

The Nature of Chain Drift

by Ludwig von Auer¹
Universität Trier

March 27, 2019

Abstract

A chained price index is said to suffer from chain drift bias, if it indicates an overall price change, even though the prices and quantities in the current period have reverted back to their levels of the base period. This type of bias is well documented in studies that apply sub-annual chaining to scanner data.

The underlying forces, however, are less well understood. Chain drift is usually explained by sales in connection with inventory behaviour of consumers. The present study shows that the chain drift problem is much broader. It arises also in everyday market conditions unrelated to sales and inventories. More specifically, chain drift is the net effect of two counteracting forces: *pendular* and *sticky* quantity reactions. The former is related to sales and the associated stocking behaviour, while the latter arises from delayed changes of purchasing habits due to search or adjustment costs. The former causes downward chain drift, whereas the latter generates upward chain drift. Therefore, the dominating force determines the direction of the chain drift. This insight explains why some empirical studies find downward chain drift, while other studies find exactly the opposite result even though sales occur.

The present paper also introduces a simple utility framework that captures pendular and sticky quantities arising from “unconventional” consumer behaviour such as stocking and delayed quantity responses to price changes. Building on this framework, a “stress test” is introduced that examines the resilience of price indices to chain drift. In the literature, various rolling window GEKS indices have been proposed as a solution to the chain drift problem. The stress test reveals that some variants are more immune to chain drift than others.

¹Universität Trier, Fachbereich IV - VWL, Universitätsring 15, 54286 Trier, Germany; Email: vonauer@uni-trier.de

1 Introduction

The conventional approach to compute the price change between a base period 0 and a comparison period T uses only the prices and quantities of these two periods. Such index formulas are known as *direct price indices* (or bilateral price indices). In the following, they are denoted by $P^{T/0}$. Direct price indices process the prices and quantities of matched items only, that is, of items that are available in both periods. Therefore, direct indices suffer from an attrition problem: the larger the time distance between periods 0 and T , the smaller the set of matched items and the larger the potential deviation between the measured average price change of the matched items and the average price movement of the complete item universe.

As an alternative to the direct index, $P^{T/0}$, one may compute the price change between periods 0 and T by a *sequence* of adjacent direct price indices, $P^{t/(t-1)}$, and link the elements of this sequence by multiplication: $P^{1/0} \cdot P^{2/1} \cdot \dots \cdot P^{T/(T-1)}$. Such products are known as *chain indices* and its factors as *chain links*. Chain indices allow for a regular updating of the universe of items and the weights attached to them.

When all prices and quantities in the current period T revert back to their levels in the base period 0, a price index comparing the current period with the base period should indicate that no price change occurred. All reasonable direct price indices, $P^{T/0}$, satisfy this requirement. However, chained direct price indices usually violate it. This violation is known as *chain drift bias*. It is always attributed to *sales* triggering non-standard substitution behaviour of consumers. Feenstra and Shapiro (2003, p. 135) examine scanner data on canned tuna and compile a weekly chained Törnqvist index that exhibits *upward* chain drift caused by sales, while de Haan (2008, p. 19), studying scanner data on detergents, finds that sales lead to *downward* chain drift, even though both studies apply the same index formula and frequency of chaining.

These contradictory results suggest to analyze the causes of chain drift in a more systematic way. The present paper shows that the chain drift of chained direct price indices is caused by quantity and price changes that are not perfectly synchronized in time. Such asynchronous price and quantity changes can arise not only from sales, but also from everyday market situations unrelated to sales. More specifically, the first contribution of the present paper is to show that chain drift is the net effect of two counteracting forces: *pendular* and *sticky* quantity reactions to price changes. The former is caused by sales and the associated stocking behaviour, while the latter is caused by delayed changes of purchasing habits due to search or adjustment costs.

Several studies of scanner data (e.g., de Haan, 2008, p. 18; de Haan and van der Grient, 2011, p. 43) demonstrate that pendular quantities cause downward chain drift. The chain drift arising from sticky quantities, however, went largely

unnoticed.² The present paper shows that they lead to upward chain drift. In Feenstra and Shapiro (2003, p. 135), sticky quantities arise in the context of sales (delayed quantity reaction to price reduction). The upward chain drift resulting from sticky quantities dominates the downward chain drift arising from pendular quantities. Therefore, in their scanner data set the net effect is upward chain drift.

Many direct price indices can be expressed as symmetrically weighted averages of price ratios (SWAP), where the weights represent some sort of expenditure shares. These SWAP indices include also the superlative index of Törnqvist which was used also by Feenstra and Shapiro (2003) and de Haan (2008). The present paper's second contribution is to formally show that in the presence of pendular or sticky quantities all SWAP indices exhibit chain drift.

Furthermore, this study introduces a simple utility framework that captures pendular and sticky quantities arising from “unconventional” consumer behaviour such as stocking and delayed quantity responses to price changes. Building on this framework, a “stress test” is introduced that can examine the resilience of price indices to chain drift. This is the third contribution of the present paper.

As a solution to the chain drift problem, Ivancic *et al.* (2011) advocate a rolling window variant of the Gini-Éltető-Köves-Szulc (RGEKS) approach. By now, several variants of such RGEKS indices have been proposed.³ The stress test reveals that some variants are more immune to chain drift than others. This is the paper's fourth contribution.⁴

As a first step, Section 2 draws attention to direct price indices that can be expressed as symmetrically weighted averages of price ratios (SWAP), where the weights represent some sort of expenditure shares. In Section 3 it is shown that the chaining of such indices leads to chain drift and that this drift is caused by pendular or sticky quantity responses to price changes. A formal proof of this result is provided in Section 4. Various RGEKS indices have been proposed as a solution to the chain drift problem. Their commonalities and differences are explained in Section 5. Section 6 introduces a utility function that generates both, pendular quantity responses related to sales and sticky quantity responses caused by search or adjustment costs. Drawing on this utility function, Section 7 presents a simulation based stress test that shows that some RGEKS indices are better suited to curtail chain drift than others. Concluding remarks are contained in Section 8.

²A notable exception is Triplett (2003, p. 152) who points out that storage, search, and information cost may generate measurement problems for scanner data price indices.

³For compact surveys see, for example, Diewert and Fox (2017, pp. 11-13) or Van Loon and Roels (2018, pp. 7-8).

⁴Besides RGEKS indices, many other multilateral approaches have been proposed to overcome the chain drift problem. For recent surveys of these methods see, for example, de Haan and Krsinich (2014), Diewert and Fox (2017), and Chessa *et al.* (2017). In ongoing research, also these methods are exposed to our stress test.

2 Chain Drift and SWAP Indices

Let S denote the set of integers $i = 1, \dots, N$, where each integer represents one of the N items of an economy. All items are available during the base period ($t = 0$) and the comparison period ($t = T$). The period t vector of prices is denoted by $\mathbf{p}^t = (p_1^t, \dots, p_N^t)$ and the corresponding vector of quantities by $\mathbf{x}^t = (x_1^t, \dots, x_N^t)$. It is customary to interpret a direct price index, P , as a mapping of the N -dimensional vectors \mathbf{p}^0 , \mathbf{x}^0 , \mathbf{p}^T , and \mathbf{x}^T into a single positive number, $P^{T/0}(\mathbf{p}^0, \mathbf{x}^0, \mathbf{p}^T, \mathbf{x}^T)$, that measures the “overall price change” between periods 0 and T . A popular example is the Törnqvist index:

$$\ln P_{T\bar{0}} = \sum_{i \in S} \frac{1}{2} \left(\frac{p_i^0 x_i^0}{\sum p_j^0 x_j^0} + \frac{p_i^T x_i^T}{\sum p_j^T x_j^T} \right) \ln \frac{p_i^T}{p_i^0}. \quad (1)$$

Consider some sequence of periods, t , with $t = 0$ being the first and $t = T$ being the last period.

Definition 1 *A chain index that compares periods T and 0 is defined by*

$$\tilde{P}^{T/0} = P^{1/0} \cdot P^{2/1} \cdot \dots \cdot P^{T/T-1}. \quad (2)$$

Some writers (e.g., Persons, 1928, p. 101; Lent, 2000, p. 314) interpret chain drift as the ratio of the chained index to the corresponding direct index, $\tilde{P}^{T/0}/P^{T/0}$. However, Szulc (1983, p. 554) points out that ratios different from unity do not necessarily indicate bias, since a time-distant direct price index, $P^{T/0}$, is likely to be biased. However, when the prices and quantities in period T return to their levels of period 0, the chain index should give the same index number as the direct index, namely 1 (e.g., Ivancic *et al.*, 2011, p. 26). Accordingly, in this specific price-quantity-scenario, the deviation from unity is an appropriate measure of the extent of chain drift (e.g., Diewert and Fox, 2017, p. 9; Diewert, 2018, p. 25; Ribe, 2012, p. 3). This interpretation of chain drift can be formalized in the following way:

Definition 2 *The chain drift test of the direct price index, $P^{t/t-1}$, postulates that*

$$P^{1/0} \cdot P^{2/1} \cdot \dots \cdot P^{0/T-1} = 1. \quad (3)$$

When a direct price index violates the chain drift test, the extent of chain drift can be measured by the term $\left(\tilde{P}^{T/0} - 1 \right)$, where $\tilde{P}^{T/0} = P^{1/0} \cdot P^{2/1} \cdot \dots \cdot P^{0/T-1}$.⁵

⁵Condition (3) can be found in Walsh (1901, p. 401) who later calls it the circularity test (see Diewert, 1993, p. 40). Unfortunately, in the current price index literature, this label is reserved for the stricter condition $P^{2/0} = P^{2/1}P^{1/0}$. Therefore, Diewert (1993, p. 40) coins the chain drift test as the “multiperiod identity test”. However, this label may possibly convey the misleading impression that, as in the identity test, the evolution of the quantities is completely irrelevant. It is not, because the quantities must reverse to their base period values. In fact, a chain drift test without the quantity reversal postulate is considered in de Haan (2008, p. 10). He calls it the “invariance to price bouncing test”. Note that for $T = 1$ the chain drift test simplifies to the time reversal test: $P^{1/0}P^{0/1} = 1$.

The Törnqvist index formula (1) has three distinctive features: (1) it can be interpreted as an expenditure weighted average of the items' individual price changes, (2) the weight of each item i increases with its own prices (p_i^0 and p_i^T) and quantities (x_i^0 and x_i^T), and (3) the impact of the two periods 0 and T on the expenditure weights is perfectly symmetric. Therefore, we can denote the Törnqvist index as a Symmetrically Weighted Average of Price ratios (SWAP).

The Törnqvist index is not the only SWAP index. A whole class of SWAP indices exists. To define this class, let \mathbf{p}_{-i}^t and \mathbf{x}_{-i}^t be defined by $\mathbf{p}^t = (\mathbf{p}_{-i}^t, p_i^t)$ and $\mathbf{x}^t = (\mathbf{x}_{-i}^t, x_i^t)$, respectively, and let

$$g_i = g(p_i^0, x_i^0, p_i^T, x_i^T, \mathbf{p}_{-i}^0, \mathbf{x}_{-i}^0, \mathbf{p}_{-i}^T, \mathbf{x}_{-i}^T) = g(p_i^T, x_i^T, p_i^0, x_i^0, \mathbf{p}_{-i}^T, \mathbf{x}_{-i}^T, \mathbf{p}_{-i}^0, \mathbf{x}_{-i}^0) \quad (4)$$

denote an “expenditure weight” of some item i , with $g(\cdot)$ being the weighting function. Note that this function is symmetric with respect to time. We consider only weighting functions $g(\cdot)$ with

$$\frac{\partial g_i}{\partial p_i^t} > 0 \quad \text{and} \quad \frac{\partial g_i}{\partial x_i^t} > 0, \quad t = 0, T. \quad (5)$$

SWAP index formulas can be written as weighted arithmetic or geometric averages of the price ratios, p_i^T/p_i^0 , with weights, g_i , defined by (4) and (5). More formally:

Definition 3 *SWAP indices, $I^{T/0}$, can be expressed by*

$$I^{T/0} = f(P^{T/0}) = \sum_{i \in S} g_i f\left(\frac{p_i^T}{p_i^0}\right), \quad (6)$$

with either $f(z) = z$ or $f(z) = \ln z$ and with weights, g_i , defined by (4) and (5).

This class of indices includes not only the superlative Törnqvist index, but also the Walsh-Vartia, Walsh-2, Vartia, and Theil index. These SWAP index formulas and the formulas of all other price indices mentioned in this study are listed in Appendix A.⁶ A price increase of some item i ($p_i^T/p_i^0 > 1$) raises the price index number of a SWAP index, while a price reduction ($p_i^T/p_i^0 < 1$) reduces the price index number. The size of these effects increases with the weight, g_i , of item i .

⁶Diewert (2018, p. 12) defines “mean of order r price indices” as

$$P = \begin{cases} \left[\sum_{i \in S} g_i \left(\frac{p_i^T}{p_i^0}\right)^r \right]^{1/r} & \text{for } r \neq 0 \\ \prod_{i \in S} \left(\frac{p_i^T}{p_i^0}\right)^{g_i} & \text{for } r = 0. \end{cases}$$

SWAP indices are a subset of this class of indices, because they restrict r to the values 0 and 1 and they allow only for weights, g_i , that are sensitive to both prices and both quantities relating to item i . Therefore, the Laspeyres, Paasche, Drobnisch, Walsh, and Marshall-Edgeworth index are mean of order 1 indices, but not SWAP indices. The Fisher index is not a mean of order r index and, thus, not a SWAP index. Instead, it belongs to the class of *quadratic* mean of order r indices (see Diewert, 1976, p. 129). Their performance in the presence of sales is analyzed in de Haan (2008, pp. 10-11).

3 Two Sources of Chain Drift

In standard microeconomic consumer theory, consumers adjust their current purchases to the current prices. The usual assumption is that consumers substitute away from products that have become relatively more expensive. The larger the elasticity of demand, the more pronounced are these quantity reactions. Figure 1 translates this standard theory into a highly stylized example. The figure depicts the prices and quantities of some item during nine consecutive periods, $t = 0, \dots, 8$. In the depicted *Scenario 1*, the prices of the item can take on only the values “low”, “normal”, and “high” and the quantities transacted can be “small”, “normal”, or “large”.

Price changes occur at the position of the depicted weight icons. The price starts at a normal level, drops in period 1 to the lower level, stays there for another period, returns to normal in period 3, and stays there also during period 4. In period 5, the price increases to high, stays there for another period, before it drops back to normal during period 7 and remains there during period 8. The quantities purchased move exactly inversely to the prices, that is, the consumers’ quantity reactions are perfectly synchronous to the price changes. As soon as the price returns to normal, also the quantity returns to normal. In times of constant prices also the quantities remain constant. When the elasticity of demand is less than unity, expenditure shares and prices are positively correlated.

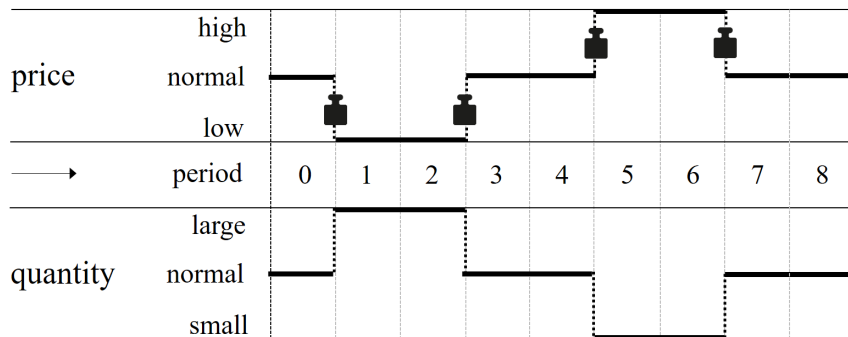


Figure 1: Synchronous Quantity Reactions to Price Changes (Scenario 1).

Due to the symmetric impact of the base and comparison period on the weights of SWAP indices, Scenario 1 generates weights such that the price decline in period 1 (its weight is related to the expenditures during periods 0 and 1) and the price increase in period 3 (its weight is related to the expenditures during periods 2 and 3) exactly offset each other. Graphically, this balanced weighting is indicated by the equal sized weight icons located at the price decline between periods 0 and 1 and the price increase between periods 2 and 3. The same is true for the price increase between periods 4 and 5 and the price decline between periods 6 and 7.

As a consequence of this balanced weighting of price increases and price declines, SWAP indices are immune to chain drift from scenarios like Scenario 1 and we get $I^{1/0} \cdot I^{2/1} \cdot \dots \cdot I^{8/7} = 1$. This unbiasedness is indicated by the horizontal arrow on the left hand side of Figure 1.

Chain drift arises, however, for the Laspeyres and Paasche index. The Laspeyres index weighs the price changes by the base period's expenditure shares only. If the elasticity of demand is below 1 and a price decline occurs, the resulting expenditure share of the comparison period is smaller than that of the base period. Since the comparison period's smaller expenditure share is not part of the weighting of the Laspeyres index, the price reduction receives a larger weight than it would receive with a weight that averages the base and comparison period expenditure shares. Conversely, a price increase receives a diminished weight. This unbalanced weighting leads to downward chain drift. If the elasticity were above 1, upward chain drift would arise. The chain drift of the Paasche index is in opposite direction to the chain drift of the Laspeyres index.

To summarize, the Laspeyres and Paasche indices are characterized by an asymmetric weighting of the expenditures of the base and comparison period. In scenarios like Scenario 1 this asymmetry causes chain drift. This is well known in the literature (e.g., Szulc, 1983, pp. 540-541). By contrast, all SWAP indices show no chain drift in Scenario 1. It would be a serious mistake, however, to conclude that these indices are immune to chain drift.

Scenario 1 corresponds to standard consumer theory as presented in introductory microeconomics textbooks. Real world consumer behaviour, however, is more complex. One important aspect sidelined by standard consumer theory is stocking behaviour. Stocks facilitate a temporary decoupling of acquisition and consumption. Households benefit from this extra flexibility in two ways. By exploiting sales, households can buy the product at a lower average price. Furthermore, households can smooth their consumption in times of volatile prices.

Sales usually lead to increased purchases, part of which are stored. If in the next period the price returns to its normal level, the purchased quantity falls *below* its normal level, because consumers first use up their extra stock. Only after the extra stock is depleted, the purchased quantity returns to its normal level. We denote such a scenario as a *pendular* quantity response to sales. This type of scenario is described, for example, in Diewert and Fox (2017, p. 9), de Haan and van der Grient (2011, p. 39), Ivancic *et al.* (2009, p. 4), and Ribe (2012, p. 3).⁷

Periods 0 to 3 of *Scenario 2* (see Figure 2) depict this case in a highly stylized form. The reduced price of period 1 triggers an immediate quantity reaction that

⁷Adapting an artificial scenario analyzed in Person (1928, p. 102), Diewert (2018, p. 27) discusses a situation that combines sales with somewhat delayed quantity responses. This scenario resembles the pattern that Feenstra and Shapiro (2003) identify in their scanner data set.

is larger than it would be without the storing motive. When in period 2 the price reverts to its normal level, the quantity drops below its normal level, because the extra stock is used up for consumption, reducing the need for new purchases. As a result of this pendular quantity response, the weight attached to the price increase between periods 1 and 2 is smaller than the weight attached to the price reduction between periods 0 and 1. Therefore, the price index level of period 2 is below that of period 0. At the end of period 2, the inventory is back to its normal level. Therefore, the purchases in period 3 increase to their normal level, even though the price is constant. The identity test says that, in the absence of any price changes, the price index is unity, regardless of any quantity changes. Because all SWAP indices satisfy the identity test, the price index level of period 3 remains on the level of period 2 and, therefore, below that of period 0. In other words, downward chain drift arises.

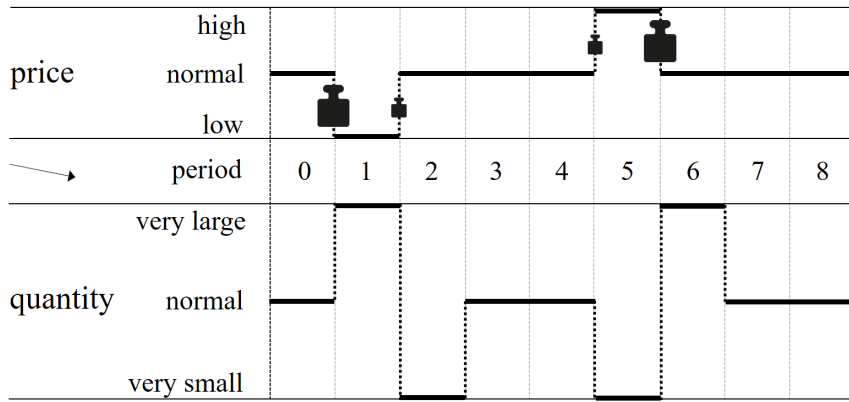


Figure 2: Pendular Quantity Reactions Caused By Sales Or Price Spikes (Scenario 2).

Periods 0 to 3 of Scenario 2 describe a storable item that consumers keep in stock. In times of unusually low prices (sales) the consumers add to their ordinary stock some extra stock and they deplete that extra stock when the price reverts to its normal level. This is the standard narrative of stocking behaviour.

However, stocking behaviour is not only relevant in times of sales, but also in times of price spikes. During such spikes consumers can plunder their ordinary stock and restock it as soon as the price returns to normal. This is depicted by periods 4 to 8 of Scenario 2. The price increases in period 5. Many consumers switch to using up their stocks. This leads to a negative quantity reaction that is larger than for items that cannot be stored. In period 6, the price returns to its normal level. This gives the consumers the opportunity to refresh their inventories. At the same time they return to their normal consumption. Therefore, the total quantity purchased exceeds the normal quantity. As a result of this pendular quantity response, the weight attached to the price reduction between periods 6 and 7 is larger than that attached to the price increase between periods 5 and 6. This leads again to downward

chain drift. At the end of period 6 the stock is back to its standard level, such that the purchases in period 7 return to their normal level even though no price change occurs between periods 6 and 7.

In sum, pendular quantities triggered by sales or by price spikes work in the same direction. Both generate downward chain drift. This is indicated by the downward pointing arrow on the left hand side of Figure 2. Pendular quantities, however, are only one of the two drivers of chain drift. The other driver operates in the opposite direction, that is, it causes upward chain drift.

In the field of industrial organization there is an extensive literature on search and switching costs and their implications for markets.⁸ Also in the field of price measurement it is well known that search and switching costs are relevant in real world consumption decisions and that they create problems for price measurement purposes (e.g., Reinsdorf, 1994, p. 137; Triplett, 2003, p. 152). Such costs can delay the consumers' substitution behaviour, such that part of the quantity response or the complete quantity response happens in a later period than the underlying price change. This is particularly true when the length of a period is relatively short (e.g., one week or one month). Feenstra and Shapiro (2003, p. 133) identify such delayed quantity responses in the context of sales. In their data set the immediate quantity response to a sale is modest, but substantially increased after, in some later period, the sale is advertised. Delayed quantity responses cause *upward* chain drift. We denote this second source of chain drift as *sticky* quantities.

Figure 3 illustrates why sticky quantities cause upward chain drift. In the depicted *Scenario 3* demand is completely price inelastic in the short-run, but price elastic in the long-run. More specifically, the complete quantity reaction to each price change is delayed by one period.

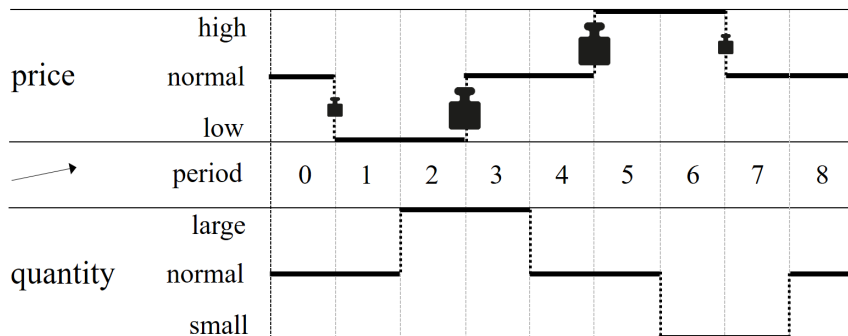


Figure 3: Sticky Quantity Reactions to Price Changes (Scenario 3).

In period 1 the price drops, while the quantity remains unchanged. Therefore, the observed expenditure during period 1 is smaller than it would be with an elastic

⁸A recent survey of this literature is Fisher Ellison (2016).

demand. As a consequence, also the weight attached to the price decline is smaller than it would be with an elastic demand. Graphically, this diminished weight is indicated by the small weight icon at the price decline between periods 0 and 1.

The complete quantity reaction to the reduced price in period 1 occurs one period delayed, that is, in period 2. The price in period 2 remains on the level of period 1. All SWAP price indices satisfy the identity test. Therefore, their price index level does not change between periods 1 and 2.

Since demand is inelastic in the short-run, the price increase between periods 2 and 3 occurs without a quantity reduction. Therefore, the weight attached to this price increase is larger than it would be with the usual quantity reduction. The large weight icon at the transition from period 2 to period 3 highlights this inflated weight.

The price increase between periods 4 and 5 is analogous to that between periods 2 and 3. Again, the price increase receives an inflated weight. The price decline between periods 6 and 7 shows the same pattern as that between periods 0 and 1. This price decline receives a diminished weight.

Overall, this unbalanced weighting of price declines and price increases leads to an upward chain drift. It is important to note that this chain drift is not caused by inventories related to sales (see periods 5 to 8), but by delayed quantity responses. The upward chain drift is indicated by the upward pointing arrow at the left hand side of Figure 3.

Theoretically, one can think of quantity reactions that are antedated by one period. This could be regarded as a “negative delay”. *Scenario 4* (see Figure 4) is a highly stylized example. The weighting effects are exactly opposite to those of Scenario 3. Price increases receive a diminished weight, whereas price declines receive an inflated weight. As a result, downward chain drift arises, indicated by the downward pointing arrow in Figure 4.

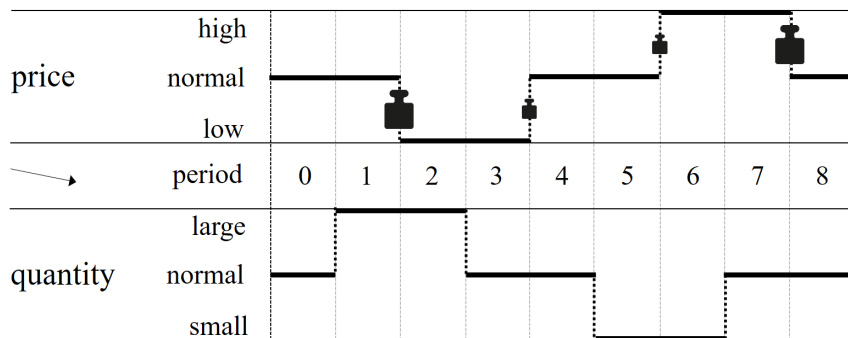


Figure 4: Quantity Reactions to Price Changes Are Antedated By One Period (Scenario 4).

In summary, sticky quantities lead to upward chain drift, whereas pendular

quantities lead to downward chain drift. The underlying problem of both cases is the asymmetric weighting of price increases and price reductions. This asymmetry is facilitated by quantity changes in times of constant prices. Since all SWAP indices satisfy the identity test, they indicate in such periods no overall price change. This suggests that a solution to the chain drift problem may come from price indices that violate the identity test. The RGEKS indices studied in Sections 6 and 7 are a prominent example. They violate the identity test.⁹ This could be interpreted as a weakness of the RGEKS indices. Alternatively, the fact that immunity to chain drift requires price indices that violate the identity test, can be interpreted as a weakness of the identity test.¹⁰

Since pendular and sticky quantities work in opposite directions, scenarios may exist in which they neutralize each other and no chain drift arises. In such scenarios, a price index that avoids chain drift from one source but not from the other, may cause larger chain drift than a price index which suffers from both sources of chain drift.

4 Formal Analysis

In this section we present a more formal analysis of the chain drift problem. We first derive a very general result that helps to identify the direction of chain drift of SWAP indices. Then we apply this general result to prove that these price indices suffer from chain drift when price and quantity changes are not fully synchronized. Since the Törnqvist index is a SWAP index and generates index numbers that are usually very close to those of the Fisher index (Diewert, 2018, pp. 16-17), the chain drift related to SWAP indices applies also to the Fisher index.

4.1 Decomposing the Relative Divergence of Two Weighted Arithmetic Means of the Same Variable

Bortkiewicz (1923, pp. 374-376) decomposes the relative divergence between the Laspeyres and Paasche index into three factors, namely the coefficient of variation of the price relatives, the coefficient of variation of the quantity relatives, and the coefficient of linear correlation between the price and quantity relatives. Szulc (1983, pp. 563-564) demonstrates that this decomposition can be generalized to any two indices that can be represented as weighted arithmetic means of the same variable, but weighted with alternative sets of weights. Consider some vector $\mathbf{r} = (r_1, r_2, \dots, r_N)$

⁹This is noted also by Ribe (2012, p. 4).

¹⁰In ILO (2004, p. 293) it is reported that the identity test is somewhat controversial. A more elaborate critique of the identity test can be found in Auer (2008, pp. 2-7).

and two weighted arithmetic means of this vector's elements,

$$I_A = \sum r_i \frac{a_i}{\sum a_j} \quad \text{and} \quad I_B = \sum r_i \frac{b_i}{\sum b_j}, \quad (7)$$

where the quotients $a_i/\sum a_j$ and $b_i/\sum b_j$ ($i, j = 1, \dots, N$) represent the different sets of weights. Defining $z_i = b_i/a_i$, we get

$$\begin{aligned} \frac{I_B - I_A}{I_A} &= \frac{\sum r_i \frac{z_i a_i}{z_j a_j} - \sum r_i \frac{a_i}{\sum a_j}}{\sum r_i \frac{a_i}{\sum a_j}} = \frac{\frac{\sum z_i a_i}{\sum a_j}}{\frac{\sum z_i a_i}{\sum a_j}} \frac{\sum r_i \frac{z_i a_i}{z_j a_j} - \sum r_i \frac{a_i}{\sum a_j}}{\sum r_i \frac{a_i}{\sum a_j}} \\ &= \frac{\sum r_i z_i \frac{a_i}{\sum a_j} - \sum r_i \frac{a_i}{\sum a_j} \sum z_i \frac{a_i}{\sum a_j}}{\sum r_i \frac{a_i}{\sum a_j} \sum z_i \frac{a_i}{\sum a_j}}. \end{aligned} \quad (8)$$

Defining

$$\bar{r} = \sum r_i \frac{a_i}{\sum a_j} \quad \text{and} \quad \bar{z} = \sum z_i \frac{a_i}{\sum a_j}, \quad (9)$$

the numerator in (8) can be re-written as

$$\begin{aligned} \sum r_i z_i \frac{a_i}{\sum a_j} - \bar{r} \bar{z} &= \sum \frac{a_i}{\sum a_j} r_i z_i - \bar{r} \bar{z} - \bar{r} \bar{z} + \sum \frac{a_i}{\sum a_j} \bar{r} \bar{z} \\ &= \sum \frac{a_i}{\sum a_j} r_i z_i - \sum \frac{a_i}{\sum a_j} r_i \bar{z} - \sum \frac{a_i}{\sum a_j} z_i \bar{r} + \sum \frac{a_i}{\sum a_j} \bar{r} \bar{z} \\ &= \sum \frac{a_i}{\sum a_j} (r_i - \bar{r}) (z_i - \bar{z}) =: cov_{rz}. \end{aligned} \quad (10)$$

Inserting (9) and (10) in (8), we get

$$\frac{I_B - I_A}{I_A} = \frac{cov_{rz}}{\bar{r} \bar{z}} = \frac{\sigma_r}{\bar{r}} \cdot \frac{\sigma_z}{\bar{z}} \cdot \frac{cov_{rz}}{\sigma_r \sigma_z} \quad (11)$$

with

$$\sigma_r = \sqrt{\sum \frac{a_i}{\sum a_j} (r_i - \bar{r})^2} \quad \text{and} \quad \sigma_z = \sqrt{\sum \frac{a_i}{\sum a_j} (z_i - \bar{z})^2}.$$

The first two quotients on the right hand side of (11) are the coefficients of variation of r_i and z_i , respectively. The third quotient is the coefficient of linear correlation between r_i and z_i . The weighted averages I_A and I_B produce identical numbers, if the values of z_i or r_i are identical for all items, or if r_i and z_i are linearly uncorrelated. Choosing $r_i = p_i^T/p_i^0$ or $r_i = \ln(p_i^T/p_i^0)$, Equations (10) and (11) can be utilized to prove that SWAP indices exhibit chain drift.

4.2 Chain Drift of SWAP Indices

Consider some general scenario A in which the prices and quantities of period T return to their values of period 0:

$$A = (\mathbf{p}^0, \mathbf{x}^0, \mathbf{p}^1, \mathbf{x}^1, \dots, \mathbf{p}^{T-1}, \mathbf{x}^{T-1}, \mathbf{p}^T, \mathbf{x}^T), \quad \text{with } \mathbf{p}^0 = \mathbf{p}^T \text{ and } \mathbf{x}^0 = \mathbf{x}^T. \quad (12)$$

The alternative scenario B differs from A only with respect to the quantities (not prices!) observed during the in-between periods, $t = 1, 2, \dots, T - 1$:

$$B = (\mathbf{p}^0, \mathbf{x}^0, \mathbf{p}^1, \check{\mathbf{x}}^1, \dots, \mathbf{p}^{T-1}, \check{\mathbf{x}}^{T-1}, \mathbf{p}^T, \mathbf{x}^T) . \quad (13)$$

The chained SWAP indices relating to scenarios A and B are denoted by $\tilde{I}_A^{T/0} = I_A^{1/0} \cdot I_A^{2/1} \cdot \dots \cdot I_A^{T/T-1}$ and $\tilde{I}_B^{T/0} = I_B^{1/0} \cdot I_B^{2/1} \cdot \dots \cdot I_B^{T/T-1}$, respectively.

Lemma 1 *A SWAP index, $I^{t/t-1}$, suffers from chain drift, if some pair of scenarios A and B defined by (12) and (13) exists, such that $\tilde{I}_A^{T/0} \neq \tilde{I}_B^{T/0}$.*

Proof: According to Definition 2, immunity to chain drift requires that $\tilde{I}_A^{T/0} = 1$. Since scenario B differs from scenario A only with respect to the quantities relating to the in-between periods ($t = 1, 2, \dots, T - 1$), immunity to chain drift also requires that $\tilde{I}_B^{T/0} = 1$. When $\tilde{I}_A^{T/0} \neq \tilde{I}_B^{T/0}$, the postulate $\tilde{I}_A^{T/0} = 1$ or the postulate $\tilde{I}_B^{T/0} = 1$ or both postulates are violated. ■

Theorem 1 *No SWAP index, $I^{t/t-1}$, is immune to chain drift.*

Proof: It suffices to identify a scenario in which all SWAP indices exhibit chain drift. Consider the time interval $t = 0, \dots, 4$ and suppose that in scenarios A and B the price evolution of item 1 is as in Figure 1 (which is identical to the price evolution in Figure 3), while the prices of all other items are constant, $p_i^t = \bar{p}_i$ ($i = 2, \dots, N$ and $t = 0, \dots, 4$). Then, $I_A^{2/1} = I_A^{4/3} = 1$ and, therefore, $\tilde{I}_A^{4/0} = I_A^{1/0} \cdot I_A^{3/2}$. Analogously, $\tilde{I}_B^{4/0} = I_B^{1/0} \cdot I_B^{3/2}$. Suppose that in scenario A the direction of the quantity changes of item 1 is as in Figure 1, that is, price and quantity changes are perfectly synchronized. Thus, every chained SWAP index yields $\tilde{I}_A^{4/0} = 1$. Suppose that in scenario B the quantities respond as in Figure 3, that is, they do not react to current price changes, but to price changes of the previous period. Since scenarios A and B satisfy (12) and (13), Lemma 1 can be invoked. It remains to be shown that $\tilde{I}_B^{4/0} \neq 1$.

Consider some SWAP index measuring the average price change between periods $t - 1$ and t . Let $z_i^{t/t-1}$ denote the ratio of item i 's weight in scenario B relative to its weight in scenario A and define $r_i^{t/t-1} = p_i^t / p_i^{t-1}$ ($i = 1, \dots, N$). In scenario A , the price reduction of item 1 in period 1, $r_1^{1/0} < 1$, leads to an immediate increase of the consumed quantity of this item. In scenario B , the quantity increase occurs one period delayed. Equations (5) imply that this delay reduces the weight of item 1 in period 1 *relative to scenario A*, $z_1^{1/0} < 1$. Conversely, in scenario A the price increase in period 3, $r_1^{3/2} > 1$, leads to an immediate quantity reduction, while the delay of this reduction occurring in scenario B increases the weight *relative to scenario A*, $z_1^{3/2} > 1$. Therefore, we have $cov_{rz}^{1/0} > 0$ as well as $cov_{rz}^{3/2} > 0$ and Equation (11) yields $I_B^{1/0} > I_A^{1/0}$ and $I_B^{3/2} > I_A^{3/2}$. Consequently, $\tilde{I}_B^{4/0} > \tilde{I}_A^{4/0} = 1$, implying upward chain drift in scenario B . ■

Instead of the sticky quantities of scenario B one could have used pendular quantities. This would have proved that SWAP indices suffer also from downward chain drift. Diewert (2018, pp. 16-17) shows that the Törnqvist and Fisher index approximate each other very closely. Therefore, also the Fisher index suffers from chain drift. Auer (2014, pp. 848-850) defines the class of Generalized Unit Value (GUV) indices (e.g., Davies, Banerjee, Lehr index). Since these indices are derived from the unit value index and this index is free of chain drift, one may hope that also these GUV indices are immune to chain drift. The stress test presented in Section 7 demonstrates that this is not the case.

5 Implications for the Analysis of Scanner Data

Storing one's favourite beer when it is on sale may reduce further purchases of beer for some weeks or months but probably not for years. Delayed quantity reactions are also rather a question of weeks than years. This implies that chaining of monthly price indices is more likely to cause chain drift than chaining of yearly price indices. Since scanner data allow for the compilation of monthly or even weekly price indices, the issue of chain drift becomes particularly relevant for this type of data source.

As a solution, Ivancic *et al.* (2011) propose a rolling window variant of the Gini-Éltető-Köves-Szulc (RGEKS) approach.¹¹ Their RGEKS index is based on the bilateral Fisher index, but could use also other direct indices such as the Törnqvist index (e.g., de Haan and van der Grient, 2011).

Results reported in Melser (2018, pp. 518-521) demonstrate that the choice of the linking procedure affects the results of RGEKS indices. Figure 5 illustrates the RGEKS approach and the various linking methods proposed in the literature. The illustration utilizes a window length of four periods. Period t is the current period. The upper grey bar represents the current window, covering periods $t - 3$ to t . The lower grey bar highlights the window associated with the previous period ($t - 1$). This old window covers the periods $t - 4$ to $t - 1$. The filled circle between the grey bars indicates the price level of period $t - 1$: P^{t-1} . This is the reference for the price level to be computed in period t : P^t . The latter is represented by the circle at the right hand side of the diagram.

Applying the GEKS index to the information contained in the current window, $[t - 3, t]$, generates for all pairs of periods (t' and t'') included in this window a price ratio $P_{[t-3,t]}^{t'/t''}$, where the superscript indicates the periods that are compared (comparison period t' , base period t''), while the subscript documents the window

¹¹The acronym GEKS honours the publications of Gini (1924), Éltető and Köves (1964), and Szulc (1964) who introduced this approach for interregional price comparisons. Balk (1981, pp. 73-74) adopts this approach to the intertemporal price measurement of seasonal products. That multilateral price indices can curtail chain drift is pointed out also in Kokoski *et al.* (1999, p. 141).

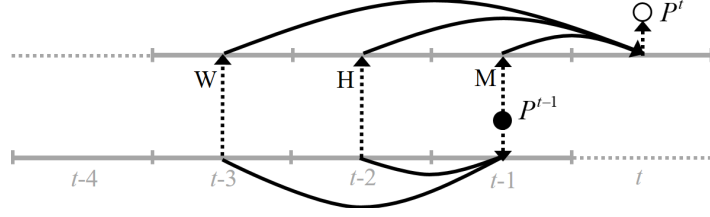


Figure 5: RGEKS Indices and Four Different Methods of Splicing.

from which the result is computed. The GEKS index computes the price ratio $P_{[t-3,t]}^{t'/t''}$ as the geometric average of all possible pairs of bilateral price indices that link the periods t'' and t' via some link period s included in the window:

$$P_{[t-3,t]}^{t'/t''} = \prod_{s=t-3}^t \left(P^{t'/s} P^{s/t''} \right)^{1/4}. \quad (14)$$

This averaging principle is applied for each pair of periods covered by the window $[t-3, t]$. The computed ratios are transitive, that is, $P_{[t-3,t]}^{t/t-2} = P_{[t-3,t]}^{t/t-1} P_{[t-3,t]}^{t-1/t-2}$.

To obtain the new price level P^t , the results of the current window must be linked to the price level P^{t-1} that was computed from the previous window. Several methods have been proposed for this linking procedure (denoted as “splicing”). They are also illustrated in Figure 5.

Ivancic *et al.* (2011) propose a “movement splice” (indicated by the “M” in Figure 5). This approach multiplies the price level P^{t-1} by the price change between periods t and $t-1$ computed from the current window:

$$P^t = P^{t-1} P_{[t-3,t]}^{t/(t-1)} \quad (\text{movement splice}). \quad (15)$$

As an alternative, Krsinich (2016, pp. 383-87) suggests the “window splice”.¹² This approach (“W” in Figure 5) uses the price ratio $P_{[t-4,t-1]}^{(t-3)/(t-1)}$ compiled by the old window to get backward from the price level P^{t-1} to the price level of period $t-3$, the first period of the current window. The result of the backward computation is multiplied by the price change between periods $t-3$ and t as compiled by the current window:

$$P^t = P^{t-1} P_{[t-4,t-1]}^{(t-3)/(t-1)} P_{[t-3,t]}^{t/(t-3)} \quad (\text{window splice}).$$

“Half splice” is another option (“H” in Figure 5). The idea and its justification is due to de Haan (2015, pp. 25-26).¹³ Here, the period linking the current and

¹²She proposes this variant in the context of a time-product dummy index which is an alternative to the RGEKS approach.

¹³He makes this proposal in the context of a time-product dummy index.

the new window is period $t - 2$ which is half way between the linking periods of movement splice (period $t - 1$) and window splice (period $t - 3$):

$$P^t = P^{t-1} P_{[t-4,t-1]}^{(t-2)/(t-1)} P_{[t-3,t]}^{t/(t-2)} \quad (\text{half splice}) .$$

Diewert and Fox (2017, pp. 17-18) introduce yet another splicing variant which they call “mean splice”. They compute the geometric average of the results obtained from the window, movement, and half splice:

$$P^t = P^{t-1} \left[P_{[t-3,t]}^{t/(t-1)} \left(P_{[t-4,t-1]}^{(t-3)/(t-1)} P_{[t-3,t]}^{t/(t-3)} \right) \left(P_{[t-4,t-1]}^{(t-2)/(t-1)} P_{[t-3,t]}^{t/(t-2)} \right) \right]^{1/3} \quad (\text{mean splice}) .$$

When the window is longer than four periods, also the results compiled from all additional possible linking periods are included in this geometric average.

Melser (2018, p. 518) develops a “mean movement splice” RGEKS approach that is illustrated in Figure 6. Like the mean splice it relies on several links, but the computational procedure is much simpler.¹⁴

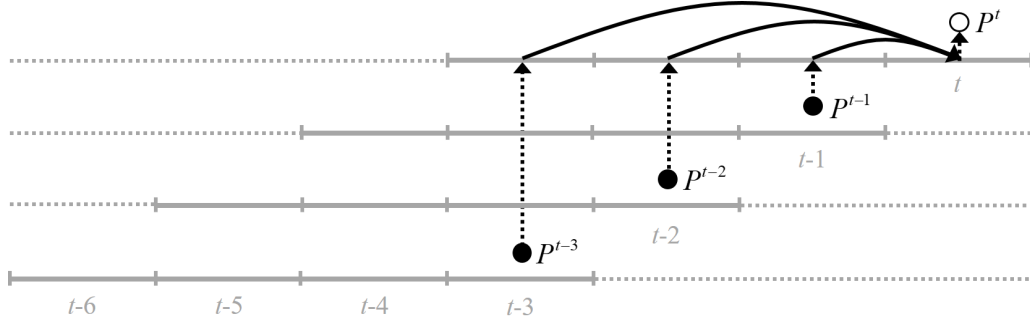


Figure 6: The Mean Movement RGEKS Index.

To obtain the current price level P^t , the mean movement RGEKS approach links the results from the current window not only to the price level P^{t-1} computed from the previous window, but also to the price levels P^{t-2} and P^{t-3} that were derived from the two preceding windows (in Figure 6 the filled black circles).¹⁵ The links to the formerly computed price levels P^{t-1} , P^{t-2} , and P^{t-3} follow the basic principle of the movement splice, that is, no backward computation from the current period to some previous period is involved. The price levels of these previous periods are directly taken from the results of the preceding windows. More specifically, the formerly computed price level P^{t-3} is multiplied by the bilateral price index $P_{[t-3,t]}^{t/(t-3)}$ obtained from the current window. Analogously, P^{t-2} is multiplied by $P_{[t-3,t]}^{t/(t-2)}$, and

¹⁴Melser (2018, p. 517) also suggests to replace the GEKS index by a weighted variant of the GEKS index. We do not explore this issue in the present paper and focus on the linking procedures.

¹⁵Chessa (2017, p. 16) applies a similar idea in the context of a Geary-Khamis index. He denotes his index as the “revisionless QU-GK method”.

P^{t-1} is multiplied by $P_{[t-3,t]}^{t/(t-1)}$. In a final step, a geometric average of these three products is computed:

$$P^t = \left[\left(P^{t-3} P_{[t-3,t]}^{t/(t-3)} \right) \left(P^{t-2} P_{[t-3,t]}^{t/(t-2)} \right) \left(P^{t-1} P_{[t-3,t]}^{t/(t-1)} \right) \right]^{1/3} .$$

The diversity of RGEKS variants raises the question whether some variant is more immune to chain drift than others. Can the mean movement RGEKS index advocated by Melser (2018) stand up to the other RGEKS indices? To provide a tentative answer, we consider two different highly stylized scenarios generating chain drift (see Figures 7 and 8). All items not depicted in the figures have constant prices during the complete time horizon.

We begin with the case of pendular quantities and assume that a Törnqvist index is applied. In Figure 7 price changes occur only from period 2 to period 3 and from period 3 to period 4. Therefore, the chained Törnqvist index yields

$$\tilde{P}^{8/0} = P^{1/0} \cdot P^{2/1} \cdot \dots \cdot P^{8/7} = P^{3/2} \cdot P^{4/3} < 1 . \quad (16)$$

This says that the price level in the final period is lower than in the first period, even though in period 8 all prices and quantities have returned to their levels of period 0. The downward chain drift arises from the stronger weight attached to the price decline between periods 2 and 3 as compared to the price increase between periods 3 and 4.

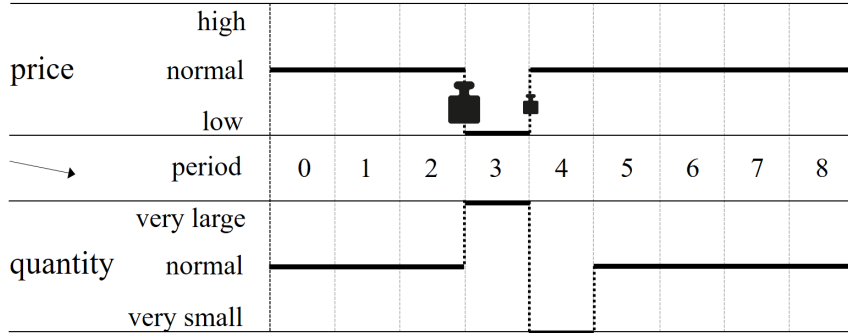


Figure 7: A Simple Scenario of Pendular Quantities.

It is straightforward but tedious to derive the final price level, P^8 , generated by the various RGEKS variants (with $P^0 = 1$). One obtains

$$\text{movement:} \quad P^8 = \left(P^{3/2} P^{4/3} \right)^{1/4} < 1 \quad (17)$$

$$\text{half:} \quad P^8 = 1 \quad (18)$$

$$\text{window:} \quad P^8 = \left(P^{3/2} P^{4/3} \right)^{1/4} < 1 \quad (19)$$

$$\text{mean:} \quad P^8 = \left(P^{3/2} P^{4/3} \right)^{1/6} < 1 \quad (20)$$

$$\text{mean movement:} \quad P^8 = \left(P^{3/2} P^{4/3} \right)^{37/243} < 1 . \quad (21)$$

Comparing result (16) to results (17) to (21) clearly confirms what advocates of RGEKS indices have emphasized before: multilateral price indices considerably reduce the chain drift arising from pendular quantities. In the simple scenario of Figure 7, the half splice RGEKS even eliminates this type of chain drift. The mean movement splice RGEKS is extremely close to the mean splice RGEKS.

As pointed out before, the reduction in chain drift comes with a violation of the identity test. Between periods 4 and 5 no price change occurs, such that the direct price index is $P^{5/4} = 1$. However, the price change as compiled by the half splice RGEKS index, say, is

$$\frac{P^5}{P^4} = (P^{2/3} P^{3/4})^{1/4} > 1 .$$

Can RGEKS indices also reduce or even eliminate chain drift caused by sticky quantities? This question has not been addressed in the literature, because sticky quantities have not been identified as a possible cause of chain drift. Figure 8 depicts a simple scenario of sticky quantities. For this scenario, the chained Törnqvist index yields

$$\tilde{P}^{8/0} = P^{1/0} \cdot P^{2/1} \cdot \dots \cdot P^{8/7} = P^{3/2} \cdot P^{5/4} > 1 .$$

In contrast to pendular quantities, sticky quantities generate upward chain drift, because the price decline between periods 2 and 3 receives a smaller weight than the price increase between periods 4 and 5.

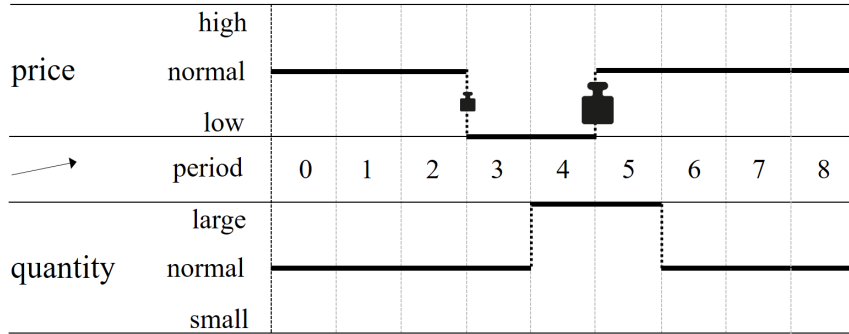


Figure 8: A Simple Scenario of Sticky Quantities.

The final price levels compiled by the various RGEKS indices are

$$\begin{aligned} \text{movement:} & \quad P^8 = (P^{3/2} P^{5/4})^{1/4} > 1 \\ \text{half:} & \quad P^8 = (P^{3/2} P^{5/3})^{1/2} > 1 \\ \text{window:} & \quad P^8 = (P^{3/2} P^{5/4})^{1/4} > 1 \\ \text{mean:} & \quad P^8 = (P^{3/2})^{1/3} (P^{5/3} P^{5/4})^{1/6} > 1 \\ \text{mean movement:} & \quad P^8 = \left[(P^{3/2})^{85/81} (P^{4/1})^{11/81} (P^{5/3} P^{5/4})^{16/27} \right]^{1/3} > 1 . \end{aligned}$$

None of the splicing variants eliminates this type of chain drift, but all of them reduce its extent. In the context of sticky quantities, the half splice is worse than the other variants. Numerically, the mean movement splice is again very close to the mean splice.

Though the scenarios depicted in Figures 7 and 8 are too stylized to draw general conclusions from them, the results suggest to investigate some aspects in more detail. Does the almost perfect match of the mean splice and the mean movement splice carry over to more complex scenarios? Is in such scenarios the half splice still very effective in eliminating chain drift caused by pendular quantities, but less effective in eliminating chain drift caused by sticky quantities?

There is an additional issue that must be addressed. The data set of Feenstra and Shapiro (2003) reveals that pendular and sticky quantities are not mutually exclusive, but can jointly arise. However, the cycle lengths of pendular quantities and sticky quantities are likely to differ. Therefore, a window length that is suitable for addressing pendular quantities is unlikely to be adequate also for addressing sticky quantities. In other words, a one-size-fits-all window may not exist. In such a situation, should one apply a shorter or rather a longer window length? Perform some RGEKS index variants better than others?

To address such questions, more general scenarios should be investigated. The usual approach is to analyze some scanner data set. The problem with such real world data sets is that the prices and quantities do not return to their base period levels. Therefore, no unassailable reference exists for assessing the extent of chain drift inherent in the various index formulas. To have some unassailable reference, we pursue a different approach. We investigate artificial price-quantity scenarios. In contrast to the scenarios in Figure 7 and 8 and the artificial scenarios studied in the existing literature, however, the quantities in our scenarios are derived from principles of utility maximization. For this purpose, the following section introduces a general but tractable utility function that is consistent with pendular quantities as well as with sticky quantities.¹⁶

6 A General Utility Function

Triplett (2003, p. 152) points out that the analysis of high-frequency data such as scanner data is an analysis of acquisitions and not of consumption: “To confront the household behavior recorded in high-frequency data requires a theory that adequately describes search, storage, shopping, and other household activities that

¹⁶Diewert and Fox (2017, p. 41) assume that the quantities are generated from CES preferences. Since these CES preferences are not compatible with pendular or sticky quantities, the authors manually adjust some quantities such that they reflect a pattern of pendular quantities. The issue of sticky quantities is not considered.

drive a wedge between acquisition periodicity and consumption periodicity.” Taking up the challenge, Feenstra and Shapiro (2003, pp. 126-130) develop a model of utility maximization that captures stocking behaviour, though not sticky quantities. In their model, consumers have perfect foresight of future prices.

The present paper follows a completely different approach. Since scanner data items such as detergents can be interpreted as differentiated goods, we use a myopic Dixit-Stiglitz CES utility function and amend it to allow for sticky and pendular quantities. In this model the consumers are not concerned with the future. Stocking directly contributes to current utility, while strong deviations from former purchasing behaviour cause “disutility” (e.g., search or adjustment costs). This disutility is captured by a feature that is borrowed from habit formation models. This feature penalizes strong volatility of consumption. As a consequence, price changes trigger quantity changes that are distributed over several periods.

Consider some period t and some representative household endowed with income m and stocks \bar{s}_i ($i = 1, \dots, N$) inherited from period $t - 1$. The budget constraint of the household in period t is

$$m + \sum_{i=1}^N p_i \bar{s}_i = \sum_{i=1}^N p_i c_i + \sum_{i=1}^N p_i s_i, \quad (22)$$

where p_i is the price of item i during period t , c_i denotes the consumed quantity of item i during period t , and s_i is the stock of item i retained for period $t + 1$. For simplicity, it is assumed that, if the household wishes, it can sell in period t units of the inherited stock \bar{s}_i at the price p_i . The purchased quantity of item i in period t is

$$x_i = c_i + s_i - \bar{s}_i. \quad (23)$$

Assume that the household’s utility function is given by

$$u = C^\alpha S^{1-\alpha} \quad (24)$$

where $0 < \alpha \leq 1$ and

$$C = \left[\sum_{i=1}^N (c_i - \gamma \bar{c}_i)^\theta \right]^{1/\theta} \quad (25)$$

$$S = \left[\sum_{i=1}^N (s_i)^\phi \right]^{1/\phi}, \quad (26)$$

with θ and ϕ being smaller than unity, \bar{c}_i being the consumption of item i in period $t - 1$, and $0 \leq \gamma < 1$ being a stability condition. One can show that $-1/(1 - \theta)$ and $-1/(1 - \phi)$ approximate the price elasticities of demand and of stocking, respectively. Therefore, it is reasonable (though not necessary for the derivation of the results) to assume that $\theta < \phi$. The larger ϕ , the less important it is for the consumer

which specific item is stored. The utility function (24) with its subutilities defined by (25) and (26) generates pendular quantities caused by stocking behaviour (s_i and \bar{s}_i) and sticky quantities caused by “habit formation” (\bar{c}_i).

In Appendix B it is shown that the maximisation of utility function (24) subject to the constraint (22) yields the following purchasing behaviour:

$$x_i^* = \underbrace{\gamma \bar{c}_i + \alpha \bar{m} p_i^{\frac{-1}{1-\theta}} P_C^{\frac{\theta}{1-\theta}}}_{c_i^*} + \underbrace{(1-\alpha) \bar{m} p_i^{\frac{-1}{1-\phi}} P_S^{\frac{\phi}{1-\phi}}}_{s_i^*} - \bar{s}_i, \quad (27)$$

with

$$P_C = \left(\sum_{i=1}^N p_i^{\frac{-\theta}{1-\theta}} \right)^{-\frac{(1-\theta)}{\theta}} \quad \text{and} \quad P_S = \left(\sum_{j=1}^N p_j^{\frac{-\phi}{1-\phi}} \right)^{-\frac{(1-\phi)}{\phi}}$$

denoting price indices of consumption and of stocking, respectively, and

$$\bar{m} = m + \sum_{i=1}^N p_i (\bar{s}_i - \gamma \bar{c}_i)$$

representing the consumer’s “net endowment”. The variables x_i^* , c_i^* and s_i^* in (27) are the optimal values of the variables x_i , c_i , and s_i in Equation (23).¹⁷

The demand function (27) can be used to compute for any price scenario (and income m , stocks \bar{s}_i , and former consumption \bar{c}_i) the corresponding quantities purchased by utility maximizing consumers. These quantities depend on the parameters of the utility function. Choosing $\alpha = 1$ and $\gamma = 0$ yields consumers that neither undertake stocking ($\bar{s}_i = 0$) behaviour nor are subjected to habit formation. Therefore, neither pendular nor sticky quantities arise. The demand function (27) simplifies to

$$x_i^* = m p_i^{\frac{-1}{1-\theta}} P_C^{\frac{\theta}{1-\theta}}.$$

Stocking without habit formation arises for $\alpha < 1$ and $\gamma = 0$. In this case, the demand function (27) simplifies to

$$x_i^* = \bar{m} p_i^{\frac{-1}{1-\theta}} \left(\alpha P_C^{\frac{\theta}{1-\theta}} + (1-\alpha) P_S^{\frac{\phi}{1-\phi}} \right) - \bar{s}_i.$$

Habit formation without stocking arises for $\alpha = 1$ and $\gamma > 0$. The resulting purchases are

$$x_i^* = \gamma \bar{c}_i + \bar{m} p_i^{\frac{-1}{1-\theta}} P_C^{\frac{\theta}{1-\theta}}.$$

In the next section, the demand function (27) is used to compile the quantities corresponding to price scenarios that can give rise to pendular and/or sticky quantities.

¹⁷If each item i got its own habit formation parameter γ_i , this parameter would replace γ in Equation (27). Merely the steady state solution (see Appendix B) would be more complex. Individualizing also the stocking behaviour (α_i instead of α) leads to considerably more cumbersome terms.

7 Simulations and Implications

The simulation comprises $T = 120$ periods ($t = 1, 2, \dots, 120$). In each period, the representative consumer can choose among the same $N = 40$ items ($i = 1, \dots, 40$). Each item receives a randomly drawn “base price” between 2.00 and 5.00. The simulation starts with a phase-in interval of 10 periods during which the prices of all N items are sold at their base price. This interval is followed by a core interval of 100 periods in which some randomly drawn items exhibit price changes triggering quantity reactions. Three different price scenarios are considered. The first relates to *pendular* quantities caused by sales, the second to *sticky* quantities caused by search and adjustment cost, and the third is a combination of the first two scenarios (*hybrid*). After the core interval, there is a phase-out interval of 10 periods in which all items are sold at the base price. Therefore, all quantities return to the levels that had prevailed at the start of the phase-in interval.

The quantities corresponding to these three price scenarios are derived from the demand function (27), where the elasticity parameters are set to $\theta = 0.5$ and $\phi = 0.6$ and the endowment to $m = 1000$. The \bar{c}_i -levels (former consumption) and \bar{s}_i -levels (stocks inherited from the preceding period) relevant for period 1 are set to the steady-state equilibrium values corresponding to the base prices.¹⁸

The three price scenarios differ with respect to the prices during the core interval and with respect to the parameters α (importance of consumption relative to stocking) and γ (relevance of past consumption for present consumption decisions). More specifically, the three price scenarios are defined in the following way:

Pendular: During the core interval of this scenario, a quarter of the items exhibits cyclical sales patterns, while all other items keep their base price throughout the complete time horizon. Every cycle starts with a price reduction. In the next period the price returns to its basic level and stays there until the next cycle starts. Each of the sales items has its own fixed price reduction (randomly drawn between 10% and 40%) and its own fixed cycle length (randomly drawn between 4 and 12 periods). Also the period of the start of the first cycle differs between the items. It occurs within the first ten periods of the core interval and is randomly drawn. The parameter values determining the relevance of stocking and adjustment costs are set to $\alpha = 0.8$ and $\gamma = 0$.

Sticky: Half of the items exhibit price changes during the core interval.¹⁹ On average, the price of such an item changes every tenth period. The periods of change are randomly drawn and the new price can deviate from the base

¹⁸These values are computed from the steady state formulas derived in Appendix B.

¹⁹These items are selected from the subset that was not subject to price changes in the pendular quantities scenario.

price by a percentage drawn from the interval $[-25\% , +25\%]$. The parameter values determining the relevance of stocking and adjustment costs are set to $\alpha = 1$ and $\gamma = 0.3$.

The scanner data analyzed by Feenstra and Shapiro (2003, p. 130) show that sales and sticky quantities are not mutually exclusive. This suggests to simulate a scenario in which both, pendular and sticky quantities are present. As in the other two scenarios, the quantities are computed from demand function (27).

Hybrid: The price movements of the pendular quantities scenario (10 items) and the sticky quantities scenario (20 other items) are combined. The 10 remaining items have constant prices during the complete time interval. The parameter values determining the relevance of stocking and adjustment costs are set to $\alpha = 0.8$ and $\gamma = 0.3$.

For all three price scenarios, the index numbers of the chained bilateral SWAP indices, the other chained bilateral indices, and the various RGEKS variants are compiled. The price level of period $t = 1$ is set equal to 100 and the extent of chain drift is measured as the deviation of the price level in period $t = 120$ from 100.

Table 1 shows that chained direct SWAP indices generate the expected results. Pendular quantities lead to considerable downward chain drift, while sticky quantities generate considerable upward chain drift.²⁰ In the pendular quantities scenario, the Törnqvist index reaches 25.15% downward chain drift, while the Walsh-Vartia index exhibits 37.78% downward chain drift. In the sticky quantities scenario the variation of the chain drift is much smaller. All listed SWAP indices show an upward chain drift of roughly 7.5%. The results of the hybrid scenario sit somewhere between the other two scenarios.

Table 1: Chain Drift of SWAP Indices (in %).

	Pendular	Sticky	Hybrid
Törnqvist	-25.15	7.41	-1.62
Walsh-2	-37.69	7.52	-2.29
Walsh-Vartia	-37.78	7.53	-2.31
Theil	-34.19	7.49	-2.07
Vartia	-33.93	7.49	-2.08

²⁰The prices and quantities from which these index numbers have been computed are documented in Tables 7 to 10 in Appendix C.

Table 2 documents the chain drift of the other chained price indices mentioned in the paper. Again, the pendular quantities scenario generates a large downward chain drift that varies between the indices. The upward chain drift arising from sticky quantities is similar to that of the SWAP indices. Only the Laspeyres index and the Paasche index do not fit into this overall picture. The Laspeyres index exhibits massive upward chain drift, while the Paasche index shows considerable downward chain drift.²¹

Table 2: Chain Drift of Some Other Direct Price Indices (in %).

	Pendular	Sticky	Hybrid
Laspeyres	468.94	29.76	337.07
Paasche	-88.94	-11.53	-77.68
Fisher	-20.66	7.15	-1.24
Drobisch	-18.48	7.19	0.22
Walsh	-37.87	7.53	-2.33
Marshall-Edgeworth	-20.70	7.15	-1.28
Banerjee	-20.70	7.15	-1.28
Davies	-22.80	7.28	-1.40
Lehr	-44.79	7.88	-3.10

RGEKS indices are expected to remove the chain drift bias arising from pendular quantities. The performance of the various RGEKS indices, however, may depend on the choice of the linking method and the window length (e.g., Melser, 2018, p. 517). Therefore, we examine different linking methods and different window lengths. Table 3 presents the results for various linking methods and for window lengths of 4, 8, 12, and 24 periods. The results of many other window lengths and linking options are documented in Tables 4, 5, and 6 in Appendix C.

Overall, the RGEKS approach with a sufficiently large window effectively curtails chain drift bias. On average, it is more successful in the context of pendular quantities than in the context of sticky quantities. In the scenario of pendular quantities, the variation between the linking methods (variation within a column) is less monotone than in the sticky quantities scenario. In particular, the deviation between movement splice and window splice is striking. This reinforces the arguments of Diewert and Fox (2017, pp. 17-18) and de Haan (2015, pp. 25-26) in favour of a more “balanced” approach such as the mean splice or the half splice. The half splice is particularly effective in the removal of chain drift caused by pendular quantities.

²¹Note that these results related to the Laspeyres and Paasche index are not unexpected, because the elasticity of demand is larger than unity.

Table 3: Chain Drift of RGEKS Indices (in %).

	Pendular				Sticky				Hybrid			
	4	8	12	24	4	8	12	24	4	8	12	24
Mean Move.	-2.10	0.32	0.02	0.12	3.93	1.77	1.12	0.49	2.52	1.29	0.95	0.54
Mean	-2.10	0.32	0.03	0.11	3.93	1.77	1.12	0.49	2.52	1.29	0.94	0.53
Movement	-5.05	-1.57	-2.09	-0.49	3.79	1.57	1.11	0.42	0.89	0.50	0.46	0.26
Half	0.04	0.31	0.67	0	4.08	1.83	0.87	0.43	3.35	0.75	0.32	0.34
Window	-1.22	2.50	1.69	3.03	3.91	2.16	2.20	2.65	3.33	4.32	5.05	6.68

The RGEKS index numbers arising from the hybrid scenario are listed on the right hand side of Table 3.²² Since for large windows the bias arising in the sticky quantities scenario is more persistent than that related to the pendular quantities scenario, the bias of the hybrid scenario resembles that of the sticky quantities scenario.

The results also show that the mean movement splice RGEKS index produces virtually the same numbers as the mean splice RGEKS. Since the computational burden of the mean movement splice RGEKS index is much smaller, it is an attractive option for the analysis of scanner data.

Except for the window splice, the chain drift bias tends to shrink, as the applied window expands. However, it should be kept in mind that in the applied simulation no item attrition occurs. With item attrition, large windows may inject a dose of assortment bias (e.g., Auer, 2018, p. 84).²³

A large battery of robustness checks has been conducted for the three scenarios. As expected, the chain drift bias increases with γ , the parameter driving the extent of stickiness in the quantities. The qualitative results, however, remain untouched. The parameter α determines the relevance of consumption relative to stocking. Increasing α reduces the relevance of stocking and, therefore, the bias arising from pendular quantities.²⁴ Again, the qualitative results are not affected. Also reducing the elasticity parameters θ and ϕ affect the level of chain drift, but not the qualitative conclusions. This is also true when the sales occur randomly rather than in a cyclical manner or when sales can last for more than one period.

²²The underlying price and quantity data are documented in Tables 11 and 12 in Appendix C.

²³In this context, Melser's (2018, p. 517) recommendation of weighting pairs of periods according to their similarity becomes relevant.

²⁴If α is lowered too much, while the elasticity parameter ϕ is kept on its level, negative purchases can arise. Therefore, we did not investigate such scenarios.

8 Concluding Remarks

Sales and the associated stocking behaviour give rise to pendular quantity movements. It is well known that pendular quantities create problems for sub-annual chaining of direct price indices, because such quantities generate downward chain drift bias. The present study has argued that pendular quantities are only part of the chain drift problem. In the presence of search and adjustment costs, price changes lead to delayed quantity changes. The resulting “sticky quantities” generate upward chain drift. Chain drift from pendular or sticky quantities arises not only for all members of the SWAP index family (e.g., Törnqvist, Walsh-2, Walsh-Vartia, Theil, and Vartia index), but also for the Fisher, Walsh, and Marshall-Edgeworth index and the Generalized Unit Value indices.

The present paper has developed a stress test that examines the resilience of price indices with respect to chain drift. The test is based on quantities generated by utility maximizing consumers. It has been applied to various direct price indices and to several RGEKS indices that have been proposed as a solution to the chain drift problem. These tests have shown that such RGEKS indices effectively reduce chain drift and that chain drift caused by sticky quantities is more persistent than chain drift caused by pendular quantities. In spite of its computational simplicity, the overall performance of the mean movement splice RGEKS index stands up to that of the best other RGEKS index variants.

RGEKS indices are not the only approach to curb the chain drift problem. Among these alternatives are the time-product dummy method and the Geary-Khamis approach. The stress test results obtained for the various RGEKS indices can serve as a reference for the stress test performance of these alternative approaches.

Appendix A: Index Formulas

Let \sum and \prod be the short hand notations for $\sum_{i \in S}$ and $\prod_{i \in S}$, respectively. The SWAP indices considered in this paper are defined by

$$\begin{aligned}
 \text{Törnqvist : } \ln P_{T\ddot{o}} &= \sum \frac{1}{2} \left(\frac{p_i^0 x_i^0}{\sum p_j^0 x_j^0} + \frac{p_i^T x_i^T}{\sum p_j^T x_j^T} \right) \ln \frac{p_i^T}{p_i^0} \\
 \text{Walsh-2 : } \ln P_{W2} &= \sum \frac{\sqrt{p_i^0 x_i^0 p_i^T x_i^T}}{\sum \sqrt{p_j^0 x_j^0 p_j^T x_j^T}} \ln \frac{p_i^T}{p_i^0} \\
 \text{Walsh-Vartia : } \ln P_{WV} &= \sum \sqrt{\frac{p_i^0 x_i^0}{\sum p_j^0 x_j^0} \frac{p_i^T x_i^T}{\sum p_j^T x_j^T}} \ln \frac{p_i^T}{p_i^0} \\
 \text{Theil : } \ln P_{Th} &= \sum \left[\frac{\sqrt[3]{\frac{1}{2} (p_i^0 x_i^0 + p_i^T x_i^T) p_i^0 x_i^0 p_i^T x_i^T}}{\sum \sqrt[3]{\frac{1}{2} (p_j^0 x_j^0 + p_j^T x_j^T) p_j^0 x_j^0 p_j^T x_j^T}} \right] \ln \frac{p_i^T}{p_i^0} \\
 \text{Vartia : } \ln P_V &= \sum \frac{L(p_i^T x_i^T, p_i^0 x_i^0)}{L(\sum p_j^T x_j^T, \sum p_j^0 x_j^0)} \ln \frac{p_i^T}{p_i^0}, \\
 &\text{where } L(a, b) = \frac{a - b}{\ln a - \ln b} \text{ for } a \neq b \text{ and } L(a, b) = a \text{ for } a = b.
 \end{aligned}$$

Popular price indices that are not SWAP indices include

$$\begin{aligned}
 \text{Laspeyres : } P_L &= \frac{\sum p_i^T x_i^0}{\sum p_i^0 x_i^0} \\
 \text{Paasche : } P_P &= \frac{\sum p_i^T x_i^T}{\sum p_i^0 x_i^T} \\
 \text{Fisher : } P_F &= \sqrt{P_L P_P} \\
 \text{Drobisch : } P_D &= \frac{1}{2} (P_L + P_P) \\
 \text{Marshall-Edgeworth : } P_{ME} &= \frac{\sum p_i^T (x_i^0 + x_i^T)}{\sum p_i^0 (x_i^0 + x_i^T)} \\
 \text{Walsh : } P_W &= \frac{\sum p_i^T \sqrt{x_i^0 x_i^T}}{\sum p_i^0 \sqrt{x_i^0 x_i^T}}.
 \end{aligned}$$

The GUV indices of Davies, Banerjee, and Lehr are defined by

$$P = \frac{\sum p_i^T x_i^T \sum x_i^0 \hat{z}_i}{\sum p_i^0 x_i^0 \sum x_i^T \hat{z}_i},$$

with

$$\begin{aligned}
 \text{Davies : } \hat{z}_i &= \sqrt{p_i^0 p_i^T} \\
 \text{Banerjee : } \hat{z}_i &= (p_i^0 + p_i^T) / 2 \\
 \text{Lehr : } \hat{z}_i &= (p_i^0 x_i^0 + p_i^T x_i^T) / (x_i^0 + x_i^T).
 \end{aligned}$$

Appendix B: Derivation of Demand Function

For deriving the optimal consumption c_i^* and stocking s_i^* ($i = 1, \dots, N$) it is useful to start by deriving the cost minimizing values of c_i for reaching a given subutility level C . Analogously, one can derive the cost minimizing values of s_i for reaching a given subutility level S . Afterwards, one can determine the utility maximizing values of C and S given the endowment $m + \sum_{i=1}^N p_i \bar{s}_i$ and past consumption \bar{c}_i ($i = 1, \dots, N$).

The cost minimizing values of c_i corresponding to some subutility level C are derived from the Lagrangian

$$\mathcal{L} = \sum_{i=1}^N p_i c_i - \lambda \left(\sum_{i=1}^N (c_i - \gamma \bar{c}_i)^\theta - C^\theta \right)$$

The first order conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_i} &= p_i - \lambda \theta (c_i - \gamma \bar{c}_i)^{\theta-1} = 0, & (i = 1, \dots, N) \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= \sum_{i=1}^N (c_i - \gamma \bar{c}_i)^\theta - C^\theta = 0. \end{aligned} \quad (28)$$

Each pair of conditions (28) leads to the relationship

$$p_j (c_i - \gamma \bar{c}_i)^{\theta-1} = p_i (c_j - \gamma \bar{c}_j)^{\theta-1}.$$

Exponentiating by $\theta/(\theta - 1)$ and rearranging yields

$$(c_i - \gamma \bar{c}_i)^\theta = p_i^{\frac{\theta}{\theta-1}} p_j^{\frac{-\theta}{\theta-1}} (c_j - \gamma \bar{c}_j)^\theta. \quad (29)$$

Inserting (29) in (25) gives

$$\begin{aligned} C^\theta &= \sum_{i=1}^N p_i^{\frac{-\theta}{1-\theta}} p_j^{\frac{\theta}{1-\theta}} (c_j - \gamma \bar{c}_j)^\theta \\ &= (c_j - \gamma \bar{c}_j)^\theta p_j^{\frac{\theta}{1-\theta}} P_C^{\frac{-\theta}{1-\theta}}, \end{aligned}$$

with

$$P_C^{\frac{-\theta}{1-\theta}} = \sum_{i=1}^N p_i^{\frac{-\theta}{1-\theta}}.$$

Therefore,

$$C = (c_j - \gamma \bar{c}_j) p_j^{\frac{1}{1-\theta}} P_C^{\frac{-1}{1-\theta}}.$$

Replacing the index j by the index i and rearranging yields

$$c_i = \gamma \bar{c}_i + C p_i^{\frac{-1}{1-\theta}} P_C^{\frac{1}{1-\theta}}. \quad (30)$$

Therefore, the minimum cost of subutility C is

$$\begin{aligned}
\sum_{i=1}^N p_i c_i &= \sum_{i=1}^N p_i \left(\gamma \bar{c}_i + C p_i^{\frac{-1}{1-\theta}} P_C^{\frac{1}{1-\theta}} \right) \\
&= \sum_{i=1}^N p_i \gamma \bar{c}_i + C P_C^{\frac{1}{1-\theta}} \sum_{i=1}^N p_i^{\frac{1-\theta}{1-\theta}} p_i^{\frac{-1}{1-\theta}} \\
&= \sum_{i=1}^N p_i \gamma \bar{c}_i + C P_C^{\frac{1}{1-\theta}} P_C^{\frac{-\theta}{1-\theta}} \\
&= \sum_{i=1}^N p_i \gamma \bar{c}_i + C P_C .
\end{aligned} \tag{31}$$

Analogously, the cost minimizing values of s_i corresponding to subutility level S are given by

$$s_i = S p_i^{\frac{-1}{1-\phi}} P_S^{\frac{1}{1-\phi}} \tag{32}$$

with

$$P_S^{\frac{-\phi}{1-\phi}} = \sum_{j=1}^N p_j^{\frac{-\phi}{1-\phi}}$$

and

$$\sum_{i=1}^N p_i s_i = S P_S . \tag{33}$$

Next, utility (24) is maximized subject to the budget constraint (22). The Lagrangian can be written in the form

$$\mathcal{L} = \alpha \ln C + (1 - \alpha) \ln S - \lambda (\bar{m} - C P_C - S P_S) , \tag{34}$$

with

$$\bar{m} = m + \sum_{i=1}^N p_i (\bar{s}_i - \gamma \bar{c}_i) . \tag{35}$$

From the first order conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C} &= \frac{\alpha}{C} + \lambda P_C = 0 \\
\frac{\partial \mathcal{L}}{\partial S} &= \frac{1 - \alpha}{S} + \lambda P_S = 0
\end{aligned}$$

one obtains

$$-C P_C = \frac{\alpha}{\lambda} \tag{36}$$

$$-S P_S = \frac{1 - \alpha}{\lambda} . \tag{37}$$

Inserting these expressions in the constraint

$$\bar{m} - C P_C - S P_S = 0$$

yields the third first order condition:

$$-\bar{m} = \frac{\alpha}{\lambda} + \frac{1-\alpha}{\lambda} = \frac{1}{\lambda}. \quad (38)$$

Inserting this result back in (36) and (37) gives

$$C = \alpha \frac{\bar{m}}{P_C} \quad (39)$$

$$S = (1-\alpha) \frac{\bar{m}}{P_S}. \quad (40)$$

Inserting these results in (30) and (32) yields the optimal consumption and stocking:

$$c_i^* = \gamma \bar{c}_i + \alpha \bar{m} p_i^{\frac{-1}{1-\theta}} P_C^{\frac{\theta}{1-\theta}} \quad (41)$$

$$s_i^* = (1-\alpha) \bar{m} p_i^{\frac{-1}{1-\phi}} P_S^{\frac{\phi}{1-\phi}}. \quad (42)$$

If the prices remain unchanged, c_i and s_i ($i = 1, \dots, N$) eventually reach their steady states c_i^{**} and s_i^{**} . In such a steady state we have $s_i = \bar{s}_i = s_i^* = s_i^{**}$ and, therefore, $x_i = c_i = \bar{c}_i = c_i^* = c_i^{**}$. Furthermore, (41) simplifies to

$$c_i^{**} = \bar{m} \frac{\alpha}{1-\gamma} P_C^{\frac{\theta}{1-\theta}} p_i^{\frac{-1}{1-\theta}}. \quad (43)$$

Rearranging gives

$$p_i c_i^{**} = \bar{m} \frac{\alpha}{1-\gamma} P_C^{\frac{\theta}{1-\theta}} p_i^{\frac{-\theta}{1-\theta}}.$$

Summing over all N items yields

$$\sum_{i=1}^N p_i c_i^{**} = \bar{m} \frac{\alpha}{1-\gamma} P_C^{\frac{\theta}{1-\theta}} \sum_{i=1}^N p_i^{\frac{-\theta}{1-\theta}} = \frac{\alpha \bar{m}}{1-\gamma}.$$

In the steady state, the left hand side is equal to m . Therefore,

$$\bar{m} = \frac{m(1-\gamma)}{\alpha}. \quad (44)$$

Inserting (44) in (43) yields

$$c_i^{**} = m P_C^{\frac{\theta}{1-\theta}} p_i^{\frac{-1}{1-\theta}}$$

and, therefore,

$$p_i c_i^{**} = m \left(\frac{P_C}{p_i} \right)^{\frac{\theta}{1-\theta}}.$$

Inserting (44) in (42) gives the steady-state inventory

$$s_i^{**} = (1-\alpha) \frac{m(1-\gamma)}{\alpha} p_i^{\frac{-1}{1-\phi}} P_S^{\frac{\phi}{1-\phi}}.$$

Multiplying the last equation by p_i and summing over all N items yields

$$\sum_{i=1}^N p_i s_i^{**} = \frac{1-\alpha}{\alpha} (1-\gamma) m P_S^{\frac{\phi}{1-\phi}} \sum_{i=1}^N p_i^{\frac{-\phi}{1-\phi}} = \frac{1-\alpha}{\alpha} (1-\gamma) m.$$

The larger the habit formation parameter γ , the smaller is the aggregated steady-state inventory.

Appendix C: Tables

Table 4 (“Pendular Quantities Scenario”), Table 5 (“Sticky Quantities Scenario”), and Table 6 (“Hybrid Scenario”) document the compiled RGEKS index values arising from different window lengths and different linking approaches. In the absence of chain drift, all index values would be equal to 100. The column heads indicate the applied window length, while the rowheads indicate the applied linking method. The results generated by the mean movement RGEKS (rowhead “M.Mo.”) approach are listed in the top line. The next line (rowhead “Mean”) shows the RGEKS index values arising from mean splicing. The following line (rowhead “0”) lists the results for movement splicing, while the bottom value in each column is the index value arising from window splicing. A column’s half splice value(s) can be found in the row that sits in the middle position between the movement splice row and the window splice row. Variation of the results within a column indicates that the linking method matters for the RGEKS index value. The influence of the window length can be seen from the variation within a row.

Table 4: RGEKS Indices for the Pendular Quantities Scenario.

Link	3	4	5	6	7	8	9	10	12	16	20	24
M.Mo.	94.22	97.90	99.03	99.28	99.80	100.32	100.12	100.00	100.02	100.16	100.12	100.12
Mean	94.22	97.90	99.03	99.28	99.80	100.32	100.12	100.00	100.03	100.16	100.11	100.11
0	92.53	94.95	95.53	95.61	97.28	98.43	96.68	97.01	97.91	98.91	99.19	99.51
1	95.94	100.04	100.54	100.10	99.50	100.52	101.20	99.31	99.93	100.03	100.01	99.82
2		98.78	100.71	101.02	100.57	99.98	100.84	101.42	99.71	100.33	99.65	99.84
3			99.43	100.35	100.66	100.31	99.82	100.61	99.59	100.08	99.97	100.22
4				99.43	99.68	100.04	99.80	99.41	100.70	99.73	100.16	100.16
5					101.14	100.50	100.73	100.44	100.67	100.19	100.22	100.17
6						102.50	101.41	101.53	100.75	100.35	100.70	100.07
7							100.58	99.63	99.73	99.33	99.79	99.74
8								100.70	99.65	100.24	99.56	99.96
9									100.08	100.31	100.00	100.06
10									101.69	99.70	99.60	100.03
11										100.07	99.70	100.00
12										100.31	100.63	100.02
13										100.14	100.32	100.06
14										102.67	100.13	99.98
15											100.06	99.76
16											99.62	99.99
17											100.02	100.17
18											102.88	100.15
19												100.26
20												99.88
21												99.82
22												103.03

Table 5: RGEKS Indices for the Sticky Quantities Scenario.

	3	4	5	6	7	8	9	10	12	16	20	24
M.Mo.	105.15	103.93	103.13	102.54	102.10	101.77	101.54	101.36	101.12	100.83	100.62	100.49
Mean	105.15	103.93	103.13	102.54	102.10	101.77	101.54	101.36	101.12	100.82	100.62	100.49
0	105.13	103.79	102.95	102.31	101.85	101.57	101.41	101.28	101.11	100.81	100.57	100.42
1	105.18	104.08	103.23	102.62	102.12	101.74	101.51	101.36	101.18	100.89	100.64	100.47
2		103.91	103.17	102.62	102.19	101.81	101.51	101.32	101.12	100.88	100.65	100.48
3			103.16	102.52	102.14	101.83	101.53	101.29	101.05	100.83	100.63	100.47
4				102.63	101.99	101.72	101.49	101.26	100.94	100.76	100.59	100.44
5					102.30	101.59	101.40	101.22	100.87	100.67	100.55	100.42
6						102.16	101.34	101.20	100.90	100.61	100.52	100.41
7							102.12	101.20	100.96	100.59	100.52	100.42
8								102.12	100.98	100.58	100.51	100.43
9									101.00	100.61	100.50	100.45
10									102.20	100.67	100.49	100.45
11										100.70	100.46	100.43
12										100.71	100.45	100.42
13										100.69	100.45	100.40
14										102.34	100.46	100.36
15											100.46	100.33
16											100.46	100.31
17											100.44	100.30
18											102.48	100.31
19												100.33
20												100.33
21												100.32
22												102.65

Table 6: RGEKS Indices for the Hybrid Scenario.

	3	4	5	6	7	8	9	10	12	16	20	24
M.Mo.	102.93	102.52	102.03	101.61	101.40	101.29	101.09	101.02	100.95	100.78	100.64	100.54
Mean	102.93	102.52	102.03	101.61	101.40	101.29	101.09	101.02	100.94	100.77	100.63	100.53
0	101.93	100.89	100.49	100.19	100.35	100.50	100.07	100.37	100.46	100.35	100.24	100.26
1	103.94	103.35	102.22	101.67	101.23	101.25	101.28	100.81	101.09	100.69	100.63	100.42
2		103.33	102.15	101.42	101.06	100.76	100.82	100.89	100.71	100.66	100.45	100.34
3			103.28	101.56	101.02	100.75	100.52	100.60	100.35	100.54	100.40	100.33
4				103.24	101.00	100.60	100.42	100.24	100.44	100.43	100.35	100.26
5					103.75	100.89	100.55	100.38	100.32	100.44	100.26	100.30
6						104.32	101.07	100.75	100.42	100.47	100.48	100.34
7							104.08	100.75	100.35	100.15	100.36	100.28
8								104.42	100.43	100.35	100.36	100.32
9									100.85	100.44	100.51	100.36
10									105.05	100.25	100.33	100.38
11										100.33	100.18	100.34
12										100.36	100.31	100.33
13										100.41	100.19	100.28
14										105.79	100.15	100.23
15											100.28	100.18
16											100.13	100.15
17											100.29	100.13
18											106.27	100.05
19												100.17
20												100.13
21												100.14
22												106.68

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