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Elementary aggregation: A not so elementary story!

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Background

Traditional CPI compilation paradigm:

- *Prices are aggregated without weights at the level of the elementary aggregate.*
- *These elementary price indices are then aggregated to the higher-levels using expenditure weights.*

What does “elementary aggregation” mean in the context of scanner data ?

What is an elementary aggregate ?

An elementary aggregate is

- *the smallest aggregate for which **expenditure data** are used for CPI purposes.*
- *the values of the elementary aggregates are used **to weight the price indices for elementary aggregates** to obtain higher-level indices.*
- *the range of goods and services covered by an elementary aggregate should be **relatively narrow**.*
- *Elementary aggregates also serve as **strata for the sampling of prices**.*

CPI Manual (2004)

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What is an item?

- *In practice, the item corresponds to*

  - *an **individual product**, specified by its characteristics, and for which a price is collected in an **outlet** at a given **time***

or

  - *a set of transactions which refer to **one or more individual products**, from **one or more places of purchase**, over a **period of time**, and for which an average price can be computed.*

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Aggregation structures (1)

Level	Aggregation method
CPI product category	Laspeyres-type
CPI product category by outlet-type and/or by region **= EA**	Laspeyres-type
Items	Jevons
Sub-items	Average price

Other fixed weights may be used to aggregate the prices of the sampled items.

Aggregation structures (2)

Index	Formula
Jevons	$I = \left(\frac{p_1^t}{p_1^0} \cdot \frac{p_2^t}{p_2^0} \cdot \frac{p_3^t}{p_3^0} \cdot \frac{p_4^t}{p_4^0} \cdot \frac{p_5^t}{p_5^0} \right)^{\frac{1}{5}}$
Jevons + Geo. Lasp.	$I = \left(\frac{p_1^t}{p_1^0} \cdot \frac{p_2^t}{p_2^0} \cdot \frac{p_3^t}{p_3^0} \right)^{\frac{1}{3} \cdot w_1} \cdot \left(\frac{p_4^t}{p_4^0} \cdot \frac{p_5^t}{p_5^0} \right)^{\frac{1}{2} \cdot w_2} = (I_1)^{w_1} \cdot (I_2)^{w_2}$
Jevons + Lasp.	$I = w_1 \left(\frac{p_1^t}{p_1^0} \cdot \frac{p_2^t}{p_2^0} \cdot \frac{p_3^t}{p_3^0} \right)^{\frac{1}{3}} + w_2 \cdot \left(\frac{p_4^t}{p_4^0} \cdot \frac{p_5^t}{p_5^0} \right)^{\frac{1}{2}} = w_1 I_1 + w_2 I_2$

Aggregation structures (3)

Level	Aggregation method
CPI product category	Laspeyres-type
Retailer	Laspeyres-type
Product sub-category	?
Items	Multilateral
Transactions	Unit value

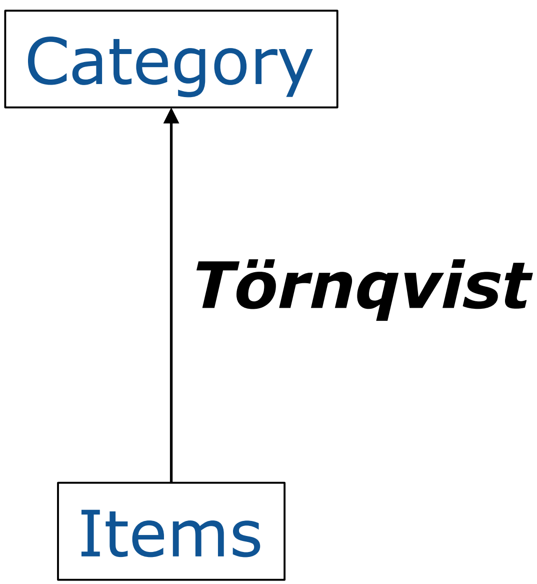
Detailed strata with possibly fixed weights may or may not be defined below the level of the CPI product category.

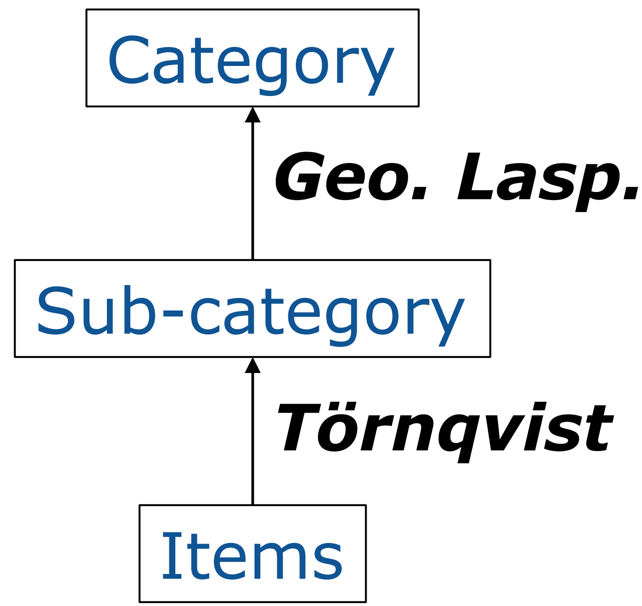
Aggregation structures (4)

Index	Formula
CCDI in 1 step, up to the category	$I = I(CCDI)$
CCDI + Geo. Lasp. in 2 steps, first to the sub-category and then to the category	$I = \prod_k (I_k(CCDI))^{w_k}$
CCDI + Lasp. in 2 steps, first to the sub-category and then to the category	$I = \sum_k w_k I_k(CCDI)$
CCDI + Törnqvist in 2 steps, first to the sub-category and then to the category	$\mathrm{I} = \prod_{\mathrm{k}} (\mathrm{I_k}(\mathrm{CCDI}))^{0.5*(\mathrm{w_k^0}+\mathrm{w_k^t})}$

What is the impact of fixed weights?

- *Some simplifying assumptions:*
  - *The set of items is constant over time.*
  - *Törnqvist instead of CCDI.*





What is the impact of fixed weights?

*The difference between the 2-step **Törnqvist + Geo. Lasp.** index and the 1-step **Törnqvist** index can be decomposed into three terms.*

Covariance(… , …)

+

Covariance(… , …)

+

Covariance (*Average price of the sub-category ,*
Deviation of the sub-category fixed weights
*from the sub-category "true" weights***)**

Empirical analysis

- *Simulations performed on Dominick's data set (Mehrhoff (2019)).*
- *Data aggregated across all stores, transformed into monthly data, using Dominick's item code.*
- *6 categories: dish detergents, soft drinks crackers, cookies, grooming products, cheese.*
- *For each category, sub-categories are constructed using the pre-defined Dominick's Commodity Code.*

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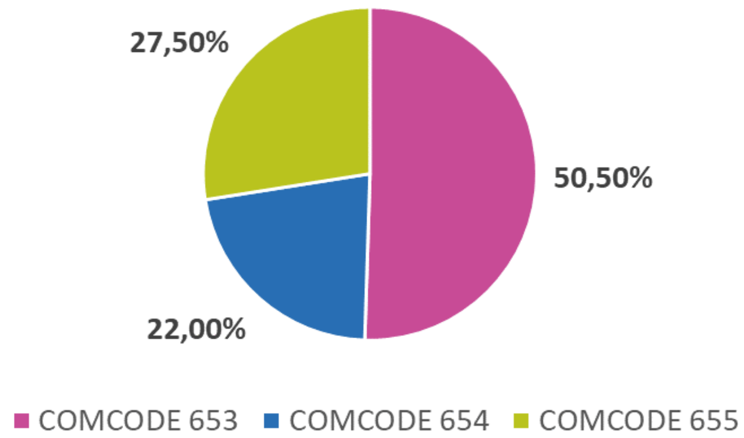
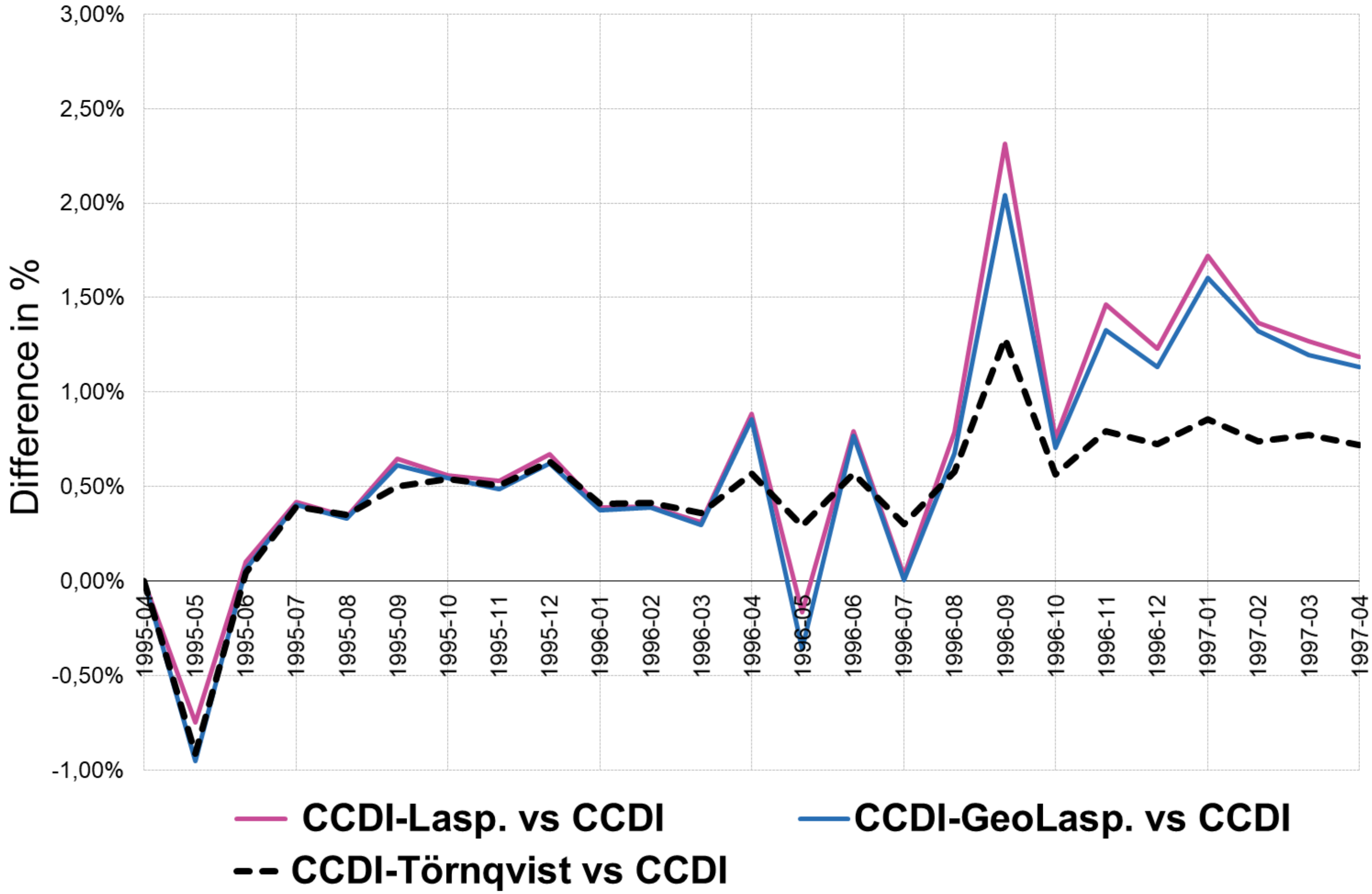
Index compilation

- *Indices compiled for 25 months (April 1995 – April 1997)*
- *1-step index up to the category : CCDI*
- *2-step index, first sub-category and then category : CCDI-Laspeyres, CCDI-Geo. Laspeyres, CCDI-Törnqvist*
- *Price reference period: April 1995*
- *Weight reference period for the fixed sub-category weights: April 1994 - March 1995*



Dividing the category **dish detergents** into three sub-categories according to the commodity code.

Example : Dish detergents



A variant of the CCDI index

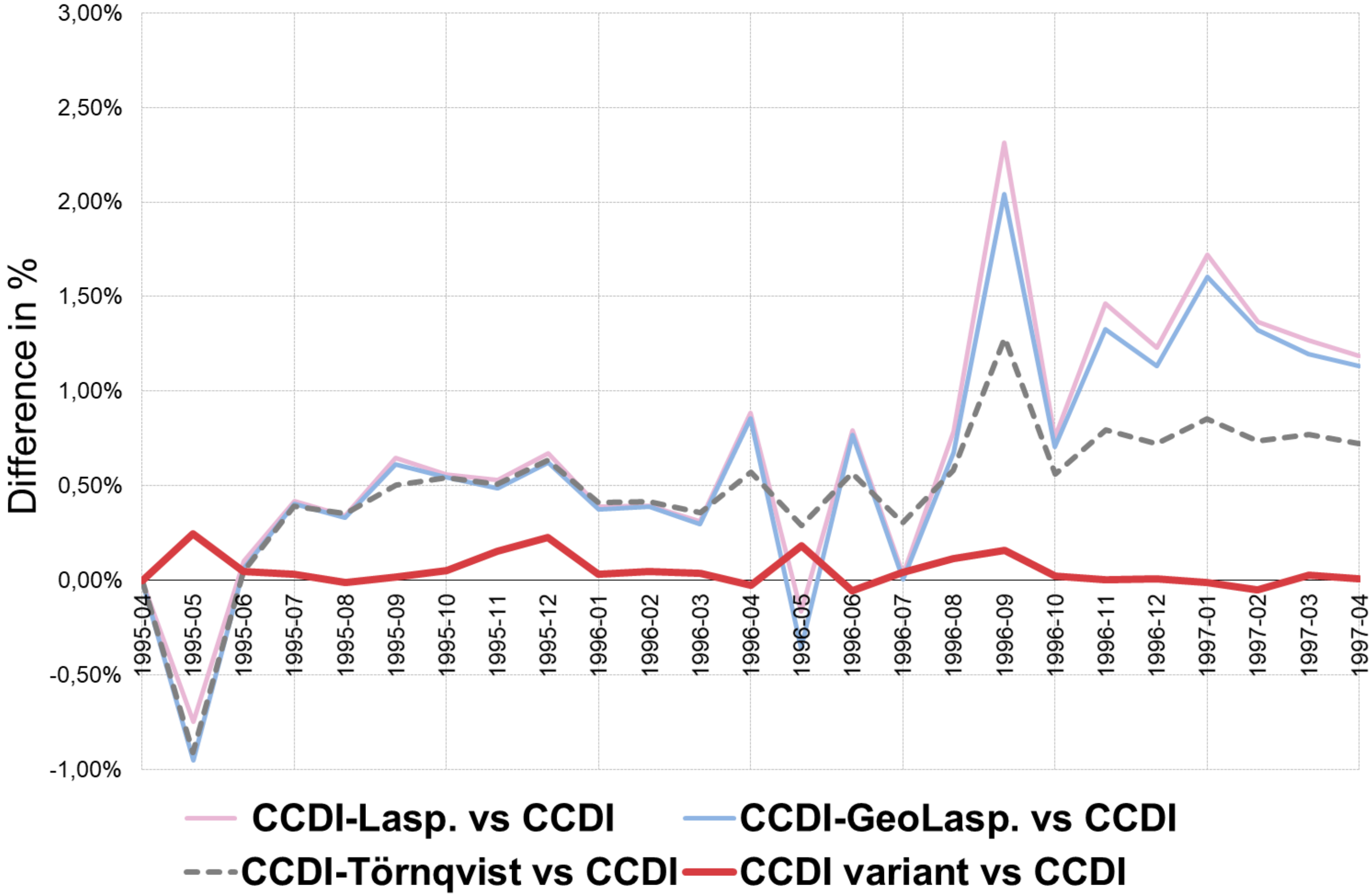
1. *Within each sub-category k, compile matched bilateral Törnqvist indices $P_{i,j}^{k}$ between any two periods i and j.*
2. *Aggregate these bilateral indices to the category level as follows.*

$$\widetilde{P_{i,j}} = \prod_{k=1}^{K} \left(P_{i,j}^{k}\right)^{0.5*\left(w_{i,j}^{k}(i)+w_{i,j}^{k}(j)\right)}$$

$$w_{i,j}^{k}(i) = \frac{\sum_{Item \in S_i^k \cap S_j^k} p_{Item}^{i} q_{Item}^{i}}{\sum_r \sum_{Item \in S_i^r \cap S_j^r} p_{Item}^{i} q_{Item}^{i}} \qquad w_{i,j}^{k}(j) = \frac{\sum_{Item \in S_i^k \cap S_j^k} p_{Item}^{j} q_{Item}^{j}}{\sum_r \sum_{Item \in S_i^r \cap S_j^r} p_{Item}^{j} q_{Item}^{j}}$$

3. Use the indices $\widetilde{P_{i,j}}$ as building blocks to compile a CCDI index ($\widetilde{P_{i,j}}$ satisfies the time reversal test).

Example : Dish detergents

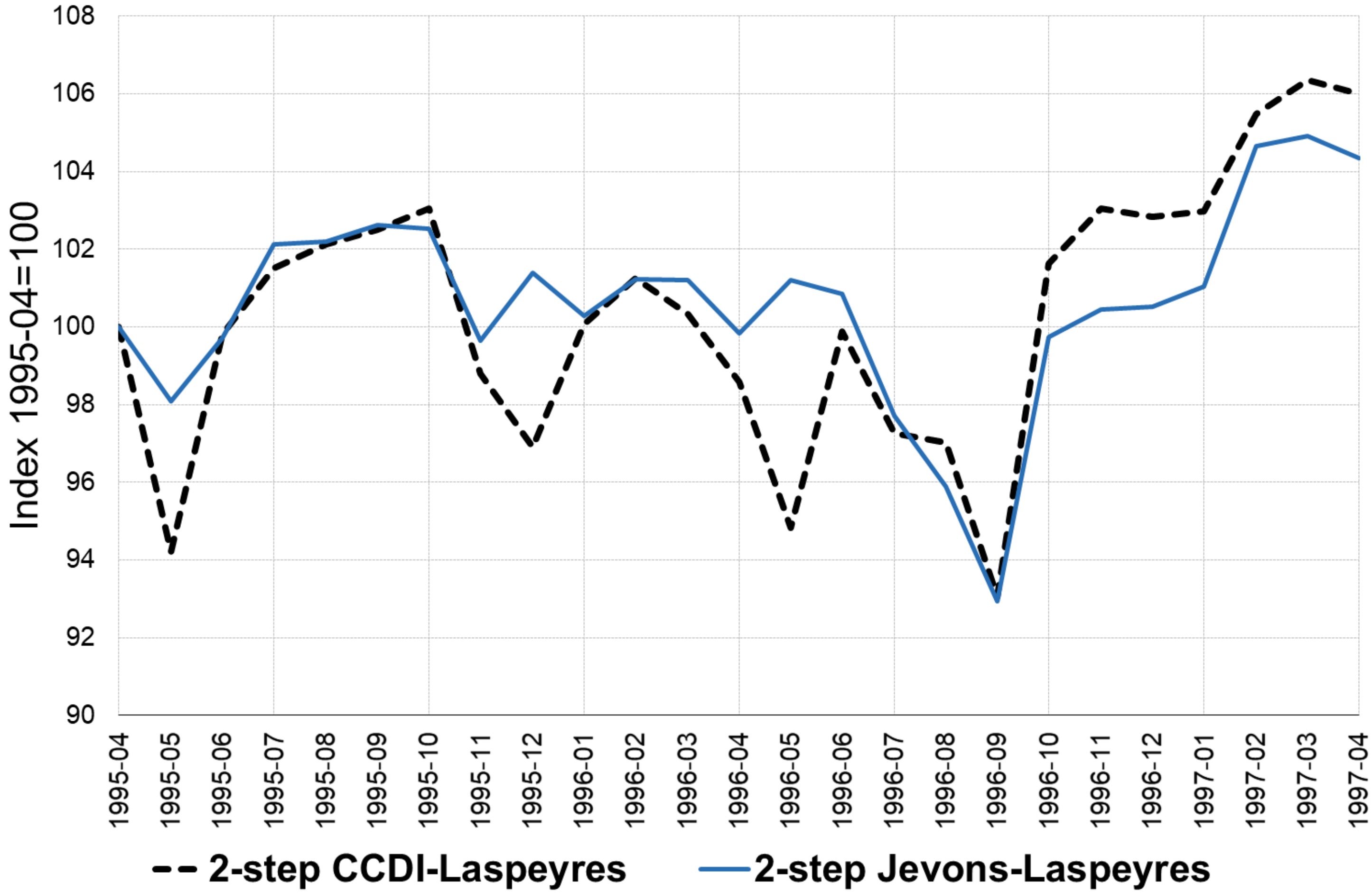


A more "standard" price index

- *Cut-off Jevons + Laspeyres index:*
  1. *Within each sub-category, consider items available in all 25 months.*
  2. *Within each sub-category, select the n=10 items with the largest sales.*
  3. *For each sub-category, compile a Jevons price index over these items.*
  4. *Aggregate the sub-category Jevons indices using a Laspeyres-type formula.*

Such an index tends to be closest to a CCDI-Laspeyres index.

Example : Dish detergents



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Conclusions

With scanner data one needs to:

1. *Define the item*
2. *Aggregate the prices of the items up to an intermediate level*
3. *Aggregate the intermediate elementary price indices*

We compiled various 2-step indices and compared them to a 1-step index which does not take into account an intermediate level.

Conclusions

- *How "narrow" should categories be constructed at the first stage of aggregation?*

- *Focus on the product dimension: what about the outlet dimension?*

- *"Consistency in aggregation", multilateral methods and dynamic universe?*

- *A more standardized way for describing elementary aggregation in a CPI?*