticity of substitution (CES) is empirically estimated for some cases found in scanner data for a fixed basket.The CES assumption is in a sense *the* economic theoretical motive for applying the Jevons formulation to elementary aggregates. The study relies on consumption structures found in Swedish data, however, the analysis is neither theoretically complete nor exhaustive but rather a case study and merely for the lowest level substitution, in elementary aggregates, i.e. not between groups.

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[1] As referred to in the CPI Manual (ILO, 2004).

[2] Due to cost constraints for collecting price quotations and practical impossibility of covering the complete product universe. Probability sampling has hence been the conventional way forward (Pearson, 1934).

2 The CES assumption

The elasticity of substitution is the change in purchased quantities of items relative to each other when any of the prices change, leading to new *relative* prices, as well as absolute[3] prices. This is under the assumption that the two items are substitutable for each other and hence are relevant to consider in a grouping like an elementary aggregate. Substitution may be none (zero), meaning that consumption is unaffected by changes in relative prices, or be positive (>0) and even tend to unity (→1), which means a perfect substitution relationship between the pair of items. In some cases, substitution may even exceed unity (>1), which means a larger relative consumption shift towards the relatively cheaper item than the percentage change in relative prices. The latter circumstance is in some sense an income effect – leading to excess total consumption than at the initial point, either due to substitution or stocking, i.e. intertemporal substitution.

When substitution appears to be constant for *all* pairs of items included in the aggregate under consideration, e.g. a cost of living formulation or elementary price index expression, and constant over time[4] between all pairs of items, a *constant elasticity of substitution* (CES) function can describe the relationships between consumed items. Shapiro and Wilcox (1997) provided estimates of substitution below unity for aggregate CPI with the use of a CES function. Similar to this, variations such as lower aggregation levels than CPI total and also in a multilevel context, i.e. substitution between elementary aggregates have been examined by Balk (2000), de Haan (2001) and Ivancic et al. (2010).

An implication from the theoretical assumption for CES is that the average consumer exists in all given situations and discriminates accordingly between products based on relative prices. Also, they have fix predetermined taste/quality preference - which many consumers have, but far from all do.

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[3] It should borne in mind that absolute price increases, leaving relative prices unchanged, may still cause substitution beyond expectations. Examples are sweetener for sugar, apple juice for orange juice, milk with home-added cocoa for chocolate flavored milk, and so on. This may imply moving from direct substitutable to indirectly substitutable items and can be considered on higher-level aggregates than the elementary. C.f. Balk (2000) and de Haan (2001).

[4] The time-invariance is supposedly only within year. The fact that this implies a constant basket appears as ignored, as well as the overlapping property between two independently sampled baskets (*t=12,y=0)* and (*t=0*,y=1) if index base is set at December year y.

2.1 The Jevons and Cobb-Douglas preferences
As explained in §20.71-86 in the CPI Manual (ILO, 2004) the Jevons form of elementary price index expression reflects Cobb-Douglas preferences, as a special case of CES.

The included items' expenditure shares (quantity *levels*[5] are not considered) are assumed representative for the base period so the Jevons Index formulates in log-scale as

$$\sum_{i=1}^{n} s_i^0 \ln\left(\frac{P_i^t}{P_i^0}\right) \qquad (1)$$

where s_i^0 is the base period expenditure share for item *i* included in the aggregate under consideration.

This is supposedly reflecting a "true elementary price aggregate" regarding two-period aggregate cost functions (ibid. §20.83) and approximates a Cobb-Douglas preferences price index (ibid §20.84).

Sampling and Jevons
The assumption of unitary and constant substitution validates the use of Jevons and sampling for Jevons (ibid. §20.83)[6] – but if not confirmed unitary and constant, it would imply in one sense that all substitutes must be in the basket at the same time – otherwise relative price changes offsets representativeness assumed when forming the basket from sampled items.

Heterogeneity in the aggregate
The composition of included items is allowed to be heterogeneous since the formulation is a ratio of a fixed set of items. This is indeed an assumption of constant product universe and requires mitigation through monthly chaining to cover the real product universe. The opening for heterogeneity in an elementary aggregate is however counterintuitive regarding the validity of the CES property; similar but not intuitively substitutable items are grouped and assumed substitutable – if the CES is to hold. Fortunately, the CPI Manual (ibid §9.7) defines criteria that approaches the construction of elementary aggregates from a practical point of view – independent of index formulation.

A constant product universe
Further, in a motivating as-if scenario, it is stated (ibid §20.86) that if expenditure shares are known for both base and comparison periods and applied in a weighted Jevons index, with narrowly defined unit

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[5] If quantities are emphasized, then the Cobb-Douglas preferences' utility is interchanged for the Leontief preferences' utility, hence dissolving the need for the Jevons formulation. This is relevant when products are directly substitutable, i.e. no marginal trade-off is needed.

[6]

values, the ideal type of index is achieved. Implicitly, this requires a constant product universe (which is not a COLI) – a seemingly strong simplification of reality. Both Balk (2000) and de Hahn (2001) examine the situation for a changing universe,
and results are intriguing for index methods that do not account for a changing product universe, bias may

, as does de Hahn (2001). Similar to several consecutive studies in the field, they honor the fact that items change (at least nominally), i.e. matching over time is impaired through known identifiers such as the barcode c.f. (Appendix. 8.2, ILO 2004).

Homothetic preferences
It is worth to note that a regulatory condition for the Jevons is that consumer preferences are homothetic, i.e. utility is unaffected by *levels* but rather by ratios of prices when income is distributed. This is a strong, if at all relevant, assumption in itself (see e.g. Diewert, 1976) – it is more intuitive to consider for instance *Linear Expenditure Systems*, in which certain minimum quantities are always achieved for each item (on aggregate levels) and demand functions are perhaps partially stepwise.

Unit elasticities more likely than zero elasticities
The following passage can be found in the CPI Manual (§20.86, ILO 2004):

"…cross-item elasticities of substitution are much more likely to be close to unity (this corresponds to the case of Cobb-Douglas preferences) than to zero (this corresponds to the case of Leontief preferences).".

This statement levies high burden on the perfectness of the aggregate under consideration – at least in theory, and careful interpretation reveals that elasticities approach unity from zero, but not exceeding unity. An optimistic interpretation may be heuristic –values exceeding unity can be truncated to unity to validate the concept.

2.2. Estimation formula for elasticity of substitution
An estimable form for σ is provided by Balk (2000) by equating the two-period expenditure changes in a COLI $P(p^0, p^1 \,|\, I^0, I^1)$ based on a suggestion by Lloyd (1975), formulated as

$$\left[\sum_{i=1}^{2} s_i^0 \left({P_i^t}/{P_i^0}\right)^{(1-\sigma)}\right]^{1/(1-\sigma)} = \left[\sum_{i=1}^{2} s_i^t \left({P_i^t}/{P_i^0}\right)^{-(1-\sigma)}\right]^{-1/(1-\sigma)} \qquad (2)$$

for two items, i=(1, 2) in two distinct time periods t=(0,1) with respective expenditure shares s_i^t. It can readily be seen that by setting

$\sigma = 0$ the left hand side becomes a weighted Carli Index (Laspeyres) and the right hand side becomes a weighted harmonic mean, i.e. a Paasche index. Some alternations are given in Table 1. It is noteworthy that formulation (2) is a fixed base formulation – i.e. not accounting for substitution outside the universe of the enumerated items. Although both Laspeyres and Paasche are obtainable, this is not necessarily a true COLI should substitution occur outside the basket/product group in question. Also, shares are normalized to one – all items are related as shares to respective time point, leaving absolute levels unaddressed.[7]

Table 1 Outcomes of CES: altering $\boldsymbol{s_i^t}$, $\boldsymbol{\sigma}$ or $\boldsymbol{P_i^t}$ **.**

	S1=s11/s01	S2=s12/s02s	P1=P11/P01	P2=P12/P02	σ
1	2/3	4/3	2	1	2
2					
3					
4					

Base period prices are set to unity, P_i^0 **=** 1 and base period shares are split 50/50 between the two items, i.e. $\boldsymbol{s_i^0} = 0.5$ for *i=1,2*.

3. Empirical elasticities from scanner data

3.1 Selected products, product universes and coverage

The following multi-brand products are chosen for the analysis:

1) Sugar free soda beverage, 1.5 Liter,
2) Dairy product, 1 Liter,
3) Coffee, 450-500 grams, grounded, all varieties and
4) Cheese, packaged, several similar varieties.

For soda and milk, the analysis covers well known competing brands in each store, i.e. pairwise, and comprises data from several years in the actual CPI sample, hence fulfilling requirements of statistical representativeness effectuated through sampling weights (proportional to expenditure). It is assumed that the two chosen specific items are perfect substitutes, i.e. relevant for the analysis.

For grounded coffee, the sample is a census but spans merely over one index year, December (y-1) to December (y) for year 2017. It is

[7] Absolute levels may be extreme values, i.e. diminishingly small or large.

unbounded regarding brand and flavor varieties; all brands are potentially included but bounded through direct matching with the base period. The approach thus assumes a fixed basket and not a COLI regarding the product universe, i.e. only matching cases can be included on store level, at which substitution is likely to occur[8].

For packaged cheese, several varieties are included in the CPI sample from year 2017 (base December 2016). Although not exact substitutes, most varieties are somewhat exchangeable, except for consumers with strong preferences. Similar to coffee, the product is subject to price competition though discounts between brands and varieties.

Product universe
Considering the product universe over time, the CPI Manual provides the following three distinctions in §8.7:

- an intersection universe, which includes only matched items;
- a dynamic double universe, which includes all items in the base comparison period and all in the current period, although they may be of different qualities;
- a replacement universe, which starts with the base period universe, but also includes one-to-one replacements when an item from the sample in the base period is missing in the current period.

The apparent restriction for the third product category, coffee, is the embedded basket shrinkage potential. No replacements are done for outgoing items since such quality assessments requires stringency. Balk (2000) discusses a mitigating approach for basket decadence.

Coverage
The two respective items within soda and milk are not considered regarding coverage – they are in essence two perfect substitutes (except for preferences) and maintained in the CPI sample. Hence, their coverages are not analyzed as it is not a relevant measure – the items survive and are rather stable regarding expenditures, with variations of course. For coffee, coverage is relevant since it is a "take-all" sample of all items in month *m* that are bounded to the base. Coverage is a two-dimensional measure. Turnover in any month *m* can be independent of what happened in the base period December *y-1*, so the included items in month *m* have a relation to

[8] An implicit assumption for CES is that no selective price-seeking behavior exists, i.e. consumers in general stay true to the store they have entered and do not change store just for specific items' prices. Such behavior has been observed in certain populations with low time costs, c.f. Lach (2007).

either the turnover in current month *m* or the turnover in the base period, correspondingly as both

a) a turnover in *m* expressed as share of turnover of *all* items in the specific category, coffee, unbounded by matching to the base period, and
b) a turnover share in the base period.

The first variety, a), reflects in some sense the dynamics in the universe whereas variety b) reflects the basket attrition. These measures do not provide inference about the price changes – a decadent basket may well suffice to reflect inflation.

3.2 Time asynchrony in the CES formula

Having a closer look at the surroundings of σ in formulation (2), there may be an opening for asynchrony between the budget shares and the price ratios;

a) If notation is taken as monthly, (t=0, 1) represent the base month (t=0) and the comparison month (t=1 *or any m*) as usual for price quotations. If same interpretation is applied to expenditure shares, making these monthly, this deviates from common practice of applying calendar year spending (i.e. "annual weights"). Any results may individually be prone to random effects but nevertheless transparent and interpretable.

b) If notation is taken as yearly, then (t=0, 1) is in fact (y=0, 1) and price quotations will be yearly unit value prices, e.g. some arithmetic average in analogy with unit values, corresponding synchronously with annual expenditure shares. This is straightforward to compute but may seem a somewhat unsubstantiated[9] price measure – a yearly average price has dubious interpretation.

c) If notation is taken as hybrid, expenditure shares reflect the complete year corresponding to respective time point (t=0,1) while price quotations refer to the time period in question – base or comparison month. This can render unbalanced inference of the parameter – a specific month is more prone to transient price effects than is the aggregate and the two components may point in uncoordinated directions due to seasonality or campaigns.

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[9] I.e. lacks unambiguous interpretation in economic reality; indices are computed monthly and average annual prices become increasingly non-interpretable with increasing inflation.

The monthly interpretation, option (a), is chosen for the analysis, motivated by data availability and the entailing vastness: weekly data, forming monthly price, is accessible from base period for 2012 (December 2011), and endpoints, December (y-1) and December (y) still reflect different years. Option (c) and option (b) would imply substantially fewer analysis points.

3.3 Data and estimation outline

The scanner data is, per store, weekly turnover and amount of units sold per item. Data is aggregated over weeks up to monthly turnover, if matching with the base period for the specific item and store. It is then aggregated over all occurrences (stores) to obtain the total shares and prices in (2). An item may be sold only one week in one month, but three weeks in the base period (or the converse) in a specific store, and is thus still matching as both time points exist for that item in that store. The aggregation to obtain the expenditure share and monthly prices is over all the matching items in respective stores, rendering one aggregate expenditure share (summing to unity) and one aggregate price per item, used in (2). Due to disclosure, no magnitude of sample sizes (number of stores) can be revealed but it is indicated if the included stores are few (more than 10 but less than 20), feasible (at least twenty) or large (at least 40).

3.4 Results

For soda, the monthly sample covered the years 2013- to 2017, inclusively, starting with base period December 2012. Coverage was large regarding stores in the CPI sample.

For milk, the monthly sample data covered the years 2015, 2016 and 2017, starting with base period December 2014. At least half of the CPI stores were covered each month, i.e. feasibly large samples.

For coffee, the number of stores was overwhelmingly large (!) as this was outside the standard CPI sample, i.e. a *big data* approach.

For cheese, data covered 2017-2018 and was large regarding stores

Estimation results are reported in Table 2.

Table 2 Summary statistics on estimations of σ

Product	#estimates	Mean	Median	Std. dev.	Share σ <0
Soda	144	3.6	2.05	10.35	22%
Dairy	72	9.68	1.34	63.1	44%
Coffee	36	2.56	2.92	2.03	11%
Cheese	42	4.21	4.05	1.41	-

Note: Column with *#estimates* refers to number of estimated σ over all time points and included retail chains (one estimate per retail chain and period).

Seen in Table 2, the mean values for σ exceeds unity by far for all products. Although a substantial share of estimates fall below zero for soda and milk, it is seen that this is not as substantiated for coffee where coverage is abundant. It is possible that the presumably perfect substitutes are not relevant in statistical meaning – something more unlikely for milk than for the soda beverage. For both Soda and Coffee, results are similar to those of Ivancic et al. (2009:

Regarding coverage, Graph 1 illustrates the coverage, aggregated over retail chains, for coffee. It is expressed as relative to the base and to the current month *m*, as outlined above.

Some remarks on the estimation

Although the results are indicative, and in some cases inconclusive, precaution should be taken when interpreting them. As pointed out by e.g. Henningsen & Henningsen (2012), CES estimation *"is generally considered problematic due to convergence problems and unstable and/or meaningless results"*. Some meaningless/non-interpretable results are observed here, mainly due to the directional/non-causal relationships between price ratios and consumption share changes as illustrated in Table 2. This is most likely due to specific strong, and non-homothetic preferences blurring causal relationships necessary to obtain CES.

Similar findings are pointed out by Ivancic et al. (2009) regarding estimation results that may contradict economic theory when using high frequency (weekly) micro data. Their findings comprise both negative elasticities as well as positive, with large variance, and in many cases not rejecting the null hypothesis of elasticities equivalent to one (σ =1) or even strictly larger than one, (σ>1). Their results derive from standard *t*-tests, giving support for the use of Jevons over simple means like Carli or Dutot, and that for some item categories, not even the Jevons may be appropriate in order to adequately *"capture the 'true' level of consumer substitution"*.

The fixed basket pitfall: non synchronized campaigns

An implication from having a sample-based fixed basket is the limitation of the sample. This accentuates in the presence of non-synchronized price reductions, i.e. campaigns. In such cases, only the included items' campaigns the corresponding months occur in index calculations. Should there be any substitution at all, or even excess substitution (as seen here), the samples reflect only included items' price alternations. All other months in which other items have price alterations are by construction not possible to account for. Hence, the sample-based limited fixed basket, as applied in practice, misses out on this price alternation mechanism in COLI.

Conclusions

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Appendix

Extracts from the ILO (2004) Manual for CPI:

9.36 The Jevons index does not imply, or assume, that expenditure shares remain constant. Obviously, the Jevons can be calculated whatever changes do, or do not occur in the expenditure shares in practice. What the economic approach shows is that if the expenditure shares remain constant (or roughly constant), then the Jevons index can be expected to provide a good estimate of the underlying cost of living index.

9.34 On the basis of the economic approach, the choice between the sample Jevons and the sample Carli rests on which is likely to approximate the more closely to the underlying cost of living index: in other words, on whether the (unknown) cross-elasticities are likely to be closer to unity or zero, on average. In practice, the crosselasticities could take on any value ranging up to plus infinity for an elementary aggregate consisting of a set of strictly homogeneous items, i.e., perfect substitutes. It should be noted that in the limit when the products really are homogeneous, there is no index number problem, and the price ‘‘index’’ is given by the ratio of the unit values in the two periods, as explained later. It may be conjectured that the average cross-elasticity is likely to be closer to unity than zero for most elementary aggregates so that, in general, the Jevons index is likely to provide a closer approximation to the cost of living index than the Carli. In this case, the Carli index must be viewed as having an upward bias.
9.35 The insight provided by the economic approach is that the Jevons index is likely to provide a closer approximation to the cost of living index for the elementary aggregate than the Carli because, in most cases, a significant amount of substitution is more likely than no substitution, especially as elementary aggregates should be deliberately constructed in such a way as to group together similar items that are close substitutes for each other.
9.36 The Jevons index does not imply, or assume, that expenditure shares remain constant. Obviously, the Jevons can be calculated whatever changes do, or do not occur in the expenditure shares in practice. What the economic approach shows is that if the expenditure shares remain constant (or roughly constant), then the Jevons index can be expected to provide a good estimate of the underlying cost of living index. Similarly, if the relative quantities remain constant, then the Carli index can be expected to provide a good estimate, but the Carli does not actually imply that quantities remain fixed.
9.37 It may be concluded that, on the basis of the economic approach as well as the axiomatic approach,

the Jevons emerges as the preferred index in general, although there may be cases in which little or no substitution takes place within the elementary aggregate and the Carli might be preferred. The index compiler must make a judgement on the basis of the nature of the products actually included in the elementary aggregate.